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Investigation of driving properties for Formula Student

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Abstract: This paper deals with the investigation of driving properties such as rolling resistance of the chassis during the cornering and the effect of anti-roll bars on this phenomenon. This variable has a significant influence on the change of loading forces at tire contact patches, which further affects the transfer of input torque to the wheels. This process of torque transfer is performed by a limited slip differential and the traction in turns can be improved in this case by a change of differential's setting.

Keywords: Formula Student; roll angle; anti-roll bar; acceleration; limited slip differential

1 Introduction

The racing tracks of Formula Student disciplines are supposed to be flat with a lot of turns and without the potholes or other vertical road irregularities. That is the reason to focus the design and suspension settings of the car more to rolling than to vertical vibrations. The car behaviour during the tests is mainly affected by spring rates, the existence of anti-roll bars in the suspensions and the settings of dampers for fast transient states. This suspension setting can eliminate the change of the vertical load of wheels by minimization of chassis roll angle during the cornering and keep the lateral force at tire contact patches as much equal as possible at both sides of the car for better traction in turns. The worst case is the loss of the tire contact with a road at one side during the cornering.

The second way how to reach better elapsed time per circuit is to investigate the properties and settings of the most used type of differential for racing cars – limited slip differential (LSD) [5]. This differential allows to nonsymmetrically split input torque to the half-shafts based on different coefficients of road friction at tire contact patches or traction ability in turns or in its end, where the sharpest acceleration is demanded.

Above mentioned problems are introduced in this paper. The calculation of roll angle is possible to derive analytically and the results can be used for the suspension design or as a basis for further extended simulations of the whole car. The complex model of Formula Student car was modelled in commercial multi-body programme MSC Adams Car [4] and is depicted in Fig. 1. This model was used for verification of analytical approach of roll angle calculation and investigation of the effect of LSD's setting on driving properties.

2 Analytical calculation of anti-roll bar's setting

It is possible to simplify the real car by replacing it with a linear model of one degree of freedom – roll angle – with reduced masses and stiffness. The reduced masses by subsystems are generally sprung (chassis) and unsprung masses (suspensions) [3]. This simplification allows to analytically derive the equation for equilibrium position, which represents the steady-state behaviour during the cornering with a constant radius. The subsystems are defined by weight, longitudinal and vertical position of the centres of gravities. The subsystems are labelled with the superscript s for the sprung and un for the unsprung masses. The subscripts F and R define the front and rear side of the car. For further simplification of

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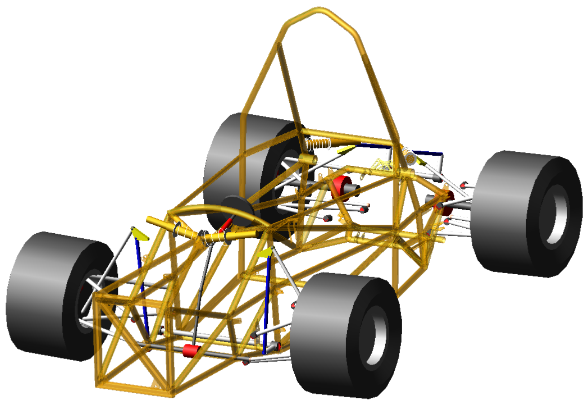

Figure 1: Computational model – Adams Car

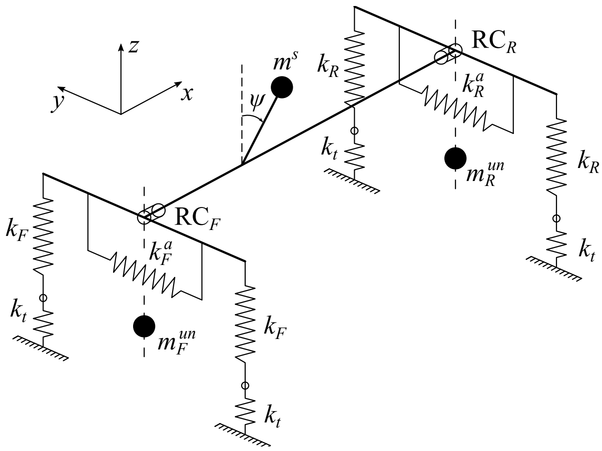

Figure 2: Computational model – analytical

model description, the front and rear sides are replaced by i subscript. It is supposed, that the car is ideally laterally symmetric.

The basic parameter, which describes every suspension is a roll centre RC [3]. The roll centre lies at the intersection of lateral wheel centre plane and longitudinal symmetry plane of the car. During the cornering with constant lateral acceleration a_{lat}, the centrifugal force rolls the chassis about the roll axis, which connects the front and rear roll centre. The distance between the centre of gravity of sprung mass and the roll axis is called effective moment arm h_0.

