Solving of the Switched Circuits by Combination Diakoptic Methods and by Combined Transformation Method

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Abstract – **This paper deals with application of the diakoptic method in analyse of the switched circuits. There are a few methods for assembling of the matrix which is circuit described. Combination diakoptic method which combine the combined transformation method and the decomposition of the reduction formula into the capacitance matrix is described below. A simple illustrative example is given, too.**

Keywords- Switched circuits, diakoptic method, matrix method, biphase switching

I. Introduction

Many methods have been developed to analyse switched circuits [1], [5], [6], [11], [12]. There are many methods to derive the capacitance matrix describing the switched capacitor circuits in the two-phase switching, too. Some of them are compiled resultant matrix which is subsequently reduced which is very unclear. Other methods compiled the resulting matrix of reduced matrices. Therefore, it is necessary to decompose the reducing relationship. One possibility of the decomposition is the application of the sum matrices, the other use of classical diakoptic methods. Decomposition can be derived in different ways again. Some methods originated with diakoptics which was introduced by Kron. A common problem of the decomposition methods is the complexity of the algorithms and the high level of the abstraction used.

Diakoptic method decomposes the circuit into several subcircuits. Each subcircuit is described by its matrix $\mathbf{Y_i}$ [4]. The resulting matrix is composed of sub-matrices (for example) $\mathbf{Y_1}$, $\mathbf{Y_2}$ connected by the incidence matrices $\mathbf{I_1}$, $\mathbf{I_2}$ (1).

$\mathbf{Y}_1$	$\mathbf{I}_1$
$\mathbf{I}_2$	$\mathbf{Y}_2$

(1)

While nonregular elements reduce each of the submatrices separately (2)

$$\mathbf{Y}_1 = \widetilde{\mathbf{Y}}_1^{[(+)>(-):o>0]} \qquad (2)$$
$$\mathbf{Y}_2 = \widetilde{\mathbf{Y}}_2^{[Z>X:X>Y]}$$

as is shown in Figure1 for example for VFA and CCII+, reduction relationship is compiled for each of the submatrix separately. It means, that is subsequently decomposed between all submatrices.

The conductivity matrix **G** and/or capacitance matrix **C** is used to solve of the switched circuits.

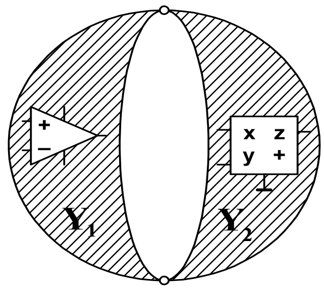

Figure 1. The circuit is decomposed into two subcircuits

II. Assembly of the Reduction Relation

The reduction formula is assembled using methods combined transformation [5], which is described the effect of switches and operational amplifiers. Consider Fig.2, where the phase are marked as odd (O) and even (E), while the nodes are numbered as 1, 2, 3.

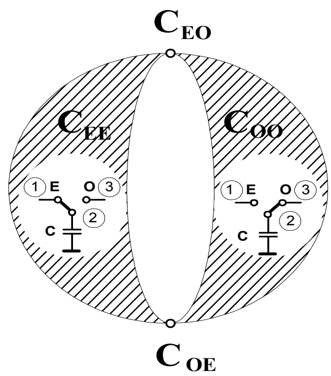

Figure 2. Switched circuit for example

In E phase are connected nodes 1. and 2., in O phase nodes 2. and 3. Thus reduction formula is compiled for integrated i.e. undecomposed circuit unlike diakoptics methods. This formula is in following form (3)

$$\mathbf{C} = \tilde{\mathbf{C}}^{[1E.=2E,2O.=3O.:1E.=2E,2O.=3O]} \tag{3}$$

where index before the colon is used to reduction of the rows and the index after colon is used to reduction of the columns of the capacitance matrix [2], [4], [5], [8], [10]. Consider SC circuit from Fig.3.

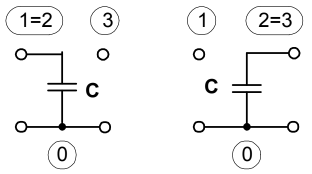

Figure 3. Circuit from Figure 2 in E and in O phase

In E phase are connected nodes 1. and 2., in O phase nodes 2. and 3. [12]. Thus reduction formula is compiled for integrated i.e. undecomposed circuit unlike diakoptics methods. This formula is in form (3), too.

The reduction formula is decomposed into partial formulas for each phase in follows step.

