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Kinematical Ruled Surfaces based on Interrelated Movements in Triads of Contacted Axoids

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ABSTRACT

Kinematical ruled surfaces are constructed by generating the line's motion of a moving ruled surface during its movement along a fixed ruled surface [Spr02a]. The main condition of constructing kinematical ruled surfaces is that a moving axoid contacts with a fixed axoid along their common generating line in each of their positions during the movement of one axoid along another. A lot of well-known kinematical ruled surfaces are constructed on the base of certain pairs of contacted axoids such as “plane – cylinder”, “plane – cone”, “cylinder – cylinder”, “cone – cone”, etc. [Kri06a]. A new model of constructing kinematical ruled surfaces based on interrelated movements in the triads of contacted axoids is proposed in this research. Geometrical models, analytical representations, and computer visualization of the new constructed kinematical surfaces for some cases of triads of contacted axoids “plane – cylinder – cylinder”, “plane – cone – cone”, “cylinder – cylinder – cylinder”, “cone – cone – cone” (Fig. 1), and for matched triads of one-sheet hyperboloids of revolution are developed in this paper. Figures of the triads of contacted axoids and corresponding constructed kinematical ruled surfaces have been developed with the help of the software application AMG (“ArtMathGraph”) [Con07a].

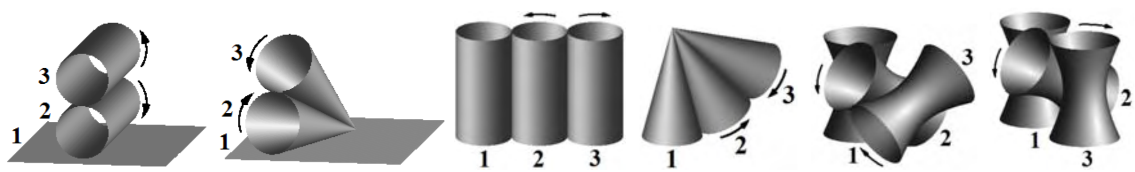

Figure 1

Keywords

Geometrical Modeling, Computer Graphics, Kinematical Surfaces.

1. INTRODUCTION

Kinematical ruled surfaces as a result of one generating line’s motion of the moving ruled surface during its movement along the fixed ruled surface in the cases of certain pairs of contacted axoids are well-known ruled surfaces [Kri06a]. New abilities for constructing kinematical ruled surfaces are originated on the base of the model of interrelated movements in the triads of the contacted axoids, where one of them is a fixed axoid (**1**) and two other are moving axoids (**2**, **3**). The substance of this model consists in the correspondence between the movement of axoid **3** along axoid **2** and the movement of axoid **2** along fixed axoid **1**. The movement of axoid **2** along fixed axoid **1** is accompanied by the backward motion of axoid **3** along axoid **2**, so that the positional relationship of axoids **1, 2, 3** during interrelated movements in the triads is fixed. Examples of the application of the proposed model for constructing rotational ruled surfaces are realized on the base of triads: “plane – cylinder – cylinder”, “plane – cone – cone”, “cylinder – cylinder – cylinder”, “cone – cone – cone” (Part 2.1-2.4). Examples of constructed kinematical ruled surfaces on the base of interrelated movements in the triads of one-sheet hyperboloids of revolution are also realized (Part 3).

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2. ROTATIONAL RULED SURFACES BASED ON MODELS OF TRIADS OF CONTACTED AXOIDS

2.1 Model of triad “plane – circular cylinder – circular cylinder”

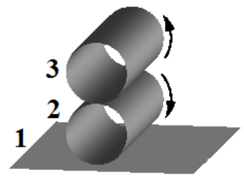


Figure 2

The triad “plane – circular cylinder – circular cylinder” is shown in Fig. 2. In this system of contacted axoids the movement of cylinder **2** along fixed plane **1** is accompanied by the backward motion of cylinder **3** along cylinder **2**, so the axis of moving cylinder **3** is located in the common plane with both axis of cylinder **2** and common generating line of cylinder **2** and plane **1** right along during interrelated movements in this triad of axoids.

Geometrical model of constructing a kinematical ruled surface, generated by one of generating lines of moving cylinder **3**, is presented as a superposition of interrelated movements: rotational movement of moving cylinder **3** around its axis and translational movement of the axis of cylinder **3** along plane **1**. As a result of successive transformations of coordinates, parametric representation (in parameters u, v) of the rotational ruled surface, generated by one of generating lines of moving cylinder **3** in the fixed coordinate system oxyz, connected with fixed axoid **1** (the common generating line of cylinder **2** and plane **1** is lying in axis ox) is:

$x = v$;

$y = u + b\cos\varphi$;

$z = 2a + b(1+\sin\varphi)$,

where $\varphi = u / b$,

a – radius of moving cylinder **2**,

b – radius of moving cylinder **3**.

