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Torsion dissipated energy of hard rubbers as function of hyperelastic deformation energy of the Yeoh model

P. Šulc[a,*], L. Pešek[a], V. Bula[a], J. Košina[a], J. Cibulka[a]

[a] *Institute of Thermomechanics of the CAS, v. v. i., Dolejškova 1402/5, 182 00 Prague, Czech Republic*

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Abstract

In this paper, we are proposing a new formulation of dissipated energy of hard rubbers as a function of the deformation energy expressed by the Yeoh hyperelastic model. Torsion deformation is considered as a planar deformation of a simple shear on the surface of a cylinder. Thus the deformation energy is dependent only on the first invariant of strain. Based on the experiment, a "hyperelastic proportional damping" (HPD) is proposed for hard rubbers under finite strains. Such damping is analogical to the model of proportional damping in the linear theory of viscoelasticity, i.e. the dissipated energy is proportional to the deformation energy multiplied by the frequency of dynamic harmonic loading. To obtain the experimental data, samples of hard EPDM rubbers of different harnesses were dynamically tested on a torsional test rig for different frequencies and amplitudes. The Yeoh model is chosen since the deformation function is dependent only on the first strain invariant for the description of the simple shear of a surface cylinder. The Yeoh constants are evaluated by curve fitting of the analytical stress function to the experimental torsion stress-deformation curve. The constants are used to express the deformation energy of the Yeoh model for specific cases of tested rubbers. The coefficients of hyperelastic proportional damping are evaluated on the basis of experimental results.

Keywords: first strain invariant, simple shear, Yeoh model, dissipated energy, deformation energy

1. Introduction

Unlike conventional structural materials, rubber materials under dynamic loading exhibit a non-linear time-varying behaviour due to the size of straining, creep, temperature and aging [8] and [9]. Tests of rubbers with higher hardness Sh 50-80 [5] were performed in the laboratories of IT AS CR in recent years [10]. The dynamic tests of hard rubbers require usually a costly long-term operation of heavy hydraulic machines. Therefore we have started to look for realization of the tests in laboratory conditions with the lighter laboratory technique. Currently we have been developing a torsional dynamic test rig for torsional straining of hard rubber samples with a circular cross-section [12]. The reason for torsion straining was that hard rubber materials are softer in torsion than in pressure and therefore it is easier to achieve larger strains. Furthermore, the shape changes are smaller in this case compared to pressure loading when so-called barrelling effect arises due to the incompressibility of the material.

The torsional test rig should serve to dynamic material tests of hard synthetic rubbers for determination of the thermo-viscous-elastic material characteristics under small as well as finite strains, different amplitudes, frequencies and temperatures. To describe the stress of the rubber at torsional load we used its analogy with simple shear stress on the surface of a cylinder sample.

*Corresponding author. Tel.: +420 266 053 121, e-mail: sulc@it.cas.cz.
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This paper deals with the deformation analysis of cylindrical samples at larger shear strains (about 30%) which led to finding the relationship between dissipated energy and strain energy at larger torsion deformations. The tested rubbers are a viscoelastic material and we propose to model its dissipation energy on the basis of excitation frequency and strain level using deformation energy defined by the theory of hyperelasticity. Therefore the function of energy dissipation of hard rubbers under finite strains is defined as a hyperelastic proportional damping (HPD) similar to modelling of damping in the linear theory of viscoelasticity. So, the dissipated energy is expressed as the product of the function of deformation energy, excitation frequency and coefficient of HPD. Nowadays, hyperelastic materials, such as elastomers, are extensively employed in a wide range of applications.The coefficients of HPD relating a dissipated energy to a strain energy were then evaluated for tested rubbers on the basis of experimental results. The constitutive relation for an incompressible, homogeneous and isotropic material at finite strains can be derived from a strain energy density function of hyperelastic models. These hyperelastic models considered as isotropic incompressible materials are used for the description of the deformation energy.

For the material testing, we choose the temperature of $-20\,^{\circ}$C which makes the influence of the amplitudes of the excitation torque on the values of the loss factor most evident. We tested two samples of isoprene butadiene rubber (EPDM) of different hardness (Sh70 and Sh85) with excitation amplitudes of torque moment in a range from 0.2 Nm to 9.2 Nm. Excitation frequencies were 2 Hz and 5 Hz. Stress-strain curves, deformation and dissipated energies were evaluated for different amplitudes of the excitation torque moment.