Consider now the circuit from Fig.2 in both phases E and O separately, as in shown in Fig.3.

In E phase are connected the nodes 1. and 2., therefore circuit in E phase can be described by reduction formula (4).

$$\mathbf{C}_{\mathbf{EE}} = \tilde{\mathbf{C}}_{\mathbf{EE}}^{[1E.=2E.:1E.=2E]} \tag{4}$$

In O phase are connected nodes 2 and 3, therefore circuit in o phase can be described by the reduction formula (5).

$$\mathbf{C}_{\mathbf{OO}} = \tilde{\mathbf{C}}_{\mathbf{OO}}^{[2O.=3O.:2O.=3O]} \tag{5}$$

Comparing the relationships of (3), (4), (5) is clear that the reduction relation (3) is decomposed into relations (4), (5) follows:

$$\begin{aligned} &\tilde{\mathbf{C}}^{[1E.=2E,2O.=3O.:1E.=2E,2O.=3O]} = \\ &= \tilde{\mathbf{C}}_{\mathbf{EE}}^{[1E.=2E.:1E.=2E]} + \tilde{\mathbf{C}}_{\mathbf{OO}}^{[2O.=3O.:2O.=3O]} \end{aligned} \tag{6}$$

As we can see, the diakoptic method is applied into reduction formula, not into schematic diagram of the circuit and/or into matrix by which is circuit described, the reduction formula is decomposed in this case.

The rule for the decomposition of the reduction relation follows from a comparison (6) of the relationships (3), (4), (5) follows: In sub-reduction formulas are used only indexes of this phase that are indexes of this submatrix, too. For example in submatrix $\mathbf{C}_{\mathbf{EE}}$ (where first index is E and second E) is used reduction of the rows (i.e. indexes before the colon are 1E, 2E) and the columns (i.e. indexes after colon are 1E, 2E), too. In submatrix $\mathbf{C}_{\mathbf{OO}}$ (where first index is O and second O) is used only reduction of the rows (i.e. indexes before the colon are 2O, 3O), and the columns (i.e. indexes after colon are 2O, 3O, too).

Relationships (4), (5) are describing transfer in E and O phase only. Transfer between phases E and O is described in general formula (1) by the two incidence matrices **I**. These matrices are $\mathbf{C}_{\mathbf{EO}}$ and $\mathbf{C}_{\mathbf{OE}}$ in relationship (7) where time different between phases E and O is described by the member $z^{-1/2}$.

$$\begin{bmatrix} \mathbf{Q}_{\mathbf{E}} \\ \mathbf{Q}_{\mathbf{O}} \end{bmatrix} = \begin{bmatrix} \mathbf{C}_{\mathbf{EE}} & -z^{-\frac{1}{2}}\mathbf{C}_{\mathbf{EO}} \\ -z^{-\frac{1}{2}}\mathbf{C}_{\mathbf{OE}} & \mathbf{C}_{\mathbf{OO}} \end{bmatrix} \cdot \begin{bmatrix} \mathbf{V}_{\mathbf{E}} \\ \mathbf{V}_{\mathbf{O}} \end{bmatrix} \tag{7}$$

Consider circuit from Fig.4.

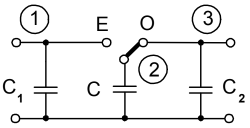

Figure 4. The circuit for example

Thus the matrix described circuit from Fig.4 is (8)

$$\tilde{\mathbf{C}} = \begin{bmatrix} C_1 & & \\ & C & \\ & & C_2 \end{bmatrix} \tag{8}$$

partial matrices after application (9)

$$\begin{aligned} &\mathbf{C} = \tilde{\mathbf{C}}^{[1E.=2E,2O.=3O.:1E.=2E,2O.=3O]} = \\ &= \tilde{\mathbf{C}}_{\mathbf{EE}}^{[1E.=2E.:1E.=2E]} + \tilde{\mathbf{C}}_{\mathbf{OO}}^{[2O.=3O.:2O.=3O]} + \\ &+ \tilde{\mathbf{C}}_{\mathbf{EO}}^{[1E.=2E.:2O.=3O]} + \tilde{\mathbf{C}}_{\mathbf{OE}}^{[2O.=3O.:1E.=2E]} \end{aligned} \tag{9}$$

are in form (10)

$$\begin{aligned} \mathbf{C}_{\mathbf{EE}} &= \begin{bmatrix} C_1 + C & \\ & C_2 \end{bmatrix} \\ \mathbf{C}_{\mathbf{OO}} &= \begin{bmatrix} C_1 & \\ & C + C_2 \end{bmatrix} \\ \mathbf{C}_{\mathbf{EO}} &= \begin{bmatrix} C_1 & C \\ & C_2 \end{bmatrix} \\ \mathbf{C}_{\mathbf{OE}} &= \begin{bmatrix} C_1 & \\ C & C_2 \end{bmatrix} \end{aligned} \tag{10}$$

