Modified Summary Graph Method Solving of the SC Circuits

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***Abstract* – As there are also graph methods of circuit analysis in addition to algebraic methods, it is clearly possible in theory to carry out an analysis of the whole switched circuit in two-phase switching exclusively by the graph method as well. For this purpose it is possible to plot a Mason graph of a circuit, use transformation graphs to reduce Mason graphs for all the four phases of switching, and then plot a summary Mason-Coates graph from the transformed graphs obtained this way. First we draw nodes and possible branches, obtained by transformation graphs for transfers of EE (even-even) and OO (odd-odd) phases. Branches obtained by transformation graphs for EO and OE phase are drawn, while their resulting transfer is multiplied by $\pm z^{-\frac{1}{2}}$. In the next step, this summary Mason-Coates graph is transformed into Mason graph. Finally this graph can be interpreted by the Mason's relation to provide transparent voltage transfers.**

Keywords – switched capacitors; transformation graph; Mason-Coates graph; summary graph; Mason graph; Mason's formula; voltage transfer.

I. INTRODUCTION

Analysis of electric circuits is necessary not only for computing of circuit properties, but it is also important for understanding their principles. The computer methods are a powerful tool for symbolic analysis of circuit parameters [1]. But it is advantageous to have a tool capable to clear and simply symbolic analysis, too. The graphs methods can be considering as this tools. Thanks to its clarity, the graphic method is extremely suitable even for understand of these networks. A clearly arranged set of the transformation graphs derived for different types of the switching circuits can be used for analyzing switched capacitor networks and for understand of course, too. The M-C signal flow graphs are used for design [2] and analysis [3] continuous time circuits and periodically switched linear circuits, too. The transformation graphs are commonly used for assembly the final matrix considering all phases [4] to solving electronics circuits and matrix is calculated by algebraic minors [5]. It means that this method is a combination of graph and numerical methods, both. But solving is possible by graphs only in Full graph method. It assumes that the core of the solution is implemented through a graph.

II. PRINCIPLE OF THE METHODS

Switched capacitors circuits solving [6] by means of nodal charge equation method system [7], [8], [9] leads generally to an equation system (1).

$$\begin{bmatrix} C_{EE} & -z^{-\frac{1}{2}}.C_{EO} \\ -z^{-\frac{1}{2}}.C_{OE} & C_{OO} \end{bmatrix} \begin{bmatrix} V_E \\ V_O \end{bmatrix} = \begin{bmatrix} Q_E \\ Q_O \end{bmatrix} \quad (1)$$

This system can be used for plotting a summary graph of the SC circuit, which will thus be plotted by first drawing the nodes and possible branches, obtained by transformation graphs for transfers of EE (even-even) and OO (odd-odd) phases, as shown in Fig. 1 [10].

Figure 1. The first step of the construction: EE and OO transfers

This process involves using only elementary mathematical operations. Transformation by the transformation graphs is evaluated by simple relationships $C = a^V.\tilde{C}.a^Q.\alpha$ where a^V, a^Q assumes values of 0 or 1, α is +1 or -1.

In the next step, branches obtained by transformation graphs for EO and OE phase are drawn between these nodes, while their resulting transfer is multiplied by $-z^{-\frac{1}{2}}$ or $z^{-\frac{1}{2}}$ as is shown in Fig. 2 for C_1 in EO and/or C_2 in OE phase [11].

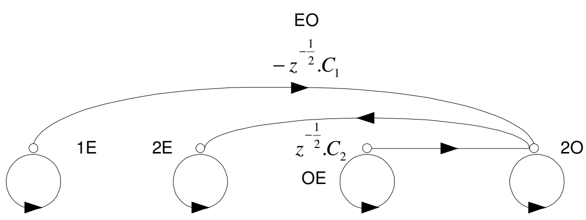

Figure 2. The second step of the construction: EO and OE transfers

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In the last step, this summary Mason-Coates graph is transformed into Mason graph. Therefore, the transfers of all branches entering into the considered node N are divided by the transfer of the self-loop considered node as is shown in Fig. 3 for transfer of the self-loop C. Transfer of the entering branch is C_1. Thus the evaluation of the summary graph is much simpler and clearer than other methods.

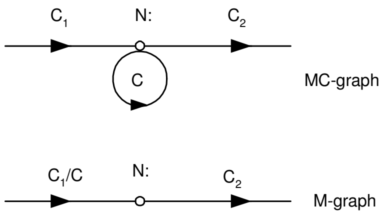

Figure 3. The construction M-graph from MC-graph

Thereby obtained summary graph is then evaluated by means of the Mason's rule for the T transfer of the graph $T = \frac{\sum p_{(i)}.\Delta_{(i)}}{1-\sum S^{(K)}}$ [10].