2. Green's approach to deformation expression

Torsion straining of the cylindrical surface is analogous to the straining in a simple shear. For simple shear deformation we express the strains in the current description

$$\begin{aligned} x_1 &= X_1 + \gamma X_2, \\ x_2 &= X_2, \\ x_3 &= X_3, \end{aligned} \tag{1}$$

where $x_{1,2,3}$ are the current coordinates, $X_{1,2,3}$ are the reference coordinates and γ is the shear strain of a segment of the cylinder. Assuming the state of plane strain, the principal strains λ_i on the surface of the cylinder are given by

$$\lambda_1 = \sqrt{1 + \gamma^2 + \gamma\sqrt{1 + \frac{\gamma^2}{4}}}, \qquad \lambda_2 = \sqrt{1 + \gamma^2 - \gamma\sqrt{1 + \frac{\gamma^2}{4}}}, \qquad \lambda_3 = 1, \tag{2}$$

where must be valid that $\lambda_1 > 1$ and $\lambda_1 > \lambda_3 > \lambda_2$. The shear size of the main strains is equal $\gamma = \lambda_1 - 1/\lambda_1$ [7].

Using equation (1), the deformation gradient tensor for simple shear can be expressed as

$$\mathbf{F} = \begin{pmatrix} 1 & \gamma & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}. \tag{3}$$

The left Cauchy Green strain $\mathbf{B}$ is determined from the deformation gradient as

$$\mathbf{B} = \mathbf{F}\mathbf{F}^{\mathrm{T}} = \begin{pmatrix} 1+\gamma^2 & \gamma & 0 \\ \gamma & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \tag{4}$$

and the principal invariants of $\mathbf{B}$ are given by

$$\begin{aligned} I_1 &= tr\mathbf{B} = \gamma^2 + 3, \\ I_2 &= \frac{1}{2}\left[(tr\mathbf{B})^2 - tr\mathbf{B}^2\right] = \gamma^2 + 3, \\ I_3 &= 1. \end{aligned} \tag{5}$$

The constitutive relation for an incompressible, homogeneous and isotropic material at finite strains can be derived from a strain energy density function W. This function can be expressed as a function of I_1 and I_2, i.e. $W(I_1, I_2)$. Then the constitutive relation for Cauchy stress is expressed in principal invariants as

$$\boldsymbol{\sigma} = -p\mathbf{I} + 2\frac{\partial W}{\partial I_1}\mathbf{B} - 2\frac{\partial W}{\partial I_2}\mathbf{B}^{-1}, \tag{6}$$

where p is the Lagrange multiplier also called hydrostatic pressure, which is associated with material incompressibility.

It has been reported in the literature devoted to hyperelasticity that the stored energy is strongly dependent on the first invariant. Materials characterized by such behaviour are called generalized neo-Hookean materials. Several strain-energy density functions defined by $W = W(I_1)$ have been proposed by Gent [2], Arruda and Boyce [1], Wineman [13], Horgan and Saccomandi [3] and Lopez-Pamies [4]. We assume herein the dependence of the strain energy density function only on the first invariant, i.e. $W = W(I_1)$ with respect to $I_1 = I_2$. Then the Cauchy stress tensor is expressed as

$$\boldsymbol{\sigma} = -p\mathbf{I} + 2\frac{\partial W}{\partial I_1}\mathbf{B}. \tag{7}$$

The Cauchy stress components for simple shear can be written according to (7) as

$$\begin{aligned} \sigma_{11} &= -p + 2(1+\gamma^2)\frac{\partial W}{\partial I_1}, & \sigma_{22} &= -p + 2\frac{\partial W}{\partial I_1}, \\ \sigma_{33} &= -p + 2\frac{\partial W}{\partial I_1}, & \sigma_{12} &= 2\gamma\frac{\partial W}{\partial I_1}. \end{aligned} \tag{8}$$

The Yeoh hyperelastic model was chosen for the description of the density of deformation energy since this model depends only on the first strain invariant. For the studied case of a simple shear, the six-parametric model was most suitable, so

$$W = \sum_{i=1}^{6} C_{i0}(I_1 - 3)^i, \tag{9}$$

where W is the strain energy density, C_{i0} are constants and I_1 represents the invariant of the left Green strain tensor [6]. Substituting (9) into (8) and after some algebra, the Cauchy shear stress σ_{12} can be written in the form

$$\begin{aligned} \sigma_{12} = 2\gamma \big(&C_{10} + 2C_{20}(I_1 - 3) + 3C_{30}(I_1 - 3)^2 + \\ &4C_{40}(I_1 - 3)^3 + 5C_{50}(I_1 - 3)^4 + 6C_{60}(I_1 - 3)^5\big). \end{aligned} \tag{10}$$

