A Geodesic Based Approach for an Accurate and Invariant 3D Surfaces Representation

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ABSTRACT

In this paper, we propose a novel 3D invariant surface representation under the 3D motion group $M(3)$. It is obtained by combining two main representations: the three-polar representation and the one defined by the radial line curves from a starting point. The retained invariant points correspond to the geometrical locations of the intersection between the two last representations. The approximation of the novel surfaces description method on the 3D discrete meshes is studied. Its accuracy for the 3D faces description and retrieval is evaluated in the mean of the Hausdorff shape distance.

Keywords

Three-polar, geodesic potential, superposition, level set, radial line, shape representation, Hausdorff, invariant, approximation, face.

1 INTRODUCTION

Actually, it is recognized that the 3D data have allowed to cross several planar images (2D) problems such as the illumination and the pose. Many domains benefit from the use of the three dimensional data like the medical one, the security and the heritage conservation.

Despite the amazing development of the 3D data scanning tools and the related technologies, the description and the analysis of 3D surfaces remain a difficult task. This comes from the lack of a natural parametrization of a 3D surface. In fact, the obtained data depend largely on the position of the scanning tool relatively to the 3D object.

In practice, the conventional representation of a 3D surface is the discrete triangulated mesh. The points cloud composing this conventional representation is not organized or partially organized. This fact made the comparison procedure between different shapes more and more difficult. Therefore, the extraction of an efficient description of the 3D surfaces has become more than necessary. In the literature, the 3D surface description methods can be grouped into four major classes: the two dimensional views, the graphs based approaches, the transformations based ones and those obtained by statistical features.

The two dimensional views based methods suppose that a 3D object corresponds to a set of 2D images. They are obtained by the projection of the 3D object from canonical points of views. The task of describing a 3D object is transformed, therefore, to a description of planar images. Many description methods can be applied on the obtained 2D images as Fourier descriptors [Vra04] and Zernike moments [Che03].

The approaches based on the graphs consist on the extraction of graphs from a 3D object. The comparison between different shapes is, therefore, reduced to a comparison between their corresponding graphs. One of the most used descriptors is the Reeb graph [Tun05]. This graph is invariant under the rotations and the translations transformations. It is also robust under the connectivity changes and the mesh simplifications. These characterizations made this last method an accurate tool for 3D surfaces description. The skeletal method [Sun03] is also known as a perfect tool for 3D surfaces description based on the graphs.

The transform based methods need first of all the conversion of the surface onto 3D voxels or a spherical grid. Many specific transformations are, then, applied to extract efficient description of the surface. The most famous methods are the 3D Radon [Dar04], 3D Fourier [Bur92] and the rotation-invariant spherical harmonics [Kaz03]. These methods of transformations are characterized by their invariance under the rotations, the translations and the scale factors. Their accuracy for the recognition and the retrieval is proved. Other works use the angular radial transform [Ric05], the spherical wavelet descriptors [Lag06] and the uniformization [Khe08] as methods of description based transformations.

The statistical features based methods consist on the extraction of numerical attributes of a 3D object which can be local or global. Many past works adopted this approach for the extraction of an efficient 3D surfaces description. We mention the pioneer work of Faugeras et al. [Fau86] based on the high curvature area determination. A 3D surface is considered as a set of points that correspond to the geometrical locations of the high curvatures values. These points are invariant under the rotations and the translations transformations. Bannour et al. [Ban00] proposed a novel pseudo-reparametrisation of 3D surfaces by the extraction of iso-curvature features. In this method of description, a 3D surface is characterized by a curves network determined by iso-curvatures computation. The surface here is not described by the points corresponding to the extremal curvatures values but by the ones corresponding to the levels set of the curvatures values. Other kind of methods use the geodesic approach which is more stable under the numerical computation errors than the ones based on the curvatures determination. Many works use the local coordinate system computed around one reference point of the surface [Sam06, Sri08, Gad12] qualified by the unipolar representation (one reference point). It consists on the set of points corresponding to the levels of the geodesic potential generated from one reference point. In recent works, Jribi et al. [Jri13, Jri14] proposed a novel representation that they qualified by the three-polar one. It is defined by the levels of the sum of the three geodesic potentials generated from three reference points. This representation has ensured a more stability under errors on the reference points positions than the unipolar one [Jri14].

We propose in this paper to extract accurate invariant points from the three-polar representation. In order to achieve this purpose, we combine the three-polar representation with the one defined by the radial line curves constructed from a starting point. The novel set of invariant points is obtained by the intersection between the two last representations. The steps of the construction of the novel representation from the 3D discrete meshes will be studied. Its accuracy for 3D faces description will be evaluated.

