# Analysis of calculation accuracy of power system electromechanical eigenvalues based on instantaneous power disturbance waveforms

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Abstract The paper presents investigation results concerning the accuracy analysis of calculating electromechanical eigenvalues of the multimachine power system (PS) state matrix based on instantaneous power disturbance waveforms when taking into account introduction of a disturbance to different units. There were analysed the instantaneous power waveforms occurring after introducing the disturbance in the form of a rectangular impulse of different height to the voltage regulation system of generators in generating units of different powers. In order to increase the computation accuracy, computations were repeated many times. The computation results were averaged. The hybrid algorithm consisting of the genetic and gradient algorithms was used for computations.

Keywords power system, electromechanical eigenvalues, transient states, waveform recovery

### I. INTRODUCTION

Stability factors [1] calculated on the basis of electromechanical eigenvalues of the PS state matrix can be used for assessment of the PS angular stability. Electromechanical eigenvalues can be determined from the PS state equations, however, the calculation results depend then on the values of the PS state matrix elements, and indirectly on the assumed models of PS elements and their uncertain parameters [2]. Electromechanical eigenvalues can also be calculated with good accuracy based on analysis of actual disturbance transients occurring in the PS after different disturbances. In that case the assumed PS model and its parameters do not influence the calculation results. Only the actual operating conditions of the system do. The aim of this paper is the accuracy analysis of calculating PS electromechanical eigenvalues on the basis of analysis of instantaneous power disturbance waveforms in PS generating units in dependence on the amplitude and location of disturbance.

### II. LINEARIZED POWER SYSTEM MODEL

The power system model linearized around the operating point is described by the state and output equations [3]:

$$\Delta \dot{\mathbf{X}} = \mathbf{A} \Delta \mathbf{X} + \mathbf{B} \Delta \mathbf{U}, \qquad \Delta \mathbf{Y} = \mathbf{C} \Delta \mathbf{X} + \mathbf{D} \Delta \mathbf{U}, \tag{1}$$

where:  $\Delta X$ ,  $\Delta U$ ,  $\Delta Y$ - deviations of the state variable vector, input vector and output variable vector, respectively.

The output quantity waveforms of the linearised PS model can be directly calculated by integrating the state equation, or by using the eigenvalues and eigenvectors of the state matrix  $\mathbf{A}$  [4]. For a disturbance in the form of impulse change in the j-th input quantity  $\Delta \mathbf{U}_j(t) = \Delta U \delta(t)$ , the waveform of the i-th output quantity (for  $\mathbf{D} = \mathbf{0}$ ) is given by:

$$\Delta Y_i = \sum_{h=1}^n F_{ih} e^{\lambda_h t}, \text{ where: } F_{ih} = \mathbf{C}_i \mathbf{V}_h \mathbf{W}_h^T \mathbf{B}_j \Delta U, \qquad (2)$$

 $\lambda_h = \alpha_h + j \nu_h$  - state matrix eigenvalue,  $F_{ih}$  - participation factor of the h-th eigenvalue in the i-th output quantity waveform,  $\mathbf{C}_i$  - i-th row of the matrix  $\mathbf{C}$ ,  $\mathbf{V}_{lo}$   $\mathbf{W}_h$  - h-th left

and right eigenvector of the state matrix respectively,  $\mathbf{B}_j$  - j-th column of the matrix  $\mathbf{B}$ .

In case of instantaneous power swing waveforms in PS, the so-called electromechanical eigenvalues associated with motion of generating unit rotors are of essential significance. They influence on instantaneous power waveforms of particular generating units in a different way, which is connected with different values of their participation factors.

# III. EXEMPLARY CALCULATIONS

Exemplary calculations were carried out for a 7 – machine PS Cigre (Fig. 1).

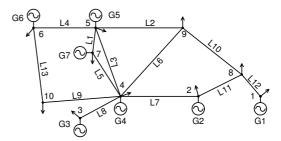


Fig. 1. Analysed 7 - machine PS Cigre

The method for calculating electromechanical eigenvalues used in investigations consists in approximation of instantaneous power waveforms in particular generating units by means of expression (2). The electromechanical eigenvalues and participation factors of particular modal components are unknown parameters of this approximation. They are selected iteratively so as to minimise the objective function value determined as the mean square error between the approximated and approximating waveform:

$$\varepsilon_{w}(\lambda, F) = \sum_{i=1}^{n} \left( \left| P_{i(m)} - P_{i(a)}(\lambda, F) \right| \right)^{2}, \qquad (3)$$

where:  $\lambda$  – vector of electromechanical eigenvalues, F – participation factor vector, index m denotes the approximated waveform, and index a denotes the

approximating waveform of the instantaneous power P calculated on the basis of the searched eigenvalues and participation factors. A hybrid algorithm consisting of genetic and gradient algorithms was used for minimisation of the objective function (3).

The PS model analysed was developed in the Matlab-Simulink environment. In calculations there were taken into account the models of: a GENROU synchronous generator with nonlinear magnetization characteristic [4], a static excitation system working in the Polish Power System [4], a IEEEG1 steam turbine [4] and a PSS3B power system stabilizer [4].

