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THE ELECTRO-THERMAL LINK FINITE ELEMENT FOR MULTIPHYSICAL ANALYSIS WITH 3D SPATIAL FUNCTIONALLY GRADED MATERIAL PROPERTIES

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Abstract: The paper presents homogenization process of electric and thermal material properties in link structure made of Functionally Graded Material (FGM) with functionally prescribed change of these material properties in all three orthogonal directions inside the bar. Numerical experiment with developed link finite element (Finite Element Method – FEM) for this class of composite materials is also presented.

Keywords: homogenization; FGM; FEM; link element; electro-thermal analysis

1 Homogenization process of real FGM bar

Let us consider a bar according to Figure 1. Its length is L [m], height h [m] and depth b [m] (rectangular cross section area A [m^2]). Let the FGM bar consists of two material components – matrix (denoted by index m) and fibre (index f) where fractions of individual constituents through the volume of the bar are functionally graded. We can consider matrix volume fraction $v_m(x,y,z)$ and fibre volume fraction $v_f(x,y,z)$ for every point (x,y,z) of the bar, so the change of material properties is 3D change. Moreover, let this change has polynomial character in every orthogonal direction $v_f(x,y,z) = \sum \varepsilon_u x^n y^m z^h$; $u = \max(r,s,l)$, $n = \{0;r\}$, $m = \{0;s\}$ and $h = \{0;l\}$, $v_m(x,y,z) = 1 - v_f(x,y,z)$, where grades of the polynomial changes r, s and l are for longitudinal and lateral directions and through the depth of the bar, respectively (ε_u represents constant coefficients for individual coefficients of the polynomial). Figure 1 also shows change of fibre volume fraction through height (h = 6.67 mm) and length (L = 200 mm) of the bar, where change through the depth of the bar is shown using parametric plot of surfaces. Thick lines in this Figure 1 represent fibre volume fraction in chosen layers through height and depth of the bar that are necessary for calculation of homogenized material properties (for electric and thermal fields) using extended mixture rule [1] and laminate theory [2].

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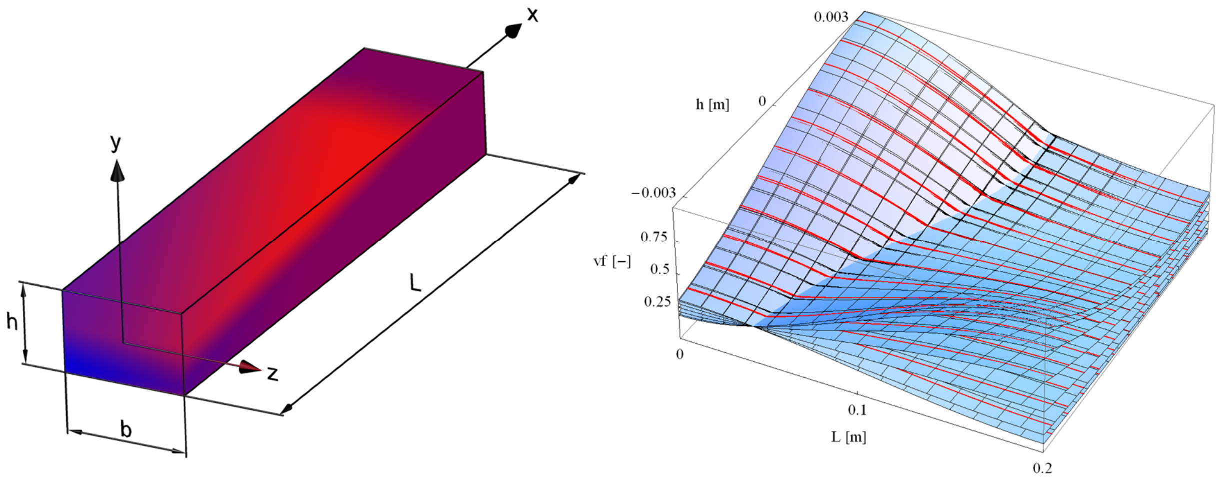

Figure 1: FGM bar with spatial change of material properties (left), change of fibre volume fraction through the length, height and depth of the bar (right).

The process of homogenization using division of the bar into layers (in lateral direction) and sublayers (through the depth of the bar) is shown in Figure 2. The result of such homogenization process is equivalent homogenized material property with polynomial change only in longitudinal direction (through height and depth of the bar the material property has constant value derived from longitudinal change). For steady-state electro-thermal analysis, the homogenization process need to be performed separately for final homogenized electric conductivity of the bar and for final homogenized thermal conductivity of the bar (both based on electric or thermal conductivities of fibre and matrix constituents of the FGM material, respectively).

