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DYNAMIC ANALYSIS OF NONLINEAR FLEXIBLE MULTIBODY SYSTEMS USING NEWMARK INTEGRATION SCHEME

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Abstract: A mathematical description of multibody mechanical systems containing flexible bodies often leads to a nonlinear set of differential–algebraic equations of motion which can be solved using various numerical integration schemes. In this paper, the possibilities of the commonly known Newmark integration scheme used on such systems are investigated and discussed.

Keywords: dynamic analysis; multibody system; Newmark integration scheme

1 Introduction

This paper deals with mechanical systems which contain several rigid or flexible bodies constrained with various joints and subjected to external force effects. The need of modelling of these so called multibody systems can be found in many engineering branches, such as transport and aviation industry, energetics or robotics. Various modelling techniques intended for including flexible behaviour of the bodies into the mathematical model of these systems were developed in the past. Since the multibody systems can undergo to large motions, the resultant mathematical description often leads to nonlinear equations of motion.

One of the most popular method for modelling of flexible bodies which are the parts of multibody systems is an *absolute nodal coordinate formulation* (ANCF) [1], which is based on the finite element method. This method leads to a linear mass matrix of the discretized body which is a big advantage in further numerical solution. The disadvantage of this method is the nonlinear expression of elastic force. Together with modelling of possible contacts and friction, the resultant model is highly nonlinear and it puts high demands on the numerical solution procedure.

Normally, the mathematical description of the flexible multibody systems leads to a set of nonlinear differential-algebraic equations (index 3 DAE). Different approaches can be used for the numerical solution, some of them convert the equations to index 1 DAE or to ordinary differential equations (ODE) [2]. In that case, the stabilization of numerical solution is needed to be used because of possible numerical drift of the solution. In this work, the well known Newmark integration scheme used for nonlinear set of index 3 DAE is investigated and discussed.

2 Newmark integration scheme for index 3 DAE

The Newmark integration scheme is a well known numerical method intended for solving the problems of structural dynamics. In past years it was also adapted to solve the problems of multibody system dynamics. In case of flexible multibody systems with the use of ANCF approach, the equations of motion are of the form

$$\left\{ \begin{array}{l} \mathbf{M}\ddot{\mathbf{q}} + \mathbf{\Phi}_{,\mathbf{q}}^T\boldsymbol{\lambda} = \mathbf{f}(\mathbf{q},\ \dot{\mathbf{q}},\ t) - \mathbf{Q}(\mathbf{q}), \\ \qquad \mathbf{\Phi}(\mathbf{q},\ t) = \mathbf{0}, \end{array} \right. \tag{1}$$

where $\mathbf{q}$ are the coordinates (including ANCF nodal coordinates), $\mathbf{M}$ is the constant mass matrix, $\mathbf{\Phi}$ is the vector of holonomic constraint equations, $\mathbf{\Phi}_{,\mathbf{q}}$ is the Jacobian matrix of constraints, $\boldsymbol{\lambda}$ is the vector of Lagrange multipliers, $\mathbf{Q}$ are the elastic forces (including nonlinear elastic forces of ANCF elements) and $\mathbf{f}$ is the vector of external forces. The approximations of velocities and position coordinates of Newmark

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method are

$$\mathbf{q}_{n+1} = \mathbf{q}_n + h\dot{\mathbf{q}}_n + \frac{h^2}{2}\left[(1-2\beta)\ddot{\mathbf{q}}_n + 2\beta\ddot{\mathbf{q}}_{n+1}\right],$$
$$\dot{\mathbf{q}}_{n+1} = \dot{\mathbf{q}}_n + h\left[(1-\gamma)\ddot{\mathbf{q}}_n + \gamma\ddot{\mathbf{q}}_{n+1}\right], \tag{2}$$

where β and γ are the positive constants [3], n is the number of discrete time step and $h = t_{n+1} - t_n$ is the time step size.