As it ensues from these parametric equations, the form of constructed kinematical ruled surface is a - independent. In other words, the kinematical ruled surface constructed on the base of triad of axoids “plane – circular cylinder – circular cylinder” (Fig. 3) is the same as the kinematical ruled surface, constructed on the base of the pair of contacted axoids “plane – circular cylinder” [Kri06a].

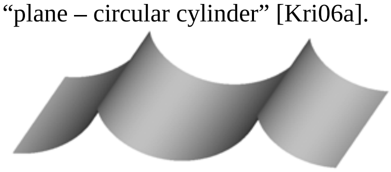


Figure 3

2.2 Model of triad “plane – circular cone – circular cone”

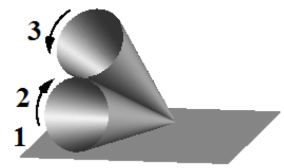


Figure 4

The triad of contacted axoids “plane – circular cone – circular cone” is shown in Fig. 4. The axoid **1** in this triad is a fixed axoid. By perfect analogy with the triad “plane – circular cylinder – circular cylinder”, described above (Part 2.1), the movement of cone **2** along fixed plane **1** is accompanied by the backward motion of cone **3** along cone **2** so that the axis of the moving cone **3** is located in the common plane with both axis of cone **2** and common generating line of cone **2** and plane **1** right along during interrelated movements in this triad of contacted axoids (Fig. 4).

As a result of successive transformations of coordinates, parametric equations (in parameters *u*, *v*) of the rotational ruled surface, generated by one of the generating lines of moving circular cone **3** in the fixed coordinate system *oxyz*, connected with fixed axoid **1**, are defined. The origin of coordinate system *oxyz* is located at the vertex of cone **2** (cone **3**).

Parametric equations of rotational ruled surface are:

$x = X\cos u - (Y\sin\theta + Z\cos\theta)\sin u$;

$y = X\sin u + (Y\sin\theta + Z\cos\theta)\cos u$;

$z = -Y\cos\theta + Z\sin\theta$,

where

$X = v\sin\alpha_3\cos\varphi$;

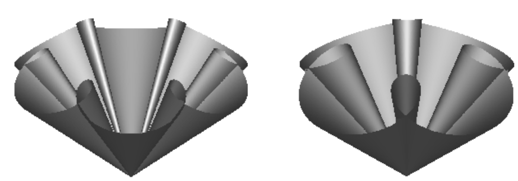
$Y = v\sin\alpha_3\sin\varphi$;

$Z = v\cos\alpha_3$;

$\theta = 2\alpha_2 + \alpha_3$; $\varphi = (1/\sin\alpha_3)u$

(α_2, α_3 – angles between the cone’s generating line and cone’s axis for circular cones **2**, **3** accordingly).

Examples of the visualization of rotational ruled surfaces constructed on the base of the triad “plane – circular cone – circular cone” are shown in Fig. 5 (cone **2** ($2\alpha_2$ = 40°), cone **3** ($2\alpha_3$ = 20°, 30°).



Cone **2** ($2\alpha_2$ = 40°), cone **3** ($2\alpha_3$ = 20°)   Cone **2** ($2\alpha_2$ = 40°), cone **3** ($2\alpha_3$ = 30°)

Figure 5

2.3 Model of triad "circular cylinder – circular cylinder – circular cylinder"

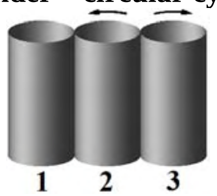


Figure 6

The triad of contacted circular cylinders is shown in Fig. 6. In this system of contacted circular cylinders, the outside surface of moving cylinder **2** revolves around the outside surface of fixed cylinder **1**. At the same time the outside surface of moving cylinder **3** revolves around the outside surface of moving cylinder **2** so the axis of moving cylinder **3** is located in the common plane with both the axis of cylinder **2** and fixed cylinder **1** right along during interrelated movements in this triad of contacted cylinders.

Geometrical model of constructing a kinematical ruled surface, generated by one of the generating lines of moving cylinder **3**, is presented as a superposition of interrelated movements: rotational movement of moving cylinder **3** around its axis and rotational movement of the axis of cylinder **3** around the axis of fixed cylinder **1** lying in axis oz of the fixed coordinate system oxyz, connected with fixed axoid **1**.