The simplified computational model [1] with one degree of freedom, roll angle ψ, is depicted in Fig. 2 and Fig. 3. The computational system is defined by the sprung m^s and unsprung m_i^{un} masses, spring rate k_t replacing the tires and coil spring rates k_i, which are linked across by rigid beams with the lengths equal to wheel tracks t_i. The rigid cross of beams is consisted of the lateral beams and the beam, which replaces the roll axis. The anti-roll bars (Fig. 4) are defined by torsional spring rates k_i^a and can be optionally linked to the lateral rigid beams. The influence of the anti-roll bars is further slightly discussed.

The loading and reaction forces are schematically shown and replaced by moments in Fig. 3. Lateral accelaration a_{lat} is supposed as positive in the negative direction of the global y-axis. Roll moment M^s expresses the roll effect of sprung mass. If the unsprung mass centre of gravity h_i^{un} is under the roll centre p_i, the roll moment M_i^{un} of centrifugal force acts against rolling of chassis (Fig. 3). The springs at the suspension are replaced by equivalent stiffness of the serial chain of coil spring and tire and generate the roll moment M_i^{spr}. The roll angle can be reduced by anti-roll bars. The relation [1] between the roll angle ψ and the twist angle θ is depicted in Fig. 4, where k_i^a is the torsional stiffness of the twist beam, a_i^a is the length of the blade and l_i^a is the length of the twist beam. The blade is supposed to be rigid, but it can be also replaced by bending stiffness. The resultant equivalent stiffness of anti-roll bar would be defined by serial chain of these two stiffness components.

Hence, the equilibrium moment equation is

$$M^s - M_F^{un} - M_R^{un} - M_F^{spr} - M_R^{spr} - M_F^a - M_R^a = 0. \tag{1}$$

After derivation of all terms in equation (1), linearization for an assumption of small roll angle and further short algebraic manipulation we get

$$\psi = \frac{m^s h_0 - m_F^{un}(p_F - h_F^{un}) - m_R^{un}(p_R - h_R^{un})}{K_F^{spr} + K_R^{spr} + K_F^a + K_R^a - m^s g h_0} a_{lat}, \tag{2}$$

where stiffness K_i^{spr}, K_i^a are calculated torsional stiffness of corresponding elements (coil springs and anti-roll bars) and g is the gravitational acceleration. The calculation of roll angle (2) can be simplified by omitting the terms of unsprung masses.

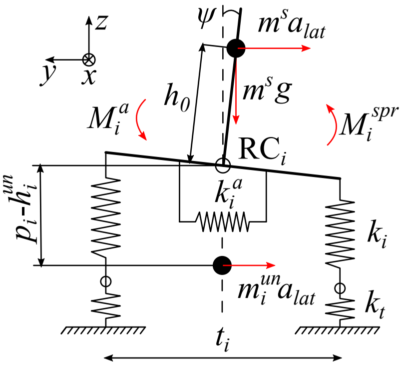

Figure 3: Loading forces and moments

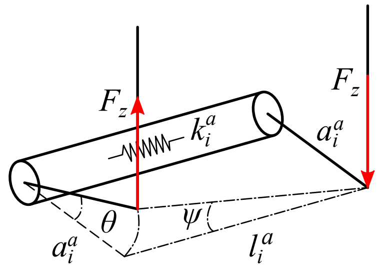

Figure 4: Model of anti-roll bar

3 Modelling of limited slip differential

During the race, the traction at one drive wheel can be affected due to weight transfer in the accelerating and cornering scenarios. Therefore it is important, that the car's driveline provides the optimal traction in all situations. The limited slip differential (LSD) improves the traction and the stability of the car since the vehicle handling is heavily influenced by the torque distribution to the drive wheels [1].

The LSD combines main features of an open differential such as allowing two wheels on the same drive axle to rotate by different angular velocities (e.g. during the cornering) and minimizing its disadvantage by limiting the traction capability in accelerating scenarios [2] (e.g. driving through a tight turn or the drive wheels are on surface with different friction coefficient). In case of usage of the open differential, the torque is transferred symmetrically to both wheels and the longitudinal traction of the car is limited by the wheel with worse grip. Thus the car has poorer acceleration performance. The LSD overcomes above-mentioned scenario by acting as a partial locked differential and allowing to transfer the torque to the wheel with more grip and enabling faster acceleration rates compared to the open differential.