In order to find the appropriate solution in each discretized time step, the Newton–Raphson iterative method within each time step is often used to minimize the force and constraint residua. This method requires the linearization of motion equations and with the use of Newmark approximations (2) it leads to the solution of a set of algebraic equations with the unknown increments of position coordinates $\Delta\mathbf{q}$ and increments of multipliers $\Delta\boldsymbol{\lambda}$. The corrections of kinematic quantities and multipliers in iteration $k+1$ are then expressed as [3]

$$\begin{aligned}
\mathbf{q}_{n+1}^{k+1} &= \mathbf{q}_{n+1}^{k} + \Delta\mathbf{q},\\
\dot{\mathbf{q}}_{n+1}^{k+1} &= \dot{\mathbf{q}}_{n+1}^{k} + \frac{\gamma}{\beta h}\Delta\mathbf{q},\\
\ddot{\mathbf{q}}_{n+1}^{k+1} &= \ddot{\mathbf{q}}_{n+1}^{k} + \frac{1}{\beta h^2}\Delta\mathbf{q},\\
\boldsymbol{\lambda}_{n+1}^{k+1} &= \boldsymbol{\lambda}_{n+1}^{k} + \Delta\boldsymbol{\lambda}.
\end{aligned} \tag{3}$$

The Newton–Raphson method requires the Jacobian matrix of residua, often called tangential stiffness matrix, and Jacobian matrix of constraints. The obtaining of tangential stiffness matrix of nonlinear system analytically can be very difficult or nearly impossible. It can be obtained numerically by finite differences, but the numerical estimation of tangential stiffness matrix in each step is computationally demanding. That is why the quasi–Newton method is often used as an alternative to the classical Newton–Raphson method [4]. In this method, the tangential stiffness matrix is numerically computed once at the simulation start and then it is modified in each iteration by some algebraic operations, see [4]. If the solution does not converge in few iterations, the tangential stiffness matrix can be numerically recomputed and the convergence of the solution can be obtained again. The quasi–Newton method can be much faster than the classical Newton–Raphson method.

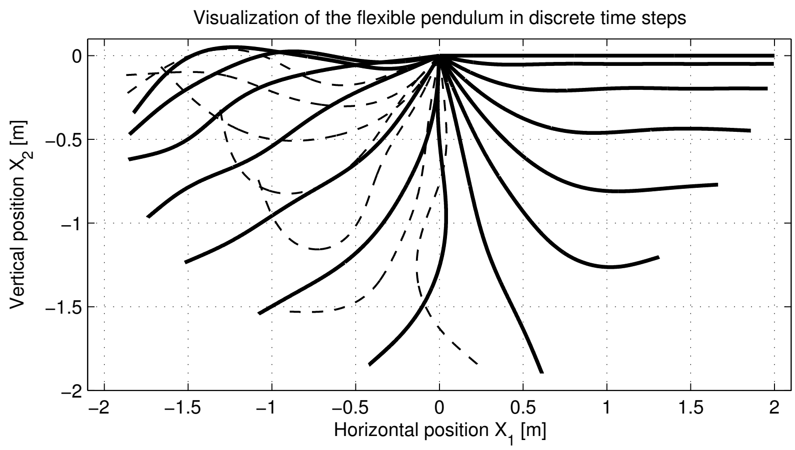

Figure 1: Visualization of the pendulum in discrete time steps.

3 Applications

The Newmark integration scheme with the quasi–Newton iterative method was implemented in MATLAB software. At first, the method was tested on a benchmark problem of a flexible pendulum.