Thus, this paper will be structured as follows: In the second section, we will present the mathematical formulation the novel representation. The similarity metric to compare between different shapes with the invariant points cloud will be exposed in the third section. In the fourth section, the approximation of the novel representation on the 3D discrete meshes will be presented. We will show in the fifth section, its accuracy for the 3D faces recognition and retrieval.

2 CONSTRUCTION OF THE NOVEL REPRESENTATION

We present in this section the mathematical formulation of the novel representation. We suppose, here, that a surface is continuous. Thus, it corresponds to a two differential manifold denoted S. Let denote by r and q two points of S. We start by presenting some differential considerations. We designate by:

- $\gamma(r,q)$: The geodesic curve joining r and q. It corresponds to the curve on the surface S having the minimal distance between r and q.
- $\widetilde{\gamma}(r,q)$: The length of the geodesic curve joining r and q.
- $U_r : S \to R$: the function that computes for each point p of S the length of the geodesic curve joining p to r. $U_r(p) = \widetilde{\gamma}(r,p)$. It is called the geodesic potential generated from the reference point r of the surface S.
- $C_U^\lambda = \{p \in S; U(p) = \lambda\}$: The geodesic level curve of value λ. It corresponds to the set of points of S which have the same value λ of the geodesic potential U. U can be a geodesic potential generated from one reference point or the sum of several geodesic potentials.

Three main steps are realized in order to obtain the novel representation:

- Construction of the three polar representation.
- Extraction of the representation defined by the radial line curves from a starting point.
- The intersection between the two last differential representations.

The obtained invariant points of the novel representation correspond to the intersection between the three polar representation and the one composed by the radial line curves.

2.1 Brief recall of the three-polar representation construction

We present, here, a brief recall of the construction process of the three-polar representation [Jri14]. It is based on the superposition of the three geodesic potentials generated from three reference points of the surface. It consists on the set of points corresponding to the levels of the sum of the three geodesic potentials. Let $\{r_i, i = 1..3\}$ be three reference points of the surface and $\{U_{r_i}, i = 1..3\}$ their corresponding potentials functions. $U_3 = \sum_{i=1}^{3} U_{r_i}$ is the geodesic potential obtained by the sum of these three geodesic potentials. Then, the three-polar representation denoted by $T_3^K(S)$ can formulated as follows:

$$T_3^K(S) = \{p \in S; U_3(p) = U_3^* + \frac{k}{K}(\alpha_K - U_3^*), k = 0..K\} \quad (1)$$
$$= \{C_{U_3}^{\lambda_k}; \lambda_k = U_3^* + \tfrac{k}{K}(\alpha_K - U_3^*), k = 0..K\}.$$

Where K is the number of the levels of the three-polar representation, α_K is the maximum of the geodesic sum, $U_3^* = min\{U_3\}$ and the integer k designates the k^{th} level of the three polar representation.
We note that this representation is invariant under the $SO(3)$ rotation group and the displacement one. The obtained three-polar representation is composed by a collection of indexed level curves according their level values.

2.2 Construction of the radial line curves representation on a 3D surface

Let q be a point of the surface S. Let denote by P_q^0 a plane that contains the point q and that intersects the surface S on a curve (radial line curve) that we call the reference radial line curve and we denote by R_q^0. The choice of R_q^0 depends on the kind of the surface.
Let call by R_q^α the radial line curve making an angle α with the reference radial curve R_q^0. It is obtained by the intersection between the plane P_q^α and the surface S. P_q^α is the plane containing the point q and having the angle α with the reference plane P_q^0. Since this plane is not unique, we choose a clockwise direction. We repeat the process of radial line curves extraction with the same angular separation.
This representation that we denote by RL^K (Radial lines) is formulated as follows:

$$RL^K(S) = \{P_q^{k\alpha} \cap S; k = 0..K\}. \quad (2)$$

We obtain, therefore, an indexed collection of radial line curves on the surface.