The disturbance was assumed in the form of a rectangular pulse of the voltage regulator reference voltage in one of generating units. The response of the system to a short rectangular pulse (of appropriately selected height and width) is close to that to the Dirac impulse. There were considered the cases of introducing the disturbance to G4 generating unit (generator rated apparent power equal to 588 MVA) and to the G6 generating unit (generator rated apparent power equal to 75 MVA). In both cases there were taken into account the pulses of the height equal to -5% and -10% of the steady value of the voltage regulator reference voltage. For instance, Fig. 2 shows the waveforms of the generator armature phase currents in G7 generating unit after occurrence of the pulse disturbance of the height equal to -10% of the steady value of the voltage regulator reference voltage in G4 and G6 generating units.

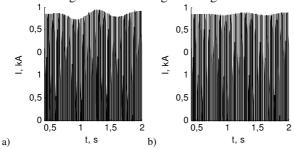


Fig. 2. Waveforms of generator armature phase currents in G7 generating unit for disturbance in G4 (a) and G6 (b) generating unit

From Fig. 2 it is evident that the disturbance in G4 generating unit of large power caused stronger swings in G7 generating unit than that in G6 generating unit of small power.

Table 1 presents the electromechanical eigenvalues calculated directly on the basis of the PS model by the Matlab-Simulink program. They are called *original eigenvalues* in the paper.

TABLE I ORIGINAL EIGENVALUES OF THE PS ANALYSED

$\lambda_1$	-0,881±j10,443	$\lambda_2$	-0,826±10,620	$\lambda_3$	-0,763±j9,669
$\lambda_4$	-0,527±j8,748	$\lambda_5$	-0,417±j7,872	$\lambda_6$	-0,189±j6,542

For instance, Fig. 3 shows the transient waveforms of the G4 generating unit instantaneous power for the disturbance in this unit obtained from simulations (full line) and recovered from the original eigenvalues (broken line). The pulse of the height equal to -5% and -10% of the steady value of the voltage regulator reference voltage was considered.

It can be noted in Fig. 3 that increase in the pulse height results in slight worsening of the quality of recovering the simulation waveform on the basis of original eigenvalues, which is caused by stronger influence of nonlinearities occurring in the PS model.

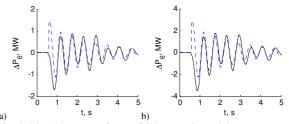


Fig. 3. Disturbance waveforms of G4 generating unit instantaneous power for pulse of -5% (a) and -10% (b) height of the voltage regulator reference voltage

Calculations of the eigenvalues were performed repeatedly on the basis of analysis of the instantaneous power waveforms of successive generating units. The results of the objective function values larger than the assumed limit value were neglected. Arithmetic means of the real and imaginary parts of the eigenvalues calculated on the basis of the instantaneous power waveforms of particular generating units were assumed to be the final result of calculations. The calculation errors of the eigenvalues for the disturbances considered are given in Table 2.

TABLE II CALCULATION ERRORS OF EIGENVALUES

-5% step of the voltage regulator reference voltage in G4 unit								
$\Delta \lambda_1$	-0,1625±j0,0119	$\Delta \lambda_2$	0,0310∓j0,0526	$\Delta \lambda_3$	0,0524∓j0,1105			
$\Delta \lambda_4$	-0,0275∓j0,0345	$\Delta\lambda_5$	0,0219∓j0,0091	$\Delta \lambda_6$	-0,0167±j0,0191			
-10% step of the voltage regulator reference voltage in G4 unit								
$\Delta \lambda_1$	-0,1803±j0,0084	$\Delta \lambda_2$	0,0414∓j0,0597	$\Delta \lambda_3$	0,0499∓j0,1537			
$\Delta \lambda_4$	-0,0080∓j0,0341	$\Delta\lambda_5$	0,0196∓j0,0226	$\Delta \lambda_6$	-0,0125±j0,0222			
-5% step of the voltage regulator reference voltage in G6 unit								
$\Delta \lambda_1$	-0,0851∓j0,0932	$\Delta \lambda_2$	-0,0263∓j0,0906	$\Delta \lambda_3$	-0,0707±j0,0915			
$\Delta \lambda_4$	-0,0371±j0,1343	$\Delta\lambda_5$	-0,0165∓j0,0569	$\Delta \lambda_6$	-0,0116∓j0,0163			
-10% step of the voltage regulator reference voltage in G6 unit								
$\Delta \lambda_1$	-0,0714∓j0,0890	$\Delta \lambda_2$	-0,0184∓j0,0598	$\Delta \lambda_3$	-0,0826±j0,0652			
$\Delta \lambda_4$	-0,0105±j0,0421	$\Delta\lambda_5$	-0,0172∓j0,0574	$\Delta \lambda_6$	-0,0115∓j0,0166			

# IV. CONCLUSION

From the investigations performed it can be stated that:

- Disturbance in a generating unit of large power results in stronger power swings in other generating units than a disturbance in a generating unit of small power. It allows easier separation of power swing waveforms from the recorded phase current and voltage waveforms as well as increase in the calculation accuracy of the eigenvalues.
- Increase in the step value of change of the voltage regulator reference voltage results in slight increase in influence of nonlinearities occurring in the system on the instantaneous power swing waveform. However, it does not cause significant worsening of the calculation accuracy of the eigenvalues.

### V. REFERENCES

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