The unknown constants C_{i0} of the Yeoh model are then evaluated using the experimental shear stress-strain curves by means of the least square method (LSM) in the form

$$\mathbf{t} \approx \mathbf{A}\mathbf{c}, \tag{11}$$

where $\mathbf{t} = [\sigma_1, \ldots, \sigma_n]^{\mathrm{T}}$ is the vector of experimental shear stresses and the matrix $\mathbf{A}$ consists of the coefficients of σ_{12} given by (10) for different values of measured γ. The vector $\mathbf{c} = [C_{10}, \ldots, C_{60}]^{\mathrm{T}}$ contains six unknown constants of the Yeoh model. By use of the LSM we get the solution as

$$\mathbf{c} = (\mathbf{A}^{\mathrm{T}}\mathbf{A})^{-1}\mathbf{A}^{\mathrm{T}}\mathbf{t}. \tag{12}$$

3. Experimental observations

The shear modulus and the loss factor for each of torque harmonic excitation amplitudes were evaluated from experimental hysteresis of deformation loops. The loss factor is a material constant that measures a dissipated energy converted to heat during a harmonic cycle. The methodology of the evaluation is presented in [10,12]. The dependences of the shear modulus on strain is shown on Fig. 1. The hyperbolic softenings can be seen for both excitation frequencies and both rubber samples.

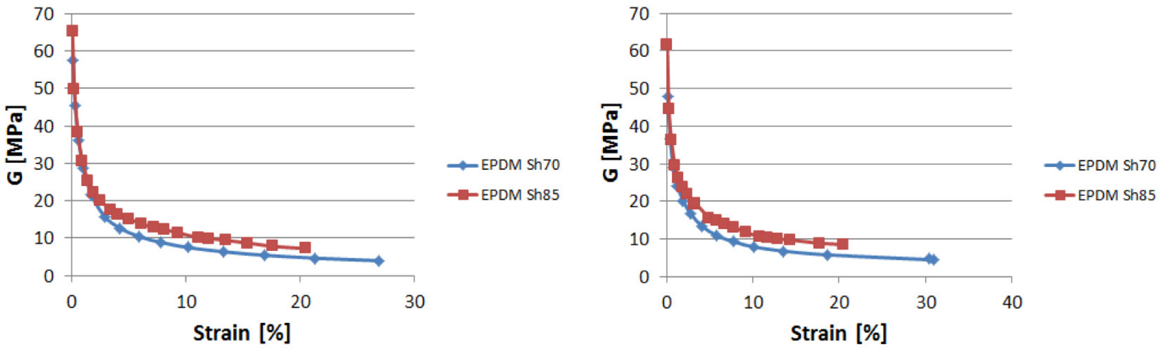

Fig. 1. The dependence of the shear modulus on strain at the temperature −20 °C for two excitation frequencies 2 Hz (left) and 5 Hz (right)

The dependencies of loss factor on strain [%] are plotted for two excitation torque frequencies (2 Hz, 5 Hz) in Fig. 2. From these dependencies, it is seen that the ratio between deformation and dissipation energy for higher values of strains is almost constant, which is typical for the proportional damping model ($B = \beta K$) for a linear viscoelastic material. Furthermore, the dependences of loss factor on the shear modulus presented in Fig. 3 show that the loss factor and therefore the dissipated energy decrease with increasing modulus for larger deformations. It means that the dissipated energy is also related to the deformation energy. Based on these observations we proposed (see below) a function of dissipation energy of hard rubbers for finite strains analogically as the proportional damping for the linear theory of viscoelasticity and extended for hyperelasticity.

