Comparison of Short Delay Measurement Methods

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***Abstract*—The article describes and compares various digital methods for measurement of short delay between fast analogue signals. A brief characterization of measured signals is given in the first chapter. In the second chapter, there are described various measurement methods. In the third chapter, there are compared various versions of each method and in the last chapter, there are the methods compared one another according to results of numerical experiments.**

I. Introduction

In many applications, such as laser rangefinders or ultrasonic flowmeters, precise measurement of short delays between analogue signals is required. A number of methods was developed for this purpose, supposing measurement of signals with sharp edges [1–3]. However if the signal has no sharp edge, it is common to use some of the analogue methods which are based on various principles. Usage of analogue measurement methods requires highly precise production and calibration of each manufactured measurement device. In recent days, there is great tendency towards digital signal processing making product development faster and production cheaper. This paper presents and compares various digital methods for delay measurement.

Short description of measured signals is given in the first chapter. The second chapter describes methods for digital measurement of analogue signal delays. The rest of the paper presents the results of experimental method comparison.

II. Signal features

Suppose we have acquired two signals (signal pair) from a sort of the sensors, for example from the ultrasonic detectors (see Fig. 1). Suppose this signal pair has following features:

- Signals appear in bursts of finite waveform length,
- both waveforms in the pair have similar envelope shape, but are time-shifted by Δt,
- bursts are non-periodic (each burst is unique),
- signals in burst are harmonically oscillating at unknown frequency f_p (same for both signals in pair),
- exact analytical description of signal envelope is not known.

An example of such waveform is in the figure 1.

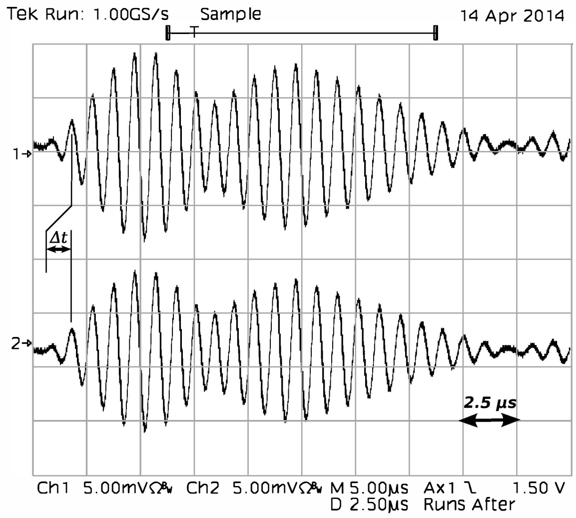

Fig. 1. Waveform example.

An example of signals pair recorded on ultrasonic flowmeter prototype. Time shift is hard to notice because $\Delta t \approx 0.01 \cdot f_p^{-1}$.

Our goal is the measurement of the time shift Δt between the two waveforms in the signal pair with high precision. The time shift can be very short ($\Delta t \approx 10^{-8} \div 10^{-11}\,\mathrm{s}$). In many applications, very fine measurement resolution (in order of $10\,\mathrm{ps}$) is required. Simple correlation methods commonly used for such measurements have resolution equal to the sampling period of the signals. To achieve required resolution in order of $10\,\mathrm{ps}$, the sampling frequencies would be unattainably high (in order of $100\,\mathrm{Gs}\cdot\mathrm{s}^{-1}$).

III. Time shift measurement methods

For time shift measurement, there are many methods. The majority of the methods require sharp edges in the signal or use analogue signal preprocessing. This paper however describes fully digital methods — the signal is sampled and then digitally processed. This chapter describes following methods:

- correlation method,
- interpolation followed by correlation,
- approximation of correlation function,
- time shift computation using Fourier transform,

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- signal approximation using least squares method and statistical evaluation.

A. Correlation method

Common method for signal delay computation is finding of the maximum of their cross–correlation function (1). This maximum of the cross-correlation function corresponds to the time shift between signals with the best similarity.

$$\begin{aligned} R_{xy}(k) &= \sum_m x(m) \cdot y(k+m) \\ L_{xy}(k) &= k \end{aligned} \tag{1}$$

The distance between samples (in time axis) is sampling period $T_s = f_s^{-1}$, where f_s is sampling frequency. Distance between points in cross–correlation function is also T_s. This means that maximal resolution of the method is equal to the sampling period T_s. Many applications need resolution that corresponds to very high sampling frequencies (10 Gs/s and higher). In these applications, it is necessary to carry out some signal interpolation in order to get better time resolution. This interpolation can be done before or after computation of the cross–correlation function.