Parametric equations (in parameters u, v) of the kinematical ruled surface, generated by one of the generating lines of moving cylinder **3** in the fixed coordinate system oxyz are:

$x = c\cos\varphi\cos u - (R + c\sin\varphi)\sin u$;

$y = c\cos\varphi\sin u + (R + c\sin\varphi)\cos u$;

$z = v$,

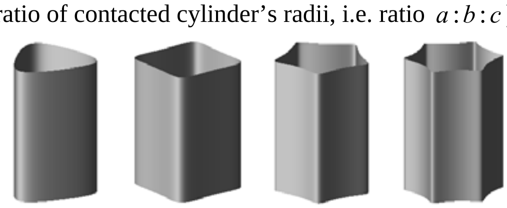
where $R = a + 2b + c$, $\varphi = -(a/c)u$,

a – radius of fixed cylinder **1**,

b – radius of moving cylinder **2**,

c – radius of moving cylinder **3**.

Examples of the computer visualization of rotational ruled surfaces constructed on the base of the triad of contacted circular cylinders are shown in Fig. 7 (ratio of contacted cylinder's radii, i.e. ratio $a:b:c$).



(3:1:1) (4:1:1) (5:1:1) (6:1:1)

Figure 7

2.4 Model of triad "circular cone – circular cone – circular cone"

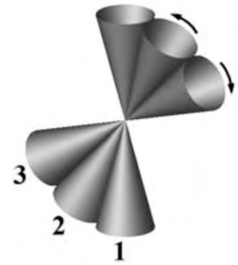


Figure 8

In the triad of contacted circular cones (Fig. 8) cone **1** is a fixed axoid. The fixed cone's axis is lying in the axis *oz* of the fixed coordinate system *oxyz*, connected with fixed axoid **1** (the origin of the coordinate system *oxyz* is located in the vertex of fixed cone **1**). By perfect analogy with the triad of contacted circular cylinders described above (Part 2.3), the movement of cone **2** along fixed cone **1** is accompanied by the backward motion of cone **3** along cone **2** so that the axis of moving cone **3** is located in the common plane with both axis of cone **2** and axis of cone **1** right along during interrelated movements in this triad of contacted cones (Fig. 8).

Parametric equations (in parameters *u*, *v*) of the rotational ruled surface, generated by one of the generating lines of moving cone **3** in the fixed coordinate system *oxyz,* are:

$x = X\cos u - (Y\cos\theta - Z\sin\theta)\sin u$;

$y = X\sin u + (Y\cos\theta - Z\sin\theta)\cos u$;

$z = Y\sin\theta + Z\cos\theta$,

where

$X = v\sin\alpha_3\cos\varphi$;

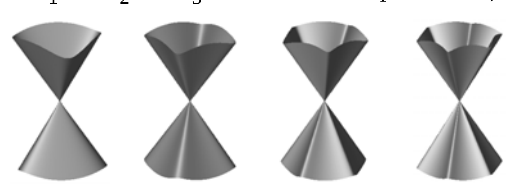
$Y = v\sin\alpha_3\sin\varphi$;

$Z = v\cos\alpha_3$;

$\theta = \alpha_1 + 2\alpha_2 + \alpha_3$; $\varphi = -(\sin\alpha_1/\sin\alpha_3)u$

(α_1 , α_2 , α_3 – angles between the cone's generating line and cone's axis for cones **1**, **2**, **3** accordingly).

Examples of the computer visualization of rotational ruled surfaces constructed on the base of the triad of contacted circular cones are shown in Fig. 9 ($\sin\alpha_1 : \sin\alpha_2 : \sin\alpha_3$ - ratio of cone's parameters).



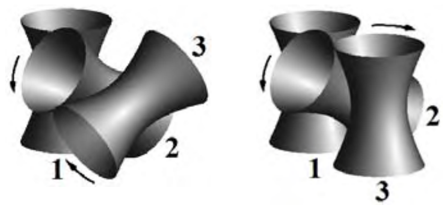
(3:1:1) (4:1:1) (5:1:1) (6:1:1)

Figure 9

3. KINEMATIC RULED SURFACES BASED ON TRIADS OF ONE-SHEET HYPERBOLOIDS OF REVOLUTION

3.1 Geometrical models of triads of one-sheet hyperboloids of revolution

Two possible variants of positional relationship of contacted axoids **1, 2, 3** in the matched triads of one-sheet hyperboloids of revolution are shown in Fig. 10. One-sheet hyperboloid of revolution **1** is a fixed axoid in both configuration variants of triads of contacted axoids.