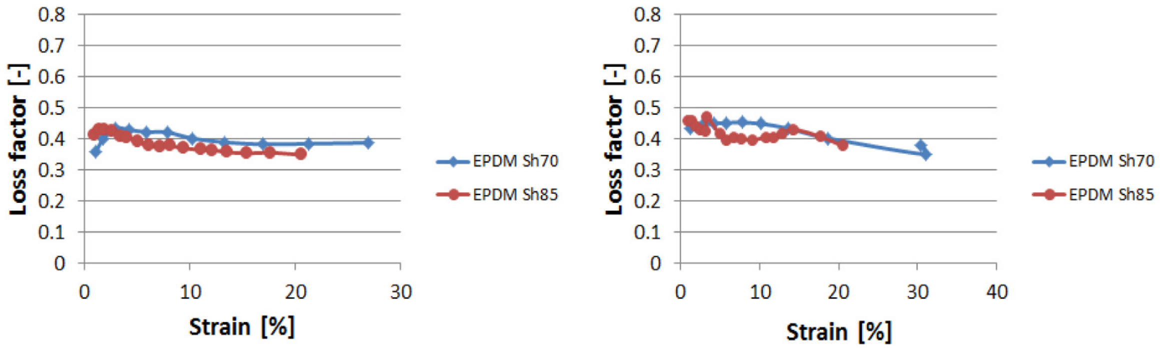

Fig. 2. The loss factor versus strain for two excitation frequencies 2 Hz (left) and 5 Hz (right) – temperature −20 °C

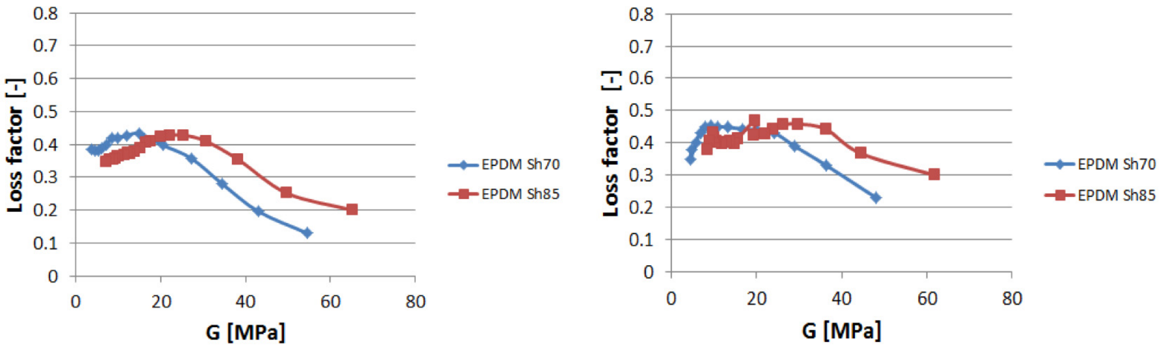

Fig. 3. The dependence of the loss factor on the shear modulus at the temperature $-20\,^\circ$C for two excitation frequencies 2 Hz (left) and 5 Hz (right)

4. Deformation energy of Yeoh hyperelastic model

To evaluate the deformation energy analytically we needed first tune the Yeoh constants to experimental shear stress-strain curve. Experimental shear stress-strain curves of selected hard rubbers were obtained by our torque test rig for a temperature $-20\,^\circ$C. The test specimen was of cylindrical shape glued at both heads to steel consoles pins mounting in the collets. By usage of the consoles the test sample is not deformed in the vicinity of the heads due to clamping. The dimensions of the test sample of rubber were: $\varnothing D = 0.03$ m, length $L = 0.095$ m.

As mentioned previously, the material isoprene butadiene rubber (EPDM) of hardness Sh70 and Sh85, the test temperature $-20\,^\circ$C and the frequency 2 Hz of torsional loading were used at first. Experimental curves of shear stress τ vs. strain (skew) γ, where the skew maximums were about 30% (sample Sh70) and 20% (sample Sh85), are shown in Fig. 4. The determined six material parameters of the Yeoh model based on LSM method [11] determined by using (10)–(12) are following:

Rubber EPDM Sh70

$C_{10} = 8.612\,9 \cdot 10^6$ Pa, $C_{20} = -5.514\,4 \cdot 10^8$ Pa, $C_{30} = 2.792\,3 \cdot 10^{10}$ Pa, $C_{40} = -7.281\,4 \cdot 10^{11}$ Pa, $C_{50} = 8.998 \cdot 10^{12}$ Pa, $C_{60} = -4.135 \cdot 10^{13}$ Pa.

Rubber EPDM Sh85

$C_{10} = 1.063 \cdot 10^7$ Pa, $C_{20} = -6.720\,7 \cdot 10^8$ Pa, $C_{30} = 4.406\,6 \cdot 10^{10}$ Pa, $C_{40} = -1.604\,7 \cdot 10^{12}$ Pa, $C_{50} = 2.912\,9 \cdot 10^{13}$ Pa, $C_{60} = -2.058\,1 \cdot 10^{14}$ Pa.

