

## On approaches, methods and problems related to wave dispersion in porous media

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The paper is devoted to modelling of wave propagation in fluid saturated porous media. The aim is to compare two approaches suitable for wave analysis in periodic structures. The homogenization-based method provides macroscopic models and relies on scale separation, such that the wave length should be much longer than the heterogeneity size. The method based on the Bloch-Floquet theory is applicable for analyzing plane wave propagation in infinite continua only, but enables to capture also effects of Bragg's scattering. Application of these two methods for computing the dispersion curves of the two-phase media with deformable and rigid skeletons is discussed in the context of two kinds of porous media: 1) deformable fluid-saturated porous media described by the Biot model at the mesoscopic scale, 2) waves in fluids saturating rigid porous structures. Computational studies based on the Bloch wave decomposition were performed for numerical models obtained using the finite element discretization of the corresponding problems imposed in the representative volume element. The results presented in terms of dispersion curves are compared with analogous results obtained using the homogenized models providing directly the shear and pressure wave responses.

*Waves in deformable fluid-saturated porous media.* The mathematical model governing the wave propagation in fluid-saturated porous media (FSPM) made of an elastic solid skeleton whose pores constitute a connected pore network was first proposed by Biot [2]. It is well known that, in (mesoscopically) homogeneous media in  $\mathbb{R}^n$ , there are  $n + 1$  principle modes of plane waves propagating as the  $n - 1$  shear waves (S), fast quasi-compressional waves (P1) and slow quasi-compressional waves (P2), cf. [7] and [6], where the homogenization based approach was reported. For the rigid double porosity media, the dispersion properties of the homogenized model were considered in [3], whereas the single porosity model was obtained in [4]. In this paper, we consider FSPM governed by the Biot model relevant to a mesoscopic scale whereby the material coefficients oscillate with a given spatial period. To analyze the wave propagation in such media, as an alternative to the homogenization based approach, we have derived a formulation based on the Bloch wave decomposition which yields a quadratic eigenvalue problem for complex wave numbers within the first Brillouin zone associated with the periodic structure.

*Waves in fluid saturating rigid periodic scaffolds.* We consider acoustic waves in a compressible fluid saturating rigid porous periodic structure. Models of homogenized media, cf. [1], were derived for two types of fluids and flow regime: a) acoustic fluid (inviscid), b) viscous fluid, but no convection flow. Obviously the case a) leads to the homogenization of the Helmholtz equation. Using homogenization in the case b) leads to the dynamic permeability

For these kinds of homogenized media, the dispersion curves are presented and compared with corresponding results computed using the Bloch wave decomposition.

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### **References**

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