The flexible pendulum was modelled using ANCF spatial thin beam elements. The pendulum is attached to the ground using a revolute joint defined by five constraints in the first node. The visualization of the pendulum in discrete time steps is shown in Fig. 1. The time step of the Newmark integration scheme was set to 0.001 s, the total simulation time was 2 s, the Newmark parameters were $\gamma = \frac{1}{2} + \alpha$ and $\beta = \frac{1}{4}\left(\gamma + \frac{1}{2}\right)^2$, where $\alpha = 0.05$ represents the added numerical damping. The maximum allowed norm of the force residuum in each time step was set to 10^{-5} and the maximum allowed norm of the constraints residuum was 10^{-7}. These two settings define the accuracy of the numerical integration. The threshold for numerical recalculation of the Jacobian matrix was set to 8 iterations. The setting of the integration scheme parameters has an influence on the overall performance of the method. The numerical recalculation of the Jacobian matrix is computationally demanding, but in some cases it helps the method to converge in less iterations. The performance of the Newmark integration scheme for nonlinear set of index 3 DAE was compared to the integration schemes implemented in MATLAB software. The default error setting for *ode* functions was used and the Baumgarte's stabilization for constraints was used. The computational times of different methods are summarized in Table 1. The Newmark integration scheme for the benchmark problem was approximately four times faster than the fastest integration scheme from *ode* family methods of MATLAB software.

Method	Newmark I3 DAE	`ode23t`	`ode23`	`ode113`	`ode45`
Comp. time [s]	**2.0**	13.8	7.9	9.3	21.2

Table 1: Computational times of tested integration schemes.

The Newmark integration scheme was also used on a more complex mechanical system which contains a thin flexible fibre, pulley and two equal weight. The fibre was discretized using 30 ANCF spatial thin beam elements. The mass of each weight is 10 kg and the weights are connected to the ends of the fibre using constraints. The fibre is laid on the fibre and a contact forces including friction are defined. The pulley has a prescribed speed of rotation as $\frac{\pi^2}{4}\sin\left(\frac{\pi t}{2}\right)$ rad/s and after $t = 4$ it stops. The time step was decreased to 0.0001 s in order to achieve good convergence of contact forces. In Figure 2, the comparison of the weight vertical speed and the pulley circumferential speed is shown. As can be seen, the sliding of the fibre occurs and it is caused by simple smoothed Coulomb's friction model, which does not include stiction phenomena. Also speed oscillations of the weight associated with the fibre flexibility occurs at the beginning of the simulation. For better understanding of the simulated mechanism, the visualizations of the mechanism in discrete time steps are shown in Figure 3. The average number of iterations needed for solution convergence in each time step is less than 4, but mainly at the beginning of the simulation the number of iterations needed for solution convergence was higher (maximum was 44 iterations).

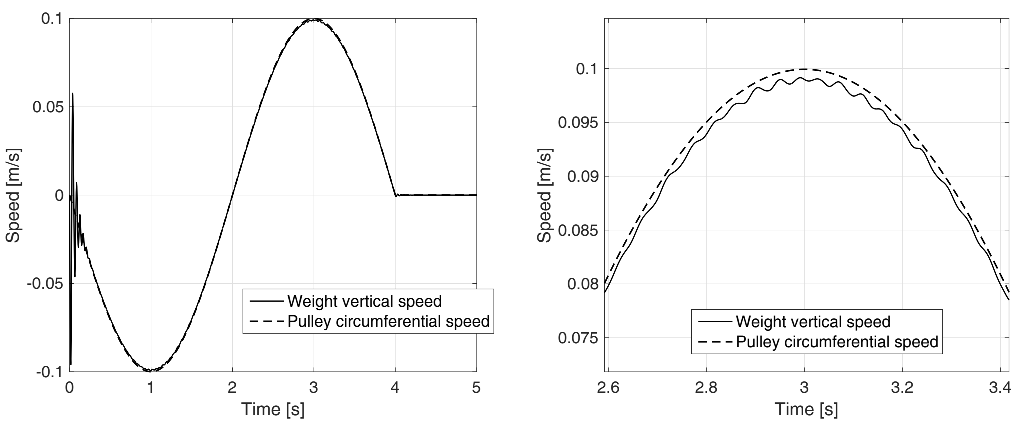

Figure 2: Comparison of the weight vertical speed and the pulley circumferential speed.