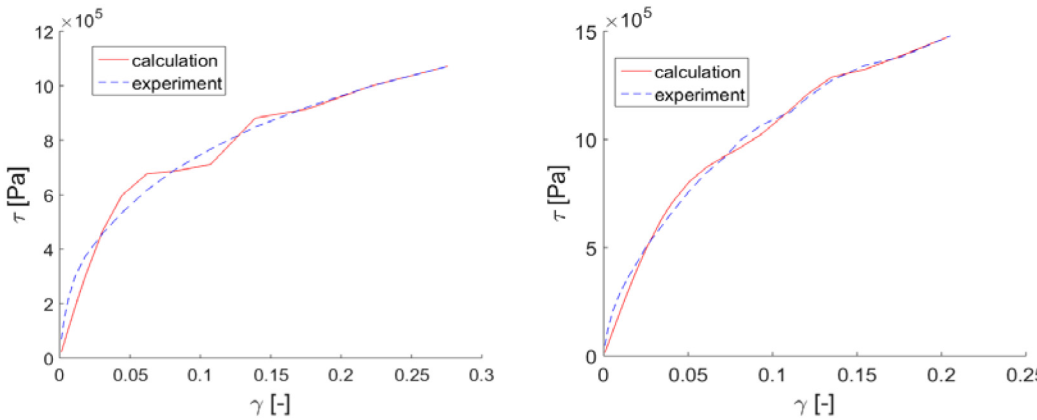

Fig. 4. Deformation curves of experiment and the six-parametric Yeoh model for EPDM Sh70 (left) and for EPDM Sh85 (right) for the frequency 2 Hz

As mentioned previously, based on experimental observations, the dissipation energy was proposed as a HPD similar to the modelling of damping in the theory of viscoelasticity. The dissipated energy U_{Dis} is then expressed as a product of the coefficient of HPD β, excitation frequency ω and deformation energy U_{Def} coming from the hyperelastic Yeoh model with six parameters:

$$U_{Dis} = \beta \cdot \omega \cdot U_{Def}. \tag{13}$$

The experimental dissipation energy was obtained from the area of the hysteresis deformation loops for each case of the excitation torque moment. The computed dissipation energies in dependence on strain for both rubbers are presented in Fig. 5. It shows the same nonlinear dependence of dissipated energy as in the case of its dependence on strain, as clear from the relation (15), see below.

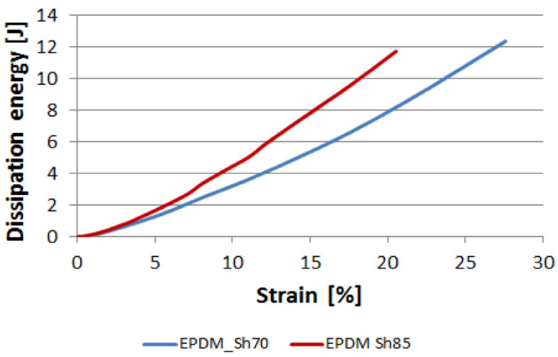

Fig. 5. The dissipated energy versus strain for EPDM Sh70 and EPDM Sh85 rubbers – temperature $-20\,°$C, excitation frequency 2 Hz

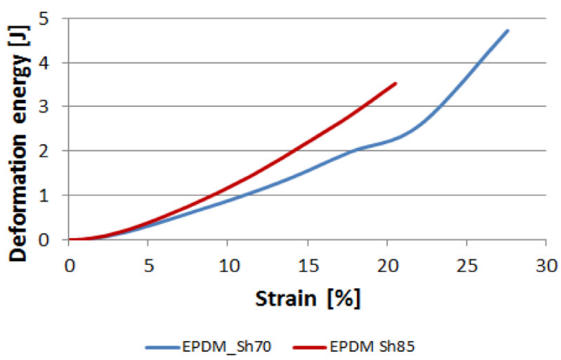

Fig. 6. The deformation energy of the Yeoh models vs. strain for EPDM Sh70 and EPDM Sh85 rubbers – temperature $-20\,°$C, excitation frequency 2 Hz