Thus the resulting capacitance matrix **C** will be in form (11).

$$\left[\begin{array}{cc|cc} C_1 + C & 0 & -z^{-\frac{1}{2}}C_1 & -z^{-\frac{1}{2}}C \\ 0 & C_2 & 0 & -z^{-\frac{1}{2}}C_2 \\ \hline -z^{-\frac{1}{2}}C_1 & 0 & C_1 & 0 \\ -z^{-\frac{1}{2}}C & -z^{-\frac{1}{2}}C_2 & 0 & C + C_2 \end{array}\right] \tag{11}$$

Thus the general rule for the decomposition of the reduction relation is follows: Consider another reduction formula in general form (12).

$$\mathbf{C} = \tilde{\mathbf{C}}^{[\mathrm{aE}\rangle \mathrm{bE,cO>dO:aE}\rangle \mathrm{bE,cO>dO}]} = \\ = \tilde{\mathbf{C}}_{\mathbf{EE}}^{[\mathrm{aE}\rangle \mathrm{bE:aE}\rangle \mathrm{bE}]} + \tilde{\mathbf{C}}_{\mathbf{EO}}^{[\mathrm{aE}\rangle \mathrm{bE:cO>dO}]} + \\ + \tilde{\mathbf{C}}_{\mathbf{OE}}^{[\mathrm{cO>dO:aE}\rangle \mathrm{bE}]} + \tilde{\mathbf{C}}_{\mathbf{00}}^{[\mathrm{cO>dO:cO>dO}]} \tag{12}$$

In other words: in submatrix $\mathbf{C_{EE}}$ (where first index is E and second E) is used reduction of the rows aE, bE only (because the indexes before the colon are aE, bE), indexes cO, dO are not used (because index O is not used as an index in this submatrix). Second index of submatrix is E, then the indexes after colon are aE, bE only, indexes cO, dO are not used. In submatrix $\mathbf{C_{EO}}$ (where first index is E and second O) is used only index E for reduction of the rows (indexes before the colon are aE, bE**,** indexes cO, dO are not used), only index O for reduction of the columns is used (because only index O is an second index of matrix, therefore indexes cO, dO are used, indexes aE, bE are not used for reduction of the columns).

Decomposition formula is in next step applying to reduction of the capacitance and/or conductance matrix. Thus relationship is reducing in these cases. If circuit contains an operational amplifier, reducing relationship is given by the sum of the reduction by switch and by operational amplifier. Consider circuit from Fig.5.

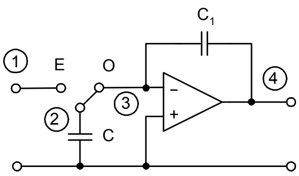

Figure 5. Circuit with switched capacitor and operational amplifier

In the even phase are connected nodes 1. and 2. (i.e. by switch will be 1E=2E:1E=2E), in odd phase are connected nodes 2. and 3. (i.e. by switch will be 2O=3O:2O=3O). Output node of operational amplifier is 4. (i.e. 4E>0, 4O>0:), input nodes is 3. (i.e. :3E>0,3O>0,2O>0). Node 1. is disconnected in odd phase (i.e.1O>0:1O>0). Thus reduction formula will be (13).

$$\tilde{\mathbf{C}}^{[1\mathbf{E}=2\mathbf{E},4\mathbf{E}>0,1\mathbf{O}>0,2\mathbf{O}=3\mathbf{O},4\mathbf{O}>0:1\mathbf{E}=2\mathbf{E},3\mathbf{E}>0,1\mathbf{O}>0,2\mathbf{O}>0,3\mathbf{O}>0]} = \\ = \tilde{\mathbf{C}}_{\mathbf{EE}}^{[1\mathbf{E}=2\mathbf{E},4\mathbf{E}>0:1\mathbf{E}=2\mathbf{E},3\mathbf{E}>0]} + \\ + \tilde{\mathbf{C}}_{\mathbf{OO}}^{[1\mathbf{O}>0,2\mathbf{O}=3\mathbf{O},4\mathbf{O}>0:1\mathbf{O}>0,2\mathbf{O}>0,3\mathbf{O}>0]} + \\ + \tilde{\mathbf{C}}_{\mathbf{EO}}^{[1\mathbf{E}=2\mathbf{E},4\mathbf{E}>0:1\mathbf{O}>0,2\mathbf{O}>0,3\mathbf{O}>0]} + \\ + \tilde{\mathbf{C}}_{\mathbf{OE}}^{[1\mathbf{O}>0,2\mathbf{O}=3\mathbf{O},4\mathbf{O}>0:1\mathbf{E}=2\mathbf{E},3\mathbf{E}>0]} \tag{13}$$