2.3 The obtained invariant points of the novel representation

From the construction of the three-polar representation, we obtain an indexed level curves of the sum the three geodesic potentials generated from their corresponding three reference points.
A collection of radial line curves indexed by their angular values according to the reference radial line curve is also obtained by constructing the radial line curves from a starting point of the surface.
By computing the intersection between the level curves of the three-polar representation and the radial line curves representation, we obtain a set of invariant points indexed by both the value of the level curves of the three-polar representation and the angle of the radial line according to the reference one.
The novel representation defined by this selection of invariant points will be denoted by $3PRL$ (Three-Polar and Radial Lines).

$$3PRL(S) = RL^K(S) \cap T_3^M(S). \quad (3)$$

3 SIMILARITY METRIC

We propose to compare between the different shapes using their corresponding novel representations. We use the well known Hausdorff shape distance introduced by Ghorbel in [Gho98, Gho12]. Let consider the real plane R^2 and the unit sphere S^2 that represent the group G of all possible normalized parametrisations of surfaces. Since the space of surfaces pieces can be seen as a set of all 3D objects assumed diffeomorphic to G, this space is assimilated to a subspace of $L^2_{R^3}(G)$ formed by all square integrated maps from G to R^3. The direct product of the Euler rotations group $SO(3)$ by the group G , acts on such space $L^2_{R^3}(G)$ in the following sense:

$$SO(3) \times G \times L^2_{R^3}(G) \to L^2_{R^3}(G) \quad (4)$$
$$\{A, (u_0, v_0), S(u, v)\} \to AS(u + u_0, v + v_0).$$

The 3D Hausdorff shape distance Δ can be written for every S_1 and S_2 belonging to $L^2_{R^3}(G)$ and g_1 and g_2 to $SO(3)$ as follows:

$$\Delta(S_1, S_2) = max(\rho(S_1, S_2), \rho(S_2, S_1)). \quad (5)$$

Where:

$$\rho(S_1, S_2) = \sup_{g_1 \in SO(3)} \inf_{g_2 \in SO(3)} \| g_1 S_1 - g_2 S_2 \|_{L^2}. \quad (6)$$

$\| S \|_{L^2}$ denotes the norm of the functional space $L^2_{R^3}(G)$.
We consider after that, a normalized version of Δ so that its variations are confined to the interval [0,1]. This distance is obtained by using the well known Iterative Closest Point (ICP) algorithm [Bes92].

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4 APPROXIMATION OF THE 3PRL REPRESENTATION ON THE MESHES OF 3D FACES

The description and the analysis of the shapes of 3D faces have achieved an increasing importance in the last few decades especially with the great development of 3D scanning tools. In all steps of the 3PRL representation construction, we supposed that the surface is a continuous two differential manifold. In the practice, the data obtained from the 3D scanning tools are discrete. They correspond to the 3D triangulated meshes known as the conventional representation of 3D surfaces. We will study here the extraction process of the novel representation on the meshes of the 3D faces.

4.1 3D mesh pre-processing

Many pre-processing steps are realized on the 3D faces meshes in order to ensure the best way to extract the 3PRL representation. The first step consists on removing non-connected parts of the 3D mesh. In fact, it is not possible to compute the geodesic distance between two points of the surface that belong to disconnected parts of a 3D mesh.

The second step consists on filling the holes on the surface. In order to have the same number of points for all faces and a finer resolution for both the level curves and the radial line ones, we make a remeshing procedure of the 3D meshes. A larger number of points is therefore obtained. All the pre-processing steps are computed by some functions provided with the VTK library (www.vtk.org).

4.2 Geodesic potential on 3D meshes

After the application of the pre-processing steps on the 3D meshes of faces, the geodesic potentials generated from the three reference points should be computed for the three-polar representation.

For a reference point, the corresponding geodesic potential consists on computing the geodesic distances between this point and each point of the 3D mesh. Several past methods have been proposed in the literature in order to compute distances on 3D meshes. We use in our work the Dijkstra algorithm [Dij59a] to compute the length of the geodesic path between two points of the surface.

4.3 Extraction of the level curves of the three polar representation

After computing the sum of the three geodesic potentials generated from the corresponding reference points, we should extract the level curves of the three-polar representation. In practice, the determination of a curve of level λ of the sum of the three geodesic potentials consists on the extraction of a trip rather than a curve. It corresponds to the set of surface points that have this sum values ($U_3 = U_{r_1} + U_{r_2} + U_{r_3}$) in the interval $[\lambda - \varepsilon, \lambda + \varepsilon]$. ε is a small positive real value.