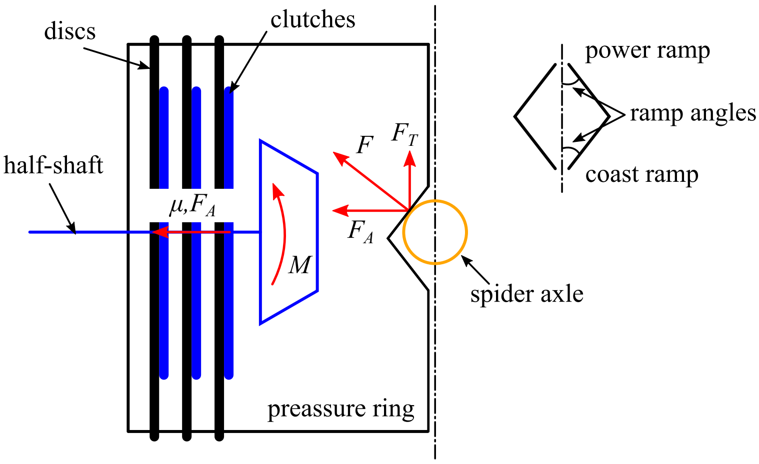

Figure 5: The scheme of LSD unit with a clutch-pack

The majority of Formula Student teams use a clutch-pack LSD design provided by Drexler Motorsport Australia. This differential has a set of discs and clutches on both sides. The clutches are depressed by pressure rings or pinion housing [5]. The contact friction pressure and corresponding loading force depend on their movement based on a ramp angle of the groove on the ring case, see Fig. 5. It can work either on accelerating (power ramp) or breaking (coast ramp) scenarios. Acute angles generate more displacement of the pressure rings and higher loading force, that causes more locking of the wheels. Figure 5 shows in detail the components of the force at the ramp contact during the acceleration.

4 Results

The results of both mentioned and investigated problems are introduced in two subsections below.

4.1 Results of anti-roll bar calculation

Above derived analytical calculation of roll angle was implemented into in-house MATLAB programme. The tires were assumed as rigid. For verification of the analytical approach and comparison to the multi-body approach (Adams Car), some testing cases were created. The first case was the configuration of the model only with the springs. Hence, the front and rear anti-roll bars were added step by step. For the most accurate results between both approaches, the simulation with constant lateral acceleration at constant radius cornering was chosen. The comparisons of the roll angle results against lateral acceleration for each simulation case are shown in Fig. 6. The results confirm, that the existence of anti-roll bars limits resultant roll angle. The differences between analytical and numerical solutions of the roll angle are minor and the results can be assumed as perfectly exact regarding the considerable simplification of the presented computational model. The comparison of the vertical force loading the anti-roll bar's blades in the multi-body approach and simple analytical calculation are slightly different (Fig. 7). It is caused by the presumption of ideally vertical load (Fig. 4) of the blades in the analytical approach.

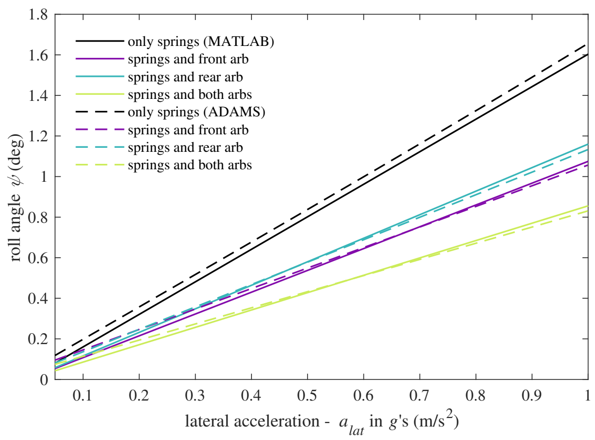

Figure 6: Results of roll angle

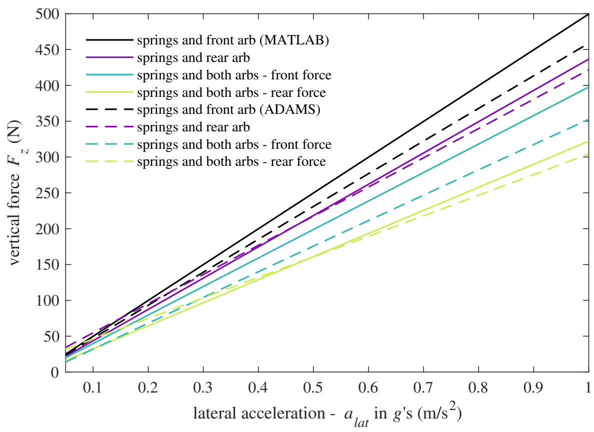

Figure 7: Results of vertical force

4.2 Parametric study of limited slip differential's setting during acceleration

The series of simulations with the Formula Student LSD model were performed in Adams Car. The main idea of the tests was to evaluate the locking percentage of the driven wheels during a rolling start (5 km/h) straight-line acceleration scenario with constant torque input. The right-side wheels were on the prescribed surface with less friction during the simulations. The goal of performed simulations was to verify the computational model with the results presented in the datasheet of Drexler Motorsport.