The capacitance matrix is (14) and the partial matrices are (15).

	1.:	2.:	3.:	4.:
1.:				
2.:		C		
3.:			C_1	$-C_1$
4.:			$-C_1$	C_1

(14)

$$\mathbf{C_{EE}} = \begin{bmatrix} C & \\ & -C_1 \end{bmatrix}, \quad \mathbf{C_{OO}} = \begin{bmatrix} -C_1 \end{bmatrix}$$

$$\mathbf{C_{EO}} = \begin{bmatrix} \\ -C_1 \end{bmatrix}, \quad \mathbf{C_{OE}} = \begin{bmatrix} C & -C_1 \end{bmatrix} \tag{15}$$

Thus the resulting matrix (16) is composed of these four sub-matrices.

$$\mathbf{C} = \left[\begin{array}{c:c|c} C & 0 & 0 \\ \hdashline 0 & -C_1 & z^{-\frac{1}{2}}C_1 \\ \hline -z^{-\frac{1}{2}}C & z^{-\frac{1}{2}}C_1 & -C_1 \end{array}\right] \tag{16}$$

III. ILLUSTRATIVE EXAMPLE

For example, the described method will be resolved by the medium size circuits with switched-capacitor . Let consider an medium size SC circuit. This circuit (biquad) with switched capacitors and op amps is shown in Fig.6.

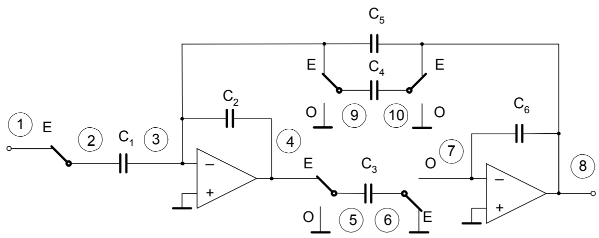

Figure 6. The medium size switched circuit

The circuit consists from 10 nodes, therefore capacitance matrix (17) consists from 10 rows and 10 columns, too.

	1.:	2.:	3.:	4.:	5.:	6.:	7.:	8.:	9.:	10:
1.:	0	0	0	0	0	0	0	0	0	0
2.:	0	C_1	$-C_1$	0	0	0	0	0	0	0
3.:	0	$-C_1$	$C_1+C_2+C_5$	$-C_2$	0	0	0	$-C_5$	0	0
4.:	0	0	$-C_2$	C_2	0	0	0	0	0	0
5.:	0	0	0	0	C_3	$-C_3$	0	0	0	0
6.:	0	0	0	0	$-C_3$	C_3	0	0	0	0
7.:	0	0	0	0	0	0	C_6	$-C_6$	0	0
8.:	0	0	C_5	0	0	0	$-C_6$	C_6+C_5	0	0
9.:	0	0	0	0	0	0	0	0	C_4	$-C_4$
10:	0	0	0	0	0	0	0	0	$-C_4$	C_4

(17)

The reduction formula (18) of this circuit is in next step decomposed into four partial formulas follows,

$$\mathbf{C}=\widetilde{\mathbf{C}}^{\left[\begin{array}{l}1E=2E,3E=9E,(4E=5E)>0,6E>0,(8E=10E)>0,1O>0,2O>0,4O>0,\\5O>0,6O=7O,8O>0,9O>0,10O>0:1E=2E,(3E=9E)>0,4E=5E,6E>0,\\7E>0,\ 8E=10E,1O>0,2O>0,3O>0,5O>0,(6O=7O)>0,\ 9O>0,10O>0\end{array}\right]}=$$

$$=\widetilde{\mathbf{C}}_{\mathbf{EE}}^{\left[\begin{array}{l}1E=2E,3E=9E,(4E=5E)>0,6E>0,(8E=10E)>0:\\:1E=2E,(3E=9E)>0,4E=5E,6E>0,7E>0,8E=10E\end{array}\right]}+$$

