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VORTEX INDUCED VIBRATION OF 1D BEAM-TYPE CONTINUA

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Abstract: The contribution focuses on the modelling of vibration of 1D beam-type continua induced by cross flow of a fluid. The influence of fluid flow is represented by the harmonic force whose frequency depends on the Strouhal number. Based on the widely used FEM formulation, vector of forces caused by the cross flow of the fluid is derived. The partitioned approach to the solution of fluid-structure interaction is described.

Keywords: Vortex-induced vibration; cross flow; beam-type continuum; fluid-structure interaction

1 Introduction

Interaction between flexible beam and the cross flow of a fluid can cause vortex-induced vibration (VIV). The phenomenon is mostly undesirable and it occurs in various fields; VIV of chimneys, towers or bride decks caused by the wind, VIV of components in the nuclear reactors caused by the cross flow of coolant, vibration of underwater ropes and cables etc. Although both structure and fluid introduces interesting behaviour during the VIV, the structural part only is studied in detail in this contribution and the effect of the fluid is taken into account only via a resultant force acting to the beam element.

2 Mathematical model

The mathematical model of 1D beam-type continuum was formulated e.g. in [1] where coefficient (mass, damping and stiffness) matrices were shown for both Euler-Bernoulli and Timoshenko beams. For the case of solitary prismatic beam element of round cross section which is fully submerged into the fluid (see Fig. 1), differential force caused by the laminar fluid flow can be expressed in the form [3]

$$dF^{(e)} = \frac{1}{2}\rho c_{rel}^{(e)2} D C_L dz, \tag{1}$$

where ρ is fluid density, D is outer diameter of the beam element, C_L is lift coefficient and c_{rel} is relative velocity between beam and the fluid. Laminar or turbulent models are suitable in dependency on the Reynolds number $Re = \frac{c_{rel}D}{\nu}$, where ν is kinematic viscosity of the fluid. If the turbulences are present, the first approximation of the forces generated by the fluid flow can be considered as a harmonic force with the amplitude defined in (1) and with the frequency

$$f_{vi} = \frac{S c_{rel}}{D}, \tag{2}$$

where S is Strouhal number. Considering angle β_e which determines the direction of dF action, the integral force acting on the node i can be decomposed into x and y components and expressed in the form

$$F_{ix} = \frac{1}{2}\rho D C_L \left(\cos\beta_e(t)\frac{L_e(t)}{2} + \cos\beta_{e+1}(t)\frac{L_{e+1}(t)}{2}\right)\sin 2\pi f_{vi,e} t, \tag{3}$$

$$F_{iy} = \frac{1}{2}\rho D C_L \left(\sin\beta_e(t)\frac{L_e(t)}{2} + \sin\beta_{e+1}(t)\frac{L_{e+1}(t)}{2}\right)\sin 2\pi f_{vi,e} t, \tag{4}$$

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where $L_e(t)$ belonging to the element e is vector based on the approximation polynomials of FEM and its particular form have been published in [2]. The effect of forces from both adjacent elements e and $e+1$ is taken into account (for the inner node of the continuum).

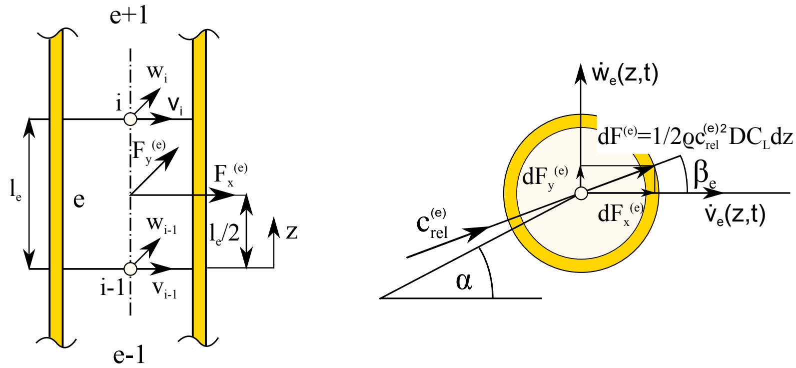

Figure 1: A side-view (left) and cross section (right) of the finite element e in the cross flow of the fluid

Using partitioned approach to the solution of fluid structure interaction and denoting $\boldsymbol{q} \in \mathbb{R}^{n,n}$ vector of generalized coordinates, the model can be written in the form

$$\boldsymbol{M}\ddot{\boldsymbol{q}}^{(k+1)}(t) + \boldsymbol{B}\dot{\boldsymbol{q}}^{(k+1)}(t) + \boldsymbol{K}\boldsymbol{q}^{(k+1)}(t) = \boldsymbol{f}_{fl}\left(\dot{\boldsymbol{q}}^{(k)}, t\right), \quad k = 0, 1, 2, \ldots, \tag{5}$$

where $\boldsymbol{M}, \boldsymbol{B}, \boldsymbol{K} \in \mathbb{R}^{n,n}$ are mass, damping and stiffness matrices, respectively. Vector of fluid forces $\boldsymbol{f}_{fl}$ depend on the velocity of the beam. For $k = 0$, the iterative procedure (5) begins with non-vibrating beam, the time behaviour of $\boldsymbol{f}_{fl}\left(\dot{\boldsymbol{q}}^{(0)}, t\right)$ is obtained and the integration in the first iteration can be performed. The iterative process goes on until a stopping criterion is satisfied.

3 Conclusion

The 1D FEM based description of beam VIV provides efficient way to describe such a complex phenomenon. The structure vibration is solved using partitioned approach. In each iteration, fluid flow forces are corrected using the time behaviour of velocity from the previous iteration. The fluid part of the interaction is not studied in detail and its influence is respected only via forces. This way of representation enables description and solution of a VIV phenomenon with reasonable computational effort.

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