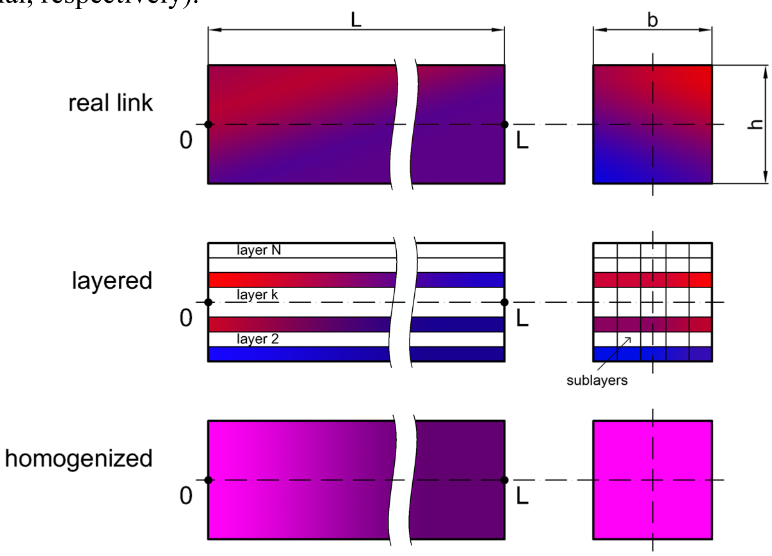

Figure 2: Homogenization process of FGM bar using layers and sublayers.

2 Semi-analytical method for solution of linear differential equations with variable coefficients and right-hand side

Procedure for solving differential equations with variable coefficients and right-hand side, which is presented in [3] is described in this chapter. General formula of such differential equation is:

$$\sum_{u=0}^{m} \eta_u(x) y^{(u)}(x) = \sum_{j=0}^{g} q_j a_j(x), \tag{1}$$

where:

m	– order of the differential equation
$y(x)$	– unknown function of independent variable x
$y^{(u)}(x)$	– u^{th} derivative of the unknown function
$\eta_u(x)$	– polynomial variable coefficient for u^{th} derivative on the left-hand side of the differential equation
g	– order of a polynomial on the right-hand side of the differential equation
q_j	– constant coefficient for j^{th} power of the right-hand side polynomial
$a_j(x) = \frac{x^j}{j!}$	– auxiliary function for the right-hand side polynomial formulation

at which $x \in \langle 0; L \rangle$, where L is the length of considered interval of unknown solution.

The solution of the differential equation with variable coefficients has the form according to [3]:

$$y^{(u)}(x) = \sum_{i=0}^{m-1} y_0^{(i)} c_i{}^{(u)}(x) + \sum_{j=0}^{g} q_j b_{j+m}^{(u)}(x), \tag{2}$$

The solution of the differential equation (2) lies in determining the transfer functions generally labelled $c(x)$ and $b(x)$ that appear in the solution. First, the functions $b_{j+m}^{(u)}(x)$ are calculated using power series and recursive process, considering $u = \{0; m\}$ and $j = \{0; g\}$. It is necessary to guarantee the convergence of the series for a given interval $x \in \langle 0; L \rangle$ for successful calculation of these functions. This is always true only for constant coefficients η_u of the differential equation. It is often necessary to divide the interval of x into the shorter sections (in our case the independent variable is geometric variable, for example $x = L$ is the length of the bar) for variable coefficients $\eta_u(x)$, and thus determine the solution also for inner region of the bar (where $x \in (0; L)$). Calculation of the transfer functions and also automatic division of the interval for non-convergence behaviour of the series is included in a computer code and listed in [3].

The differential equations suitable for presented semi-analytical solution method must fulfil the following requirements:

– one independent variable of the function
– polynomial variable coefficients of the differential equation
– polynomial character of the right-hand side of the differential equation
– known interval of the independent variable where the solution of the differential equation needs to be determined

The order of the differential equation and the order of the right-hand side are arbitrary.

The described procedure of calculation such differential equations is suitable for calculation of electric and / or thermal field within the bar (1D task with only one independent variable), where change of material properties (electric and / or thermal conductivity) has polynomial (therefore variable) character.

3 Numerical experiment – electro-thermal analysis of FGM conductor

In this chapter there will be one example of calculation of electric and thermal field in given FGM link conductor presented. The task will be calculated using our new approach and also by commercial FEM code ANSYS [4] and by numerical solver for differential equations in software Mathematica [5] due to comparison reasons.