B. Interpolation followed by correlation

Interpolation followed by correlation is a method described in the paper [4]. This method is based on common correlation method. Analogue signals are sampled (sampling period is less than required resolution of results) and interpolated to required sampling rate. On these interpolated (i.e. up-sampled) signals is applied common correlation method.

Interpolation ratio can be computed from equation (2) where f_s is sampling frequency of the original signal, f_{interp} is sampling frequency of interpolated signal and res is required resolution.

$$\frac{1}{n} = \frac{f_s}{f_{interp}} = res \cdot f_s \tag{2}$$

The interpolation can be carried out in different ways:

- approximation using analytical description of signal,
- Lagrange's (Newton's) polynomial interpolation,
- Hermitian polynomial interpolation,
- interpolation using zero–valued samples insertion and filtration,
- spline interpolation.

Because interpolation ratio can be very high, usability of this method is limited due to very high computation complexity.

C. Approximation of correlation function

Method described in [5] is another modification of correlation method increasing its resolution. Signal pair is sampled and then cross-correlation of signals is computed. The correlation function is approximated using least square method in vicinity to it's maximum. Maximum of the approximation function is analytically computed. Its position corresponds to the time shift between signals in pair.

Signal (maximal frequency f_{max}) is sampled at sampling frequency f_s ($f_s > 2 \cdot f_{max}$) and according to the equation (1) the correlation function R_{xy} is computed. It's maximum $R_{max} = \max(R_{xy})$ and the corresponding lag L_{max} are found. The lag corresponds to the time shift with resolution T_s. In the vicinity to L_{max} the correlation function is approximated by function $\widehat{R}_{xy}$ and its maximum $\widehat{R}_{max} = \max(\widehat{R}_{xy})$ is found. Having corresponding lag $\widehat{L}_{max}$ time shift Δt can be computed with resolution much higher than T_s, see equation (3).

For approximation of correlation function it is advantageous to use polynomial approximation using least squares method (at least in case of cross–correlation of signals specified in chapter II). In the neighbourhood of correlation maxima, n points in both directions are picked up. Coordinates of these points are $[L_{max-n}, R_{max-n}], \ldots, [L_{max}, R_{max}], \ldots, [L_{max+n}, R_{max+n}]$. These points are polynomially fitted by polynomial $P_m(L)$ of degree m (in the least–squares sense). Local maxima of this polynomial can be computed analytically. Local maxima $[\widehat{L}_{max}, \widehat{R}_{max}]$ nearest (in sense of $\min |L_{max} - \widehat{L}_{max}|$) to correlation function maxima $[L_{max}, R_{max}]$ determines the time shift Δt, see equation (3).

$$\Delta t = \hat{L}_{max} \cdot T_s = \frac{\hat{L}_{max}}{f_s} \tag{3}$$

Appropriate number of points n for polynomial fitting depends on the degree of polynomial (m). If the quadratic curve is fitted, it's good choice to fit it between inflexion points of the correlation function. For harmonic function the inflexion points are situated at zero crossings. In this case n is given by equation (4). For higher polynomial degree can be used more points.

$$n = \left\lfloor \frac{f_s}{4 \cdot f_p} \right\rfloor \tag{4}$$

Increasing degree of the polynomial brings also increasing ripple and approximation error in intervals between nodes. Extreme case arises when $m+1$ points ($n = m/2$) are fitted with polynomial of degree m. In this case least square method changes into Lagrange interpolation with all consequences.

D. Signal time shift computation using Fourier transform

Let us have a pair of discrete (attenuated) harmonic signals $x_1(t)$ and $x_2(t)$. Both signals have frequency f_p. If phase shift $\Delta\varphi$ is known, then it is possible to compute time shift Δt of waveforms using (5). [6]

$$\Delta t = \frac{\Delta\varphi}{\omega_p} = \frac{\Delta\varphi}{2 \cdot \pi \cdot f_p} \tag{5}$$

Phase shift $\Delta\varphi$ and frequency f_p can be easily computed using discrete Fourier transform. Phase shift computed using Fourier transform is in range

$< -\pi, +\pi >$, then the time shift is in range $\left\langle -\frac{1}{2 \cdot f_p}, +\frac{1}{2 \cdot f_p} \right\rangle$.