Use the Mason graph instead Mason-Coates graph simplifies evaluation. Therefore it is not necessary to compose a sum capacitance matrix and to express this consequently in numbers, and so it is possible to reach the final result in a graphical way.

III. EXAMPLE

Described way of a graph evaluation will be illustrated by the following example. A circuit with a switched capacitor has got the schematic wiring diagram shown in Fig. 4.

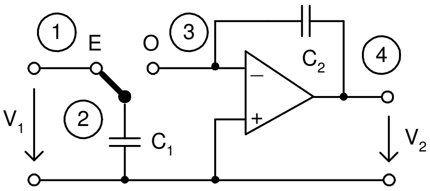

Figure 4. Schematic diagram from the example

The circuit has four nodes; therefore the starting graph in Fig. 5 has also four nodes.

The summary graph obtained from the partial transformed graphs from the Fig. 5 by the above mentioned procedure is then shown in Fig. 6.

First, the results of the transformed graphs for EE and OO phases are plotted (in case of this example only) as nodes.

In the next step, the results of the transformed graph for the EO and OE phases multiplied by $-z^{-\frac{1}{2}}$ or $z^{-\frac{1}{2}}$ are then drawn between these nodes as branches, i.e. the branch with the transfer $-z^{-\frac{1}{2}}.(-C_1)$ between the nodes $1E = 2E$, and $3O = 4O$, and the branches with the transfers $z^{-\frac{1}{2}}(-C_2)$ between the nodes 3E = 4E, and 3O = 4O.

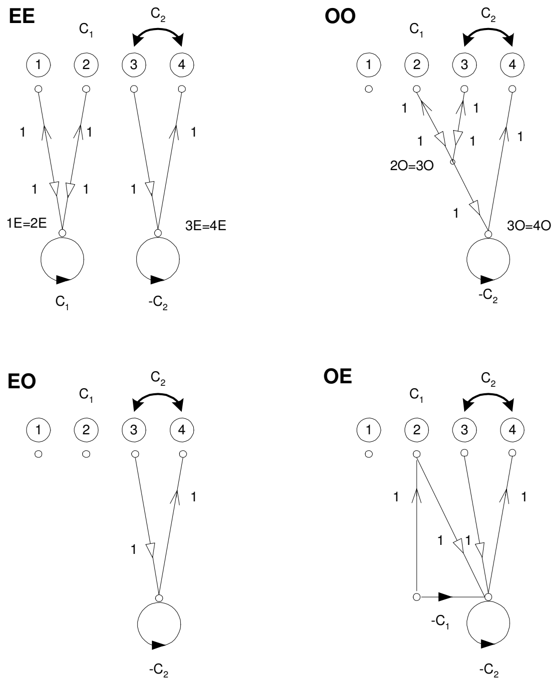

Figure 5. The transformations graphs for EE, OO, EO and OE phases

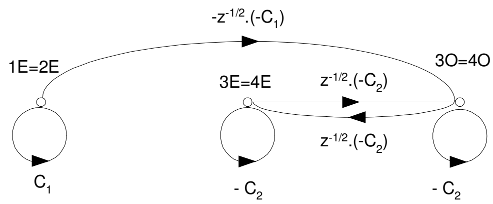

Figure 6. The summary MC-graph of the SC circuit from Fig. 5

The summary Mason-Coates graph is transformed into Mason graph in next step. Therefore, the transfer of the branch from the node $1E = 2E$ to the node $3O = 4O$ i.e. $z^{-\frac{1}{2}}C_1$ is divided by the transfer of the self-loop node $3O = 4O$ i.e. $-C_2$, thus transfer of this branch will be $-z^{-\frac{1}{2}}\frac{C_1}{C_2}$, as is shown in Fig. 8. The remaining branches will have transfer $\frac{-z^{-\frac{1}{2}}C_2}{-C_2} = z^{-\frac{1}{2}}$ because the transfer of the branches is $-z^{-\frac{1}{2}}C_2$ and the transfer of the self-loop node is $-C_2$.

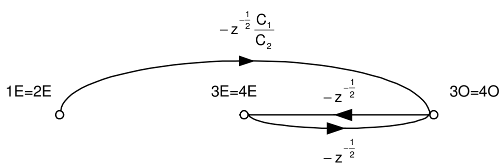

Figure 7. The summary M-graph of the circuit from Fig. 4 derived from Fig. 6.