Let us consider electro-thermal conductor with rectangular cross section, see Figure 1. Let the nodes are symbolically denoted "0" (origin of the coordinate system) and "L" (end of the bar). Its length is $L = 200$ [mm], its height is $h = 6.67$ [mm] and depth is $b = 4$ [mm]. Let the conductor consists of mixture of two component materials – matrix (index m) with constant electric conductivity $\sigma_m(x,y) = 100$ [$\mathrm{Sm^{-1}}$] and thermal conductivity $\lambda_m(x,y) = 1.33$ [$\mathrm{Wm^{-1}K^{-1}}$], and fibre (index f) with electric conductivity $\sigma_f(x,y) = 2000$ [$\mathrm{Sm^{-1}}$] and thermal conductivity $\lambda_f(x,y) = 450$ [$\mathrm{Wm^{-1}K^{-1}}$]. Volume fraction of individual components is functionally changed according to Figure 1.

Number of layers for homogenization process is $N = 11$ through the height of the bar and there are 7 sublayers through the depth of the bar. After homogenization process the homogenized electric and thermal conductivities (superscript H) are polynomial functions only in one (longitudinal) direction x:

$$\sigma^H(x) = 1130.4 - 39\,332.1x^2 + 131\,107x^3 \ [\mathrm{Sm^{-1}}]$$
$$\lambda^H(x) = 244.65 - 9287.9x^2 + 30\,959.6x^3 \ [\mathrm{Wm^{-1}K^{-1}}]$$

We assume steady-state for electro-thermal analysis. For electric analysis, in node 0 there is electric current specified (the entering current density $J(0)$ is calculated and defined). In node L there is electric potential $\varphi(L)$ specified. For thermal analysis, there is the temperature $t(0)$ and the heat flux $q(L)$ specified for the nodes 0 and L, respectively (the heat is entering the node L). We assume also non-constant (polynomial) internal heat generation $Q(x)$, convection from the link surface with polynomial convective coefficient $\alpha(x)$ and constant ambient temperature t_{amb} (the change of convection coefficient was chosen artificially very high to demonstrate accuracy of our new link finite element also under the limiting conditions). Then the boundary conditions (see Figure 3) are:

$$I(0) = 1 \ \mathrm{A}$$
$$\varphi(L) = 11 \ [\mathrm{V}]$$
$$t(0) = 10 \ [°\mathrm{C}]$$
$$q(L) = 15\,600 \ [\mathrm{Wm^{-2}}]$$
$$Q(x) = 10^4 + 3 \times 10^7 x - 75 \times 10^7 x^3 \ [\mathrm{Wm^{-3}}]$$
$$\alpha(x) = 100 + 223\,125x - 4\,568\,750x^2 + 30\,546\,875x^3 - 66\,406\,250x^4 \ [\mathrm{Wm^{-2}K^{-1}}]$$
$$t_{amb} = 10 \ [°\mathrm{C}]$$

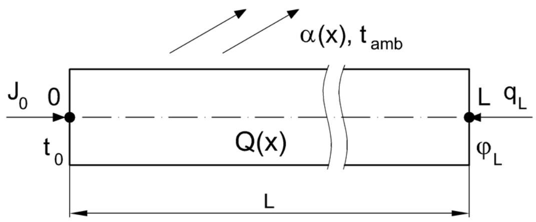

Figure 3: Boundary conditions for electro-thermal steady-state analysis.

We also created 3D model in code ANSYS, we used over 550 000 SOLID69 elements. Total number of nodes were over 570 000. The task was also solved in software Mathematica, where the differential equations for electric and thermal fields with homogenized material properties and specified boundary conditions were numerically solved. Finally, the task was also solved by our new developed two-nodal link element using derived FEM equations [6] for nodes of the link and using supplemental equations [6] for chosen points in the region of the link (41 points together). In Figure 4 we can see calculated longitudinal distribution of the electric potential and temperature in the conductor and in Figure 5 there is distribution of the electric current densities and heat fluxes for chosen layers (1st, 6th and 11th) of the 1st and 6th sublayer.