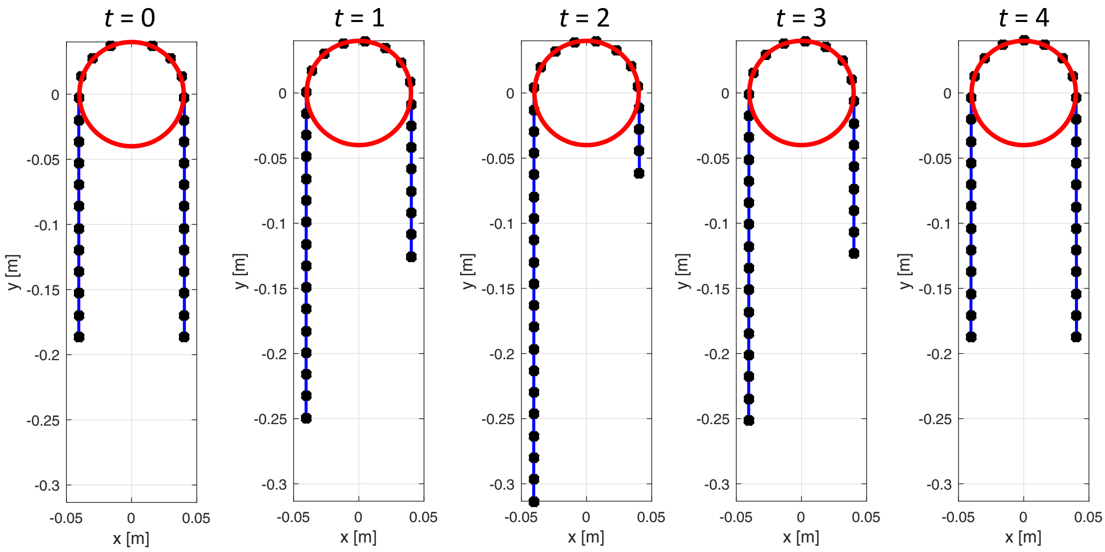

Figure 3: Visualization of the fibre and pulley in discrete time steps.

4 Conclusion

In this paper, the Newmark integration scheme intended for dynamic analysis of nonlinear flexible multibody systems was described. Because of the nonlinearities, the solution in each time step needs to be found iteratively. The common iterative procedures such as Newton–Raphson method or quasi–Newton method were mentioned in this paper. Advantages and disadvantages of proposed methods are briefly described. Then the described numerical procedure was used on two mechanical systems. The solution was obtained faster with the Newmark method than with the *ode* family methods from MATLAB which is the main advantage of the Newmark integration scheme. The main disadvantage is the sensitivity of the solution convergence to the parameters of the method and selected time step size. The performance of the method can be significantly improved by the implementation of adaptive time step [3].

The described Newmark method for nonlinear multibody systems has only the first order of accuracy. In future, the HHT method which has the second order of accuracy will be implemented. The HHT method is based on the Newmark method, but it uses modified equations of motion (1).

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References

[1] BULÍN, Radek, Michal HAJŽMAN and Pavel POLACH. Nonlinear dynamics of a cable–pulley system using the absolute nodal coordinate formulation. *Mechanics Research Communications*. 2017, **82**, 21–28.

[2] HAJŽMAN, Michal and Pavel POLACH. Application of stabilization techniques in the dynamic analysis of multibody systems. *Applied and Computational Mechanics*. 2007, **1**, 479–488.

[3] GÉRADIN, Michel and Alberto CARDONA. *Flexible Multibody Dynamics, A Finite Element Approach*. Chichester: John Wiley and Sons, 2001. ISBN 0-471-48990-5.

[4] GÉRADIN, Michel, Sergio IDELSOHN and Michel HOGGE. Computational strategies for the solution of large nonlinear problems via quasi-newton methods. *Computers and Structure*. 1981, **13**(1-3), 73–81.