By evaluating this summary graph, which is done by substitution into the Mason's formula: $T = \frac{\sum p_{(i)}.\Delta_{(i)}}{1-\sum S^{(K)}}$, we get the following final results this way . It is thus possible to express in numbers the two following transfers $\frac{V_{4E}}{V_{1E}}$ (2) and $\frac{V_{4O}}{V_{1E}}$ (3), for which it holds that:

$$\frac{V_{4E}}{V_{1E}} = \frac{\sum p_{(i)}.\Delta_{(i)}}{1-\sum S^{(K)}} = = \frac{-z^{-\frac{1}{2}}\frac{C_1}{C_2}.z^{-\frac{1}{2}}}{1-z^{-\frac{1}{2}}.z^{-\frac{1}{2}}} = -\frac{C_1}{C_2}.\frac{z^{-1}}{1-z^{-1}} \tag{2}$$

$$\frac{V_{4O}}{V_{1E}} = \frac{\sum p_{(i)}.\Delta_{(i)}}{1-\sum S^{(K)}} = = \frac{-z^{-\frac{1}{2}}\frac{C_1}{C_2}.1}{1-z^{-\frac{1}{2}}.z^{-\frac{1}{2}}} = -\frac{C_1}{C_2}.\frac{z^{-\frac{1}{2}}}{1-z^{-1}} \tag{3}$$

To compare the solution by the above graph method, we will present a calculation of the same circuit by the used method of MC-graph. It is thus possible to express in numbers the two following transfers, for which it holds that:

$$\frac{V_{4E}}{V_{1E}} = \frac{\sum p_{(i)}.\Delta_{(i)}}{V-\sum S^{(K)}\cdot V^{(K)}} = = \frac{-z^{-\frac{1}{2}}.(-C_1).z^{-\frac{1}{2}}.(-C_2)}{(-C_2).(-C_2)-z^{-\frac{1}{2}}.(-C_2).z^{-\frac{1}{2}}.(-C_2)} = = \frac{z^{-1}.C_1}{z^{-\frac{1}{2}}.C_2-C_2} \tag{4}$$

and for the second one

$$\frac{V_{4O}}{V_{1E}} = \frac{\sum p_{(i)}.\Delta_{(i)}}{V-\sum S^{(K)}\cdot V^{(K)}} = = \frac{-z^{-\frac{1}{2}}.(-C_1).(-C_2)}{(-C_2).(-C_2)-z^{-\frac{1}{2}}.(-C_2).z^{-\frac{1}{2}}.(-C_2)} = \frac{z^{-\frac{1}{2}}.C_1}{z^{-\frac{1}{2}}.C_2-C_2} \tag{5}$$

IV. CONCLUCIONS

While in case of using the graph method a graph was indicated, a transformation graph was plotted and from its results a summary MC-graph was drawn and in next step is transformed into M-graph and evaluated by the Mason's rule, after which the result was obtained by an easy simplification, in case of solving by the MC-graph calculation is somewhat more difficult.

If calculation is made "by hand", described multiple transformations (i.e. starting graph → transformation graph → transformed MC-graph → final M-graph) leads to a reduction in the number of members in the dominator and denominator and numerical operations, as we can see.

REFERENCES

[1] J. Vlach, K. Singhal, Computer methods for circuit analysis and design, 2nd ed. Van Nostrand Reynhold, New York, 2003.

[2] D. Biolek,V. Biolková, "Analysis of circuits containing active elements by using modified T-graphs," in Advances in Systems Science: Measurement, Circuits and Control, WSEAS Press, Electrical and Computer Engineering Series, 2001, pp. 279-283.

[3] D. Biolek, V. Biolková, "Flow Graphs Suitable for Teaching Circuit Analysis" in Proceedings of the 4th WSEAS International Conference on Applications of Electrical Engineering – AAE´05, Praque, 2005.

[4] T. Dostál, "The Analysis of the Active Components Containing Switched Capacitors by Nodal Voltage Method", Electronics horion, 1984, 45, no.1, pp.21-26. (in Czech)

[5] J. Čajka, J. Kvasil, Theory of Linear Networks. SNTL/ALFA, 1979, pp.55-159. (in Czech)

[6] D. Biolek, V. Biolková, "Flow Graphs for Analysis (not only) Current-Mode Analogue Blocks." Contribution to the book "Recent Advances in Circuits, Systems and Signal Processing", WSEAS Press, Electrical and Computer Engineering Series, 2002.

[7] J. Hospodka, J. Bicak, "Application for Symbolic Analysis of Linear Circuits Including Switched Circuits" in Proceedings of the 6th WSEAS International Conference, System Science and Simulation in Engineering, Venice, Italy, 2007, pp. 254-258.

[8] J. Jerabek, R. Schottner, K. Vrba, "Full Differential Universal Filter with Current Active Elements" in Proceedings of the 4th WSEAS International Conference Circuits, System and Signals (CCS´10), Corfu Island, Greece, 2010.

[9] D. Biolek, Solving Electronic Circuits, BEN Publisher Prague, 2004 (in Czech).

[10] B. Brtník, Full-Graph Solution of Switched Circuits. LAP Saarbrucken, 2011, ch.1,3.

[11] B. Brtník, "Comparison of the Full Graph Metod Switched Circuits Solution", Przegled Elektrotechniczny. Issue 6, 2012, pp. 332-336.