If the frequency f_p is high, sampling produces a large number of samples. We can reduce the amount of samples using nonuniform sampling at average sampling frequency under the Nyquist's limit ($\overline{f_s} < 2 \cdot f_p$), see [7].

E. Signal approximation using least squares method and statistical evaluation

This method is a new method developed at our university. Method wasn't published nor used in real appliance, yet. Sampled signals are divided into small elements (e.g. half–waves) and each element is approximated using least square method. For corresponding pairs of elements their time shifts are analytically computed. This set of elementary time shifts is statistically evaluated.

Let us have pair of time shifted signals x_1 a x_2. Both of them are divided in some way[1] to n elements $x_{11}, x_{12}, \ldots, x_{1n}$ a $x_{21}, x_{22}, \ldots, x_{2n}$. Each element is approximated (in sense of least squares) by appropriate function (polynomial is a good choice). We get n functions for each signal $\widehat{x}_{11}, \widehat{x}_{12}, \ldots, \widehat{x}_{1n}$ and $\widehat{x}_{21}, \widehat{x}_{22}, \ldots, \widehat{x}_{2n}$. For each pair of corresponding functions their time shift $\widehat{\Delta t}_1 \cdots \widehat{\Delta t}_n$ is computed. The algorithm for computation of time shifts depends on used approximation function, for quadratic approximation it is the distance between peaks, for linear approximation it is the distance between zero–crossings etc. The set of elementary time shifts $\{\widehat{\Delta t}_i\}$ is statistically evaluated in order to get final time shift Δt. Basic methods of statistical evaluation is computation of arithmetic mean or median of the set $\{\widehat{\Delta t}_i\}$.

IV. Experimental evaluation of methods

For appropriate method selection it is necessary to know method's features for various sampling frequencies and noise levels. Numeric evaluation of methods was carried out using simulated signal imitating signal from real appliance. The main features of simulated signal are in accordance with the signal features described in chapter II, $f_p = 1\,\mathrm{MHz}$. White Gaussian noise at various signal to noise ratios was added to the simulated signal in order to evaluate method's noise sensitivity.

Time shift computations for various time shifts were carried out for many times (5 to 1000 times, depending on computation complexity) and errors were statistically evaluated. The computations were carried out in double precision IEEE 754 floats.

Two various error behaviours were observed regarding dependence between time shift and error magnitude:

- The error depends only on method's parameters. In next chapters the error is considered as absolute (6).
- The error depends on method's parameters and on the time shift. In next chapters the error is considered as relative (7).

$$E_{abs} = \widehat{\Delta t} - \Delta t_{skut} \tag{6}$$

$$E_{rel} = \left| \frac{\widehat{\Delta t} - \Delta t_{skut}}{\Delta} t_{skut} \right| \tag{7}$$

In the following statements the results for various combinations of parameters are compared and in the next chapter (chapter V) the methods are compared one another.

A. Interpolation followed by correlation

1) Interpolation methods comparison: Comparison of following interpolation methods was carried out: interpolation using zero–valued samples insertion and filtration [8], linear interpolation and spline interpolation using natural cubic spline [9] and pchip spline [10]. Lagrangian and Hermitian interpolation aren't suitable for interpolating oscillating signals due to their properties.

Interpolation method using zero–valued samples insertion and filtration was found to get much better results than other methods.

2) Effect of filtration: Effect of signal filtration was analysed. Signal according to features described in chapter II with added white Gaussian noise at various levels was filtered using bandpass filter with cut-off frequencies $0.5 \cdot f_p$ and $2 \cdot f_p$. The signals were filtered before or after interpolation. Experiments confirm that filtration after interpolation has no effect on results, see figure 2. When filtration is carried out before interpolation, error magnitude increases rapidly.

B. Approximation of correlation function

Comparison of results for approximation of correlation function by polynomial of degree two and four has been