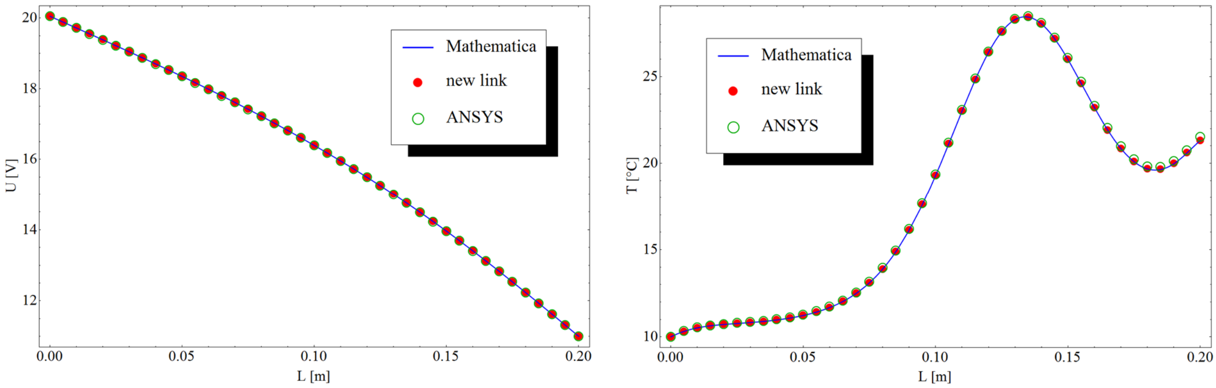

Figure 4: Longitudinal distribution of the electric potential and temperature in the conductor.

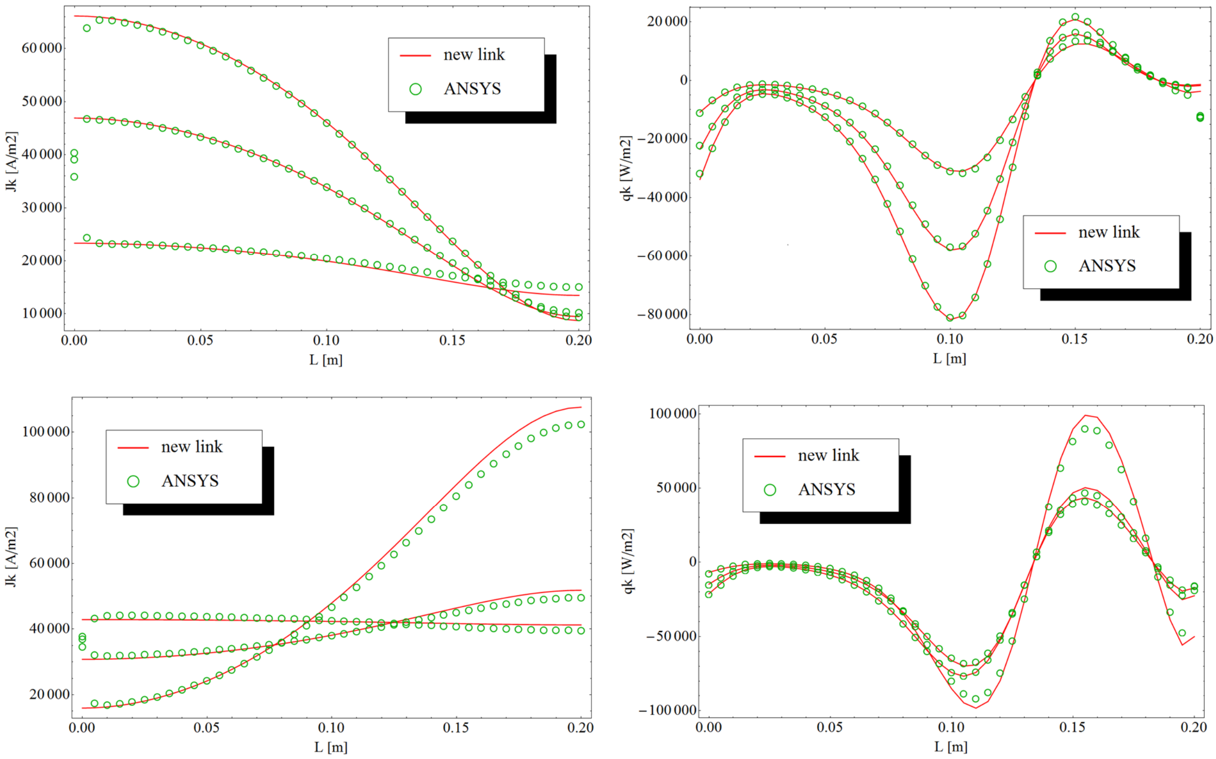

Figure 5: Electric current densities (left) and heat fluxes (right) for 1st, 6th and 11th layers within the 1st (top) and the 6th (bottom) sublayers.

4 Conclusion

We can see that obtained results correspond to ANSYS 3D simulation very well. But there is notable difference for current densities and heat fluxes results near the nodes of the bar where appropriate boundary condition was applied. It is caused due to the fact that 3D ANSYS model has to fulfil these boundary conditions for every element in this nodal area (curves of current densities and heat fluxes actually meet in one point). But our new developed finite thermal element is 1D system and the current density and heat flux are secondary variables, so the mentioned boundary conditions are fulfilled only on global level (for homogenized values $J^H(x)$ and $q^H(x)$), not for layers. This behaviour and difference in results is classic example of local effect caused by simplification of the real system into a 1D system.

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