The results show, that decreasing of the ramp angle (Fig. 5) generates more torque based on the friction in the clutch-pack and the angular velocity of the slipping wheel decreases towards the velocity of the left wheel with better grip, see Fig. 8. The results of the angular velocity of the left wheel are close for all simulation cases and the depicted dashed line is assumed as mean value through all testing cases. To verify the model, the locking percentage was calculated for all settings of the ramp angles and the results are presented in Fig. 9. The comparison of average locking percentage between the computational model and the datasheet of Drexler is shown in Tab. 1 below.

The covered distance was also calculated for the verification of computational model. It can be assumed, that the car with more locking percentage, thus better traction, is able to cover longer distance during the same simulation time. The results of simulations are written in Tab. 2. The differences between each setting of the ramp angle and the open differential could seem as minor, but the final sum of these differences could significantly influence the racing time in several laps.

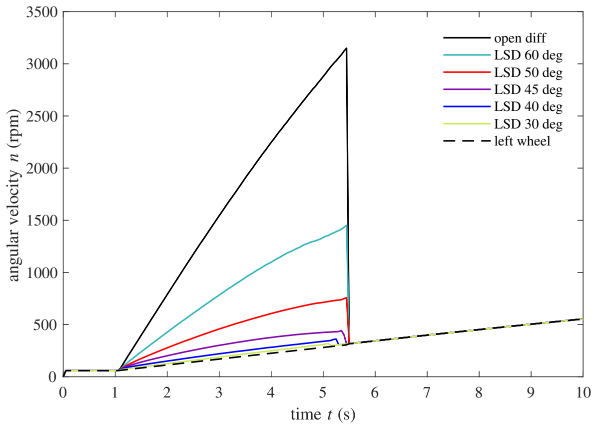

Figure 8: Angular velocities of driven wheels

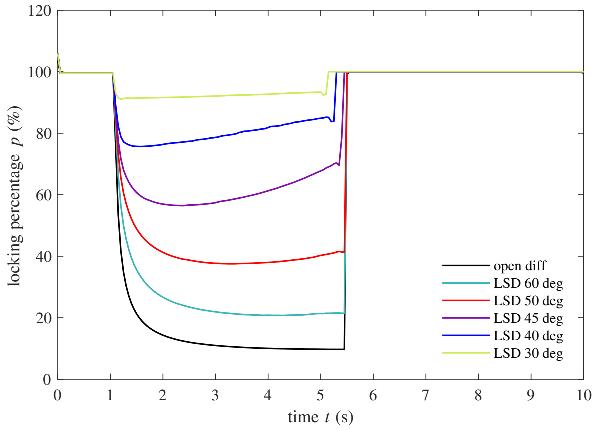

Figure 9: Results of locking percentage

	Average locking percentage	*Drexler Motorsport (locking)*
Open differential	14 %	–
LSD – ramp angle 60 deg	26 %	29 %
LSD – ramp angle 50 deg	42 %	42 %
LSD – ramp angle 45 deg	62 %	51 %
LSD – ramp angle 40 deg	80 %	60 %
LSD – ramp angle 30 deg	92 %	88 %

Table 1: Comparison of locking percentage

	Covered distance	*Relative distance to open differential*
Open differential	66.910 m	–
LSD – ramp angle 60 deg	67.139 m	+ 0.229 m
LSD – ramp angle 50 deg	67.268 m	+ 0.358 m
LSD – ramp angle 45 deg	67.327 m	+ 0.417 m
LSD – ramp angle 40 deg	67.350 m	+ 0.440 m
LSD – ramp angle 30 deg	67.351 m	+ 0.441 m

Table 2: Covered distance during acceleration

5 Conclusion

The presented paper is firstly focused on the analytical calculation of roll angle. The second part discusses the problems and effect of the limited slip differential on the acceleration.

The calculation of roll angle is derived from the basic computational model (1 DoF – roll angle) with reduced masses and stiffness. This model was solved analytically in MATLAB and numerically in Adams Car. The results of roll angle for increasing lateral acceleration are very close for both approaches and the simple linear model with one degree of freedom can be used for this type of simulation. The results of roll angle are more than satisfied, but it is necessary to be careful with the interpretation of resultant loading force on the blades and using it for structural analyses of the anti-roll bar or its chosen parts.

The performed numerical analyses of limited slip differential in Adams Car show, that the computational model and its behaviour is very close to the real unit. The model was verified through the comparison of numerical results of locking percentage and corresponding datasheet. The influence of LSD unit setting to the acceleration was also investigated and evaluated as covered distance during the simulation. Concerning the simulations, it is clear, that LSD unit improves the Formula Student performance in the dynamic tests. For future work, the variations of presented simulations can be implemented, such as accelerating and cornering at the same time.

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