4.4 Extraction of the radial line curves

The radial line curves of a surface S according to a starting point q consist on the intersection between the planes with the same angular separation and the surface. Since the 3D surface is composed by a 3D mesh, a plane does not slice the surface necessary on points (the plane could pass through the edges of triangles). Therefore, a radial curve R_q^α will be composed by a set of points of S that have Euclidean distances to the plane P_q^α less than a small positive real value ε_p.

$$R_q^\alpha = \{p \in S; de(p, P_q^\alpha) \leq \varepsilon_p\}. \tag{7}$$

with $de(p,q)$ is the Euclidean distance between two points p and q. P_q^α is the plane passing by the point q with an angular separation value equal to α according to the reference plane P_q^0. We note that the Euclidean distance between a point and a plane consists on the one between the point and its orthogonal projection on the same plane. The Fig. 1 illustrates the extraction procedure of the radial line curves on 3D meshes.

For the case of the 3D faces, the reference radial line curve corresponds to the vertical one when the face is moved to the upright position.

5 EFFECTIVENESS OF THE NOVEL REPRESENTATION FOR 3D FACES DESCRIPTION

We use in our experimentation the 3D faces data base BU-3DFE (Binghamton University - 3D Facial Expression) [Lij06]. This database contains a total of 100 subjects composed by 56 women and 44 men. For each subject, seven facial expressions are performed (neutral, disgust, happiness, angry, surprise, sadness and fear).

For the 3PRL representation, we used a total of 30 faces corresponding to six subjects of the database (3 men and 3 women). Five facial expressions are chosen for each subject including the neutral expression. We use for the construction of the three-polar representation the three reference points corresponding to the two outer corners of eyes and the nose tip [Jri14] (Fig. 2(a)). The level 0 of the sum of the three-polar representation can be composed by one or multiple points. We compute its centroid. This point will be denoted by C_e. All the level curves of the three-polar representation can be seen as curves around this point (Fig. 2(b)). C_e will be also the starting point of the radial line curves with the same angular separation (Fig. 2(c)). The Fig. 2(d) shows the invariant points of the novel 3PRL representation extracted from a 3D

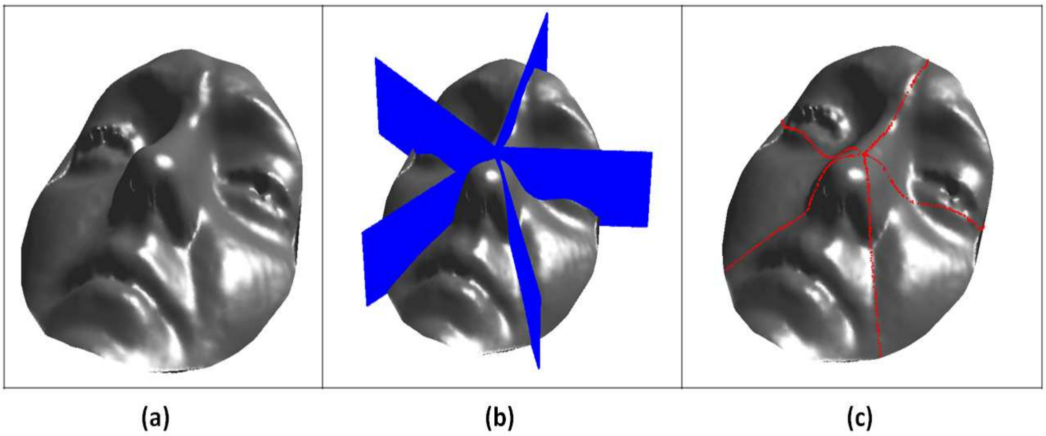

Figure 1: The extraction of radial line curves from a 3D face mesh: (a) a 3D face. (b) the planes with the same angular separation (blue color) and the face. (c) the radial line curves extracted from the surface.