Variant 1 Variant 2

Figure 10

In correspondence with the proposed model of interrelated movements in the triad of contacted axoids, as the base of constructing kinematical ruled surfaces, the movement of axoid **2** along fixed axoid **1** is accompanied by the backward motion of axoid **3** along axoid **2** (as it is shown in Fig. 10), so the positional relationship of contacted one-sheet hyperboloids of revolution **1, 2, 3** is fixed during interrelated movements in this triad of axoids. The case when interrelated movements in this triad of contacted axoids are realized, so that the center of the waist circle of moving axoid **3** is located in the common line with centers of waist circles of both moving axoid **2** and fixed axoid **1** right along during interrelated movements in this triad, has been described in this research. It is necessary to notice here that the main condition of constructing kinematical ruled surfaces based on the pairs of contacted axoids (Fig. 11) is that the moving axoid contacts with the fixed axoid along their common generating line in each of their positions during the movement of one axoid along another.

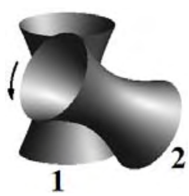


Figure 11

In the cases described above (Parts 2.1–2.4) such moving as rolling one axoid along another is sufficient to meet this main condition. At the same time such moving as rolling one axoid along another in the case of one-sheet hyperboloid of revolution as fixed and moving axoids is insufficient to meet the main condition of constructing kinematical surfaces. However, as it follows from the earlier research [Con09a], the task of constructing kinematical ruled surfaces moves in this case to feasible solution on the base of *complex moving* one axoid along another. *Complex moving* is a combination of several concerted movements of one axoid along another.

In the case of the pair of one-sheet hyperboloids of revolution (as fixed and moving axoids), the geometrical model of *complex moving* one axoid along another as the base of constructing kinematical ruled surfaces can be represented as a superposition of three interrelated movements [Kri15a]:

(1) rotational movement of the moving axoid around its axis;

(2) rotational movement of the moving axoid's axis around the fixed axoid's axis;

(3) translational movement of the moving axoid along the common generating line of both axoids.

Besides, as it was determined in the earlier research [Con09a], for fulfillment of the main condition of constructing kinematical ruled surfaces based on the *complex moving* one axoid along another in the case of the pair of different contacted one-sheet hyperboloids of revolution, the parametric condition

$$a_1^2 + c_1^2 = a_2^2 + c_2^2$$

for the matched pair of contacted axoids must be in progress.

Parameters a_1, c_1 and a_2, c_2 are parameters of the canonical equation of the matched pair of fixed (**1**) and moving (**2**) axoids accordingly.

(The canonical equation of the one-sheet hyperboloid of revolution [Kor61a]:

$\frac{x^2}{a^2} + \frac{y^2}{a^2} - \frac{z^2}{c^2} = 1$, where a – radius of waist circle).

Consequently, in the case of interrelated movements in the triad of contacted one-sheet hyperboloids of revolution (Fig. 10) as the base of constructing kinematical ruled surfaces, *complex moving* of axoid **2** along fixed axoid **1** must be accompanied by the backward *complex moving* of axoid **3** along axoid **2** as it is shown in Fig. 10.

In addition, for triads of different contacted one-sheet hyperboloids of revolution the parametric condition

$$a_1^2 + c_1^2 = a_2^2 + c_2^2 = a_3^2 + c_3^2$$

for matched triads of contacted axoids must be in progress (a_1 c_1, a_2, c_2 and a_3, c_3 are parameters of the canonical equation of matched triad of fixed (**1**) and moving (**2, 3**) axoids accordingly).

3.2 Analytical representation and computer visualization of new constructed kinematical ruled surfaces

In the geometrical model of the triad of contacted one-sheet hyperboloids of revolution (Fig. 10) the axis of fixed axoid **1** is lying in the axis oz of the fixed coordinate system oxyz, connected with fixed axoid **1** (the origin of the coordinate system oxyz is located in the center of the waist circle of fixed one-sheet hyperboloid of revolution **1**).