Total deformation energy was analytically evaluated from the deformation energy density u_{Def} multiplied by the specific torque volume V_{red}. The deformation energy density is calculated directly from the deformation energy density of the Yeoh model as

$$u_{Def} = C_{10}(I_1-3)+C_{20}(I_1-3)^2+C_{30}(I_1-3)^3+C_{40}(I_1-3)^4+C_{50}(I_1-3)^5+C_{60}(I_1-3)^6. \tag{14}$$

Then the total deformation energy is calculated from this relationship

$$U_{Def} = u_{def} \cdot V_{red}. \tag{15}$$

The total deformation energies calculated for EPDM Sh70 and EPDM Sh85 rubbers modelled by using the Yeoh six-parametric models are shown in Fig. 6. It is clear that the higher hardness of the rubber is, the higher deformation energy is obtained.

The dependence of the total deformation of the Yeoh model for the EPDM Sh85 rubber and for the both used frequencies, i.e. 2 Hz and 5 Hz, can be seen in Fig. 7. It shows that the frequency dependence is very weak at torsional harmonic load with frequencies up to 5 Hz. The constants $C_{10}, \ldots, C_{60}$ were obtained for the excitation frequency 5 Hz in the the same way as for the frequency 2 Hz.

The HPD coefficients β evaluated from (13) in dependence on strain are presented for both rubbers in Fig. 8. It is obvious that the coefficient is almost the same for both tested rubbers and it remains almost constant with the value of strain.

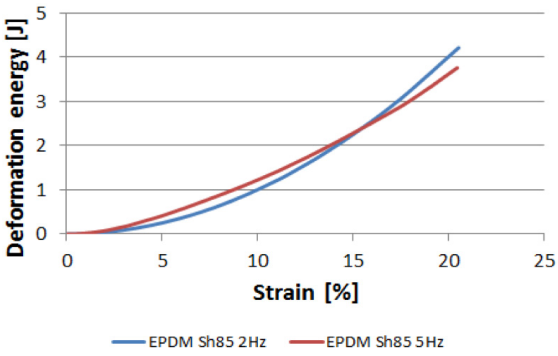

Fig. 7. The Yeoh deformation energies vs. strain for the EPDM Sh85 rubber and for excitation frequencies 2 Hz and 5 Hz and for temperature −20 °C

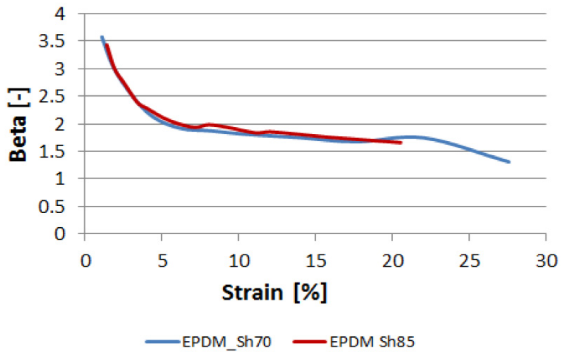

Fig. 8. The hyperelastic proportional damping coefficient β vs. strain for EPDM Sh70 and EPDM Sh85 rubbers – temperature −20 °C, excitation frequency 2 Hz

5. Conclusion

Two EPDM hard rubbers were tested on our torque test rig in order to determine their material constants and damping behaviour under different amplitudes and frequencies of harmonic excitations and under finite shear deformations. First, the shear modulus and the loss factor were evaluated for each excitation setting. Based on the experimental observations, the model of "hyperelastic proportional damping" was proposed here and the unknown coefficients of HPD were identified for both rubbers. The results of the coefficient values show that the coefficient is almost the same for both tested rubbers and it remains almost constant (independent from strain). Since the harder tested rubber has higher deformation energy, also the dissipation energy is higher. The reason is that it is related to the deformation energy by the proportional damping expression. For the simple torsion case, the shear stress expressed as a function of the first invariant of strain was in a very good agreement with the experimental results. In order to obtain a good estimation of the constants of the Yeoh model, a sufficient number of experimental data points at each load step, i.e. amplitudes and frequencies, are required. The results were obtained for one selected temperature. Due to the strong temperature dependence of hard rubber, both its dissipation and its strain energy will change with the temperature. Therefore, in order to obtain the temperature dependence of the HPD coefficients they should be evaluated separately for each temperature.

Our future research will focus on the model of dissipation. The Yeoh model will be implemented into our in-house finite element code. It will allow us to simulate the thermo-dynamic behaviour of the rubber dampers of more complicated shapes, states of stress and finite deformations.

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