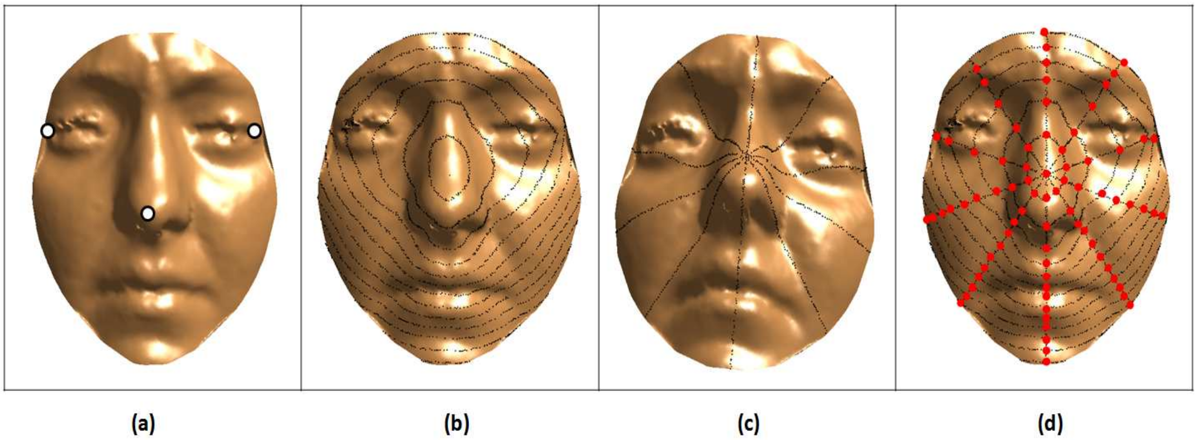

Figure 2: (a) The used three reference points. (b) the three-polar representation. (c) The radial line curves. (d) The 3PRL representation (invariant points with the red color).

face mesh.
In order to test the effectiveness of the novel representation to describe 3D faces, the Hausdorff shape distance is computed between each pair of invariant points cloud of faces representations. we obtain therefore a matrix of normalized Hausdorff shape distance. The Fig. 3 illustrates this matrix.

The table 1 summarizes the organization of the faces in this matrix.
From the observation of this matrix of distances, we can note that the novel invariant representation well describes the 3D faces. Indeed, the distances between the faces of the same person with different facial expressions are smaller than the other distances.
We make also an experimentation for the retrieval process. The Fig. 4 illustrates the obtained results.
The first column corresponds to the query subjects and the retrieved results are presented in the rest of this table. The query subjects are chosen to be the neutral faces of the six persons. From the explanation of the obtained results, we can note the effectiveness of such novel representation for the retrieval procedure. In fact only two errors exist in the retrieval process (faces with red color squares in the table.)

6 CONCLUSION

In this paper, we have introduced a novel 3D surfaces representations that we called 3PRL. It is obtained by combining two main representations: the three-polar representation and the one defined by the radial line curves from a starting point. The 3PRL representation consists on the set of invariant points corresponding to the intersection between the two last representations. The approximation of such representation on the 3D discrete meshes was studied. Its accuracy for 3D faces description and retrieval was evaluated.
The perspectives of such work are multiple. We, first,