$$+\widetilde{\mathbf{C}}_{\mathbf{EO}}^{\left[\begin{array}{l}1E=2E,3E=9E,(4E=5E)>0,6E>0,(8E=10E)>0:\\:1O>0,2O>0,3O>0,5O>0,(6O=7O)>0,\ 9O>0,10O>0\end{array}\right]}+ \qquad (18)$$

$$+\widetilde{\mathbf{C}}_{\mathbf{OE}}^{\left[\begin{array}{l}1O>0,2O>0,4O>0,5O>0,6O=7O,8O>0,9O>0,10O>0:\\:1E=2E,(3E=9E)>0,4E=5E,6E>0,7E>0,8E=10E\end{array}\right]}+$$

$$+\widetilde{\mathbf{C}}_{\mathbf{OO}}^{\left[\begin{array}{l}1O>0,2O>0,4O>0,5O>0,6O=7O,8O>0,9O>0,10O>0:\\:1O>0,2O>0,3O>0,5O>0,(6O=7O)>0,9O>0,10O>0\end{array}\right]}$$

thus the submatrices are (19).

$$\mathbf{C}_{\mathbf{EE}}=\begin{array}{c|c|c|c|}
 & 1E=2E & 4E=5E & 8E=10E \\ \hline
1E=2E & C_1 & 0 & 0 \\ \hline
3E=9E & -C_1 & -C_2 & -C_5-C_4 \\ \hline
7E & 0 & 0 & -C_6 \\ \hline
\end{array}$$

$$\mathbf{C}_{\mathbf{EO}}=\begin{array}{c|c|c|}
 & 4O: & 8O: \\ \hline
1E=2E & 0 & 0 \\ \hline
3E=9E & -C_2 & -C_5 \\ \hline
7E & 0 & -C_6 \\ \hline
\end{array}$$

$$(19)$$

$$\mathbf{C}_{\mathbf{OE}}=\begin{array}{c|c|c|c|}
 & 1E=2E & 4E=5E & 8E=10E \\ \hline
3O & -C_1 & -C_2 & -C_5 \\ \hline
6O=7O & 0 & -C_3 & -C_6 \\ \hline
\end{array}$$

$$\mathbf{C}_{\mathbf{OO}}=\begin{array}{c|c|c|}
 & 4O & 8O \\ \hline
3O & -C_2 & -C_5 \\ \hline
6O=7O & 0 & -C_6 \\ \hline
\end{array}$$

The resulting matrix (20) is composed of these four sub-matrices (19) $\mathbf{C}_{\mathbf{EO}}$, $\mathbf{C}_{\mathbf{OE}}$ are multiplied by member $z^{-1/2}$.

$$\mathbf{C}=\left[\begin{array}{ccc|cc}
C_1 & 0 & 0 & 0 & 0 \\
-C_1 & -C_2 & -C_5-C_4 & z^{-\frac{1}{2}}C_2 & z^{-\frac{1}{2}}C_5 \\
0 & 0 & -C_6 & 0 & z^{-\frac{1}{2}}C_6 \\ \hline
z^{-\frac{1}{2}}C_1 & z^{-\frac{1}{2}}C_2 & z^{-\frac{1}{2}}C_5 & -C_2 & -C_5 \\
0 & z^{-\frac{1}{2}}C_3 & z^{-\frac{1}{2}}C_6 & 0 & -C_6
\end{array}\right] \qquad (20)$$

IV. CONCLUSION

A unified method in analyzing switched capacitor circuits is presented. This method can be used for solving SC circuits by hand.

When in e.g. [6] first is compile matrix 2Nx2N size (i.e. 20x20 for circuit from Fig.6) which is reduced in next step, in the described method the reduction is carried out already in the NxN matrix (i.e. 10x10 size). Therefore, the reduction is clearer as well as a method.

Thanks its clarity it is supported understanding of the principle of switched circuit analysis as well.

The described method gives the solution a purely mathematically when, after decomposition of the reduction relation, only values are substituted.

Solved matrix has half rows and columns, i.e. computed matrix contains a quarter of elements. Reduction relationship is approximately half as is shown in (6), (9), (11), (12), (17). Although the calculation must be done four times, the resulting computational complexity is approximately half them from integrated matrix. Therefore hand solving by described method is clear and well-arranged.

ACKNOWLEDGMENT

This work is sponsored by Department of Electrical Engineering, University of Pardubice.

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