As a result of successive transformations of coordinates, the parametric equations (in parameters u, v) of a new kinematical ruled surface, generated by one of the generating lines of moving axoid **3** in the fixed coordinate system oxyz are:

$x = (X\cos\theta + Z\sin\theta)\cos u - (a_1 + 2a_2 + a_3 + Y)\sin u$;

$y = (X\cos\theta + Z\sin\theta)\sin u + (a_1 + 2a_2 + a_3 + Y)\cos u$;

$z = -X\sin\theta + Z\cos\theta$, where

$X = -a_3\sin\varphi + a_3 v\cos\varphi$;

$Y = a_3\cos\varphi + a_3 v\sin\varphi$;

$Z = c_3 v$;

$\varphi = -(a_1/a_3)u$;

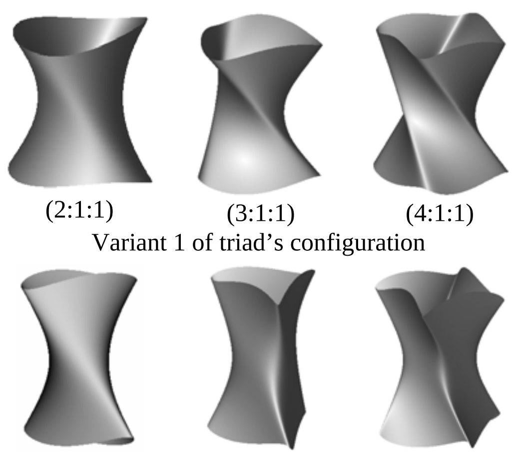
$\theta = \theta_1 + 2\theta_2 + \theta_3$ (Variant 1 in the Fig. 10),

$\theta = \theta_1 - \theta_3$ (Variant 2 in the Fig. 10);

$\theta_1 = arctg(a_1/c_1)$; $\theta_2 = arctg(a_2/c_2)$;

$\theta_3 = arctg(a_3/c_3)$.

Examples of the computer visualization of kinematical ruled surfaces, constructed on the base of both variant 1 and variant 2 (Fig. 10) of triad's configurations of contacted one-sheet hyperboloids of revolution, are shown in Fig. 12 (ratio of waist circles radius of axoids **1, 2, 3** as $a_1 : a_2 : a_3$).



(2:1:1) (3:1:1) (4:1:1)
Variant 1 of triad's configuration

(2:1:1) (3:1:1) (4:1:1)
Variant 2 of triad's configuration

Figure 12

Computer representation of figures for triads of contacted axoids and computer construction of new kinematical ruled surfaces has been realized by the previously developed software application AMG ("ArtMathGraph") [Con07a].

4. CONCLUSIONS

Thus, the new geometrical model for constructing kinematical ruled surfaces based on interrelated movements in triads of contacted axoids is developed in this research. On the base of this model, the analytical representation and computer visualization of new constructed kinematical ruled surfaces is realized for some cases of triads, so as "plane – cone – cone", "cylinder – cylinder – cylinder", "cone – cone – cone", and the matched triad of one-sheet hyperboloids of revolution. The new proposed geometrical model in the combination with the graphic ability of the previously developed software application gives improved opportunity for computer search of desirable kinematical ruled surfaces.

5. REFERENCES

[Spr02a] Sprott, K., Ravani, B. Kinematic generation of ruled surfaces. Advanced in Computational Mathematics, 17: 115-133, 2002.

[Kri06a] Krivoshapko, S.N., Ivanov, V.N. Khalabi, V.N. Analytical Surfaces. Nauka, Moscow, Russia, 2006, 544 p.

[Con07a] Rachkovskaya, G.S., Kharabayev, Yu.N., and Rachkovskaya, N.S. Computer composition of the transformed classical surfaces as the ways and means of the construction of visual models of realistic objects (The new software application "ArtMathGraph") Proceedings of the 15-th International Conference in Central Europe on Computer Graphics, Visualization and Computer Vision 2007, Plzen, Czech Republic, p.p. 29-32.

[Con09a] Rachkovskaya, G.S., Kharabayev, Yu.N. Geometric modeling and computer graphics of kinematic ruled surfaces on the base of complex moving one axoid along another (one-sheet hyperboloid of revolution as fixed and moving axoids). Proceedings of the 17-th International Conference in Central Europe on Computer Graphics, Visualization and Computer Vision 2009, Plzen, Czech Republic, p.p.31-34.

[Kri15a] Krivoshapko, S.N., Ivanov, V.N. Encyclopedia of Analytical Surfaces. Switzerland, Springer, 2015, 752 p.

[Kor61a] Korn, G.A. and Korn, T.M. Mathematical handbook for scientists and engineers. McGraw-Hill, NY, USA, 1961, 720 p.