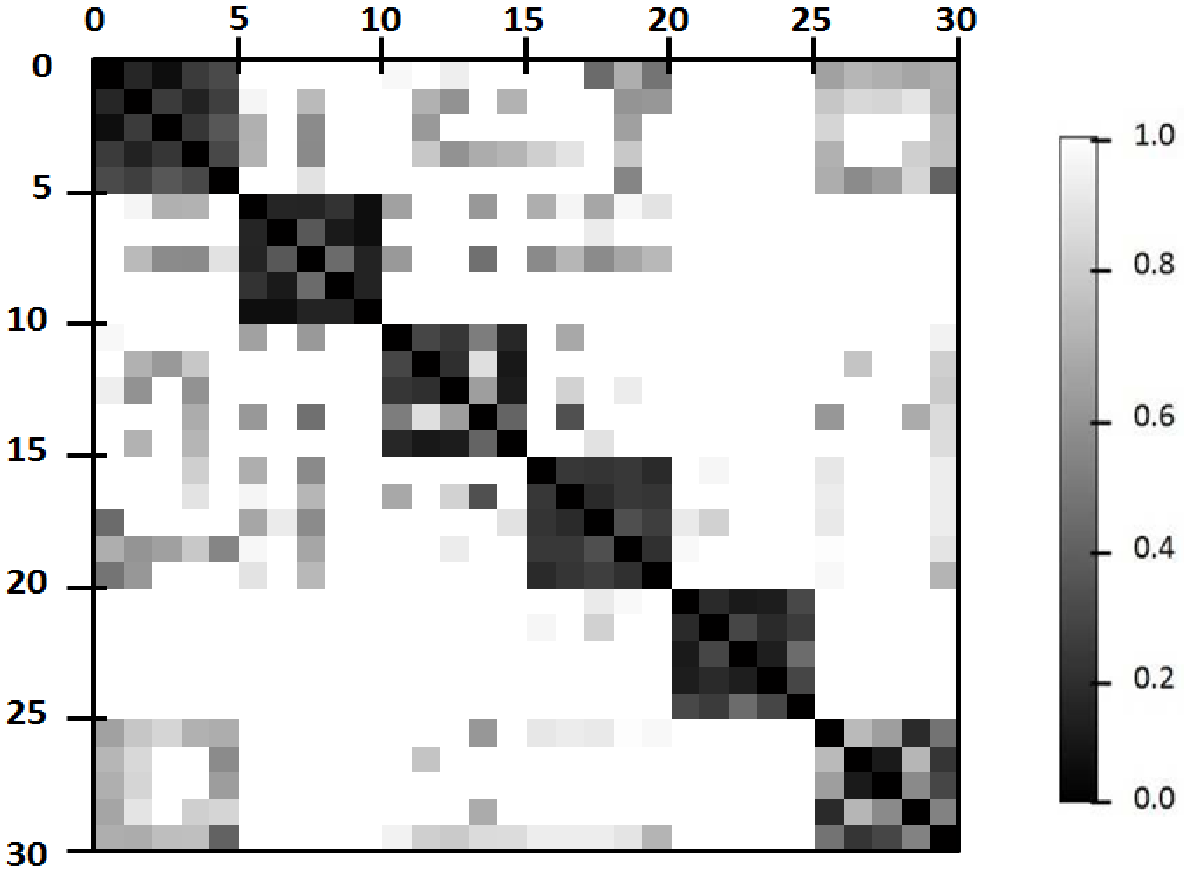

Figure 3: Matrix of pairwise normalized Hausdorff shape distance

Rows 1..5 Columns 1..5	Rows 6..10 Columns 6..10	Rows 11..15 Columns 11..15	Rows 16..20 Columns 16..20	Rows 21..25 Columns 21..25	Rows 26..30 Columns 26..30
Five faces of the subject 1	Five faces of the subject 2	Five faces of the subject 3	Five faces of the subject 4	Five faces of the subject 5	Five faces of the subject 6

Table 1: Organization of the data in the matrix

propose to experiment this novel representation on a larger number of faces. We intend also to make a study of the optimal numbers of levels of the three-polar representation and of the used radial line curves. It will be also interesting to define the optimal number of the reference points.

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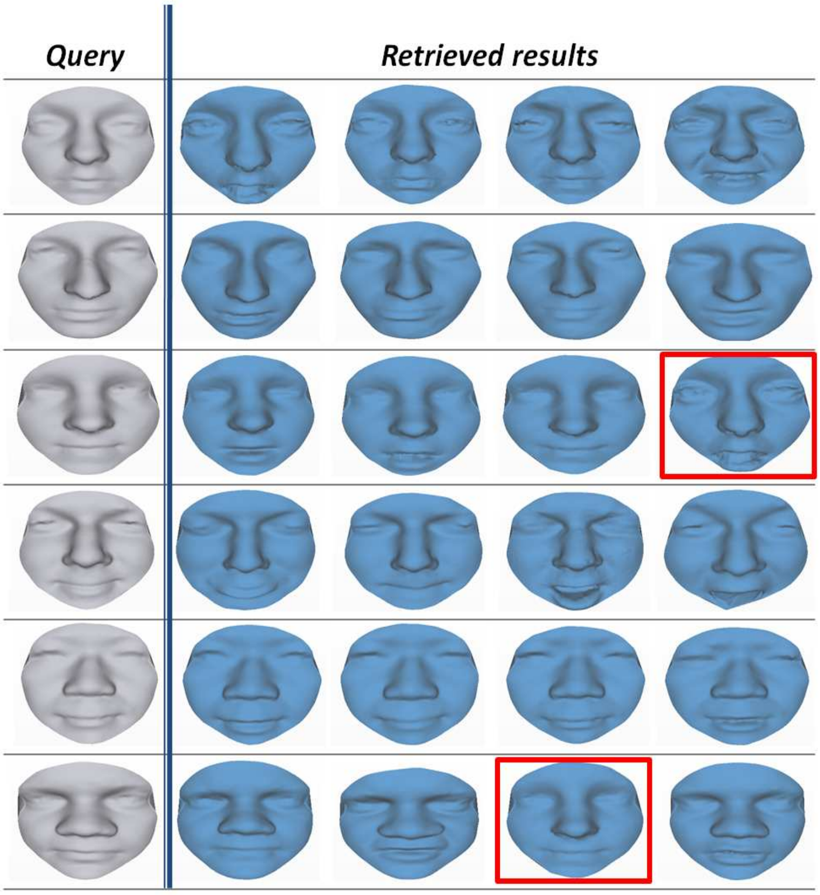

Figure 4: The retrieved results from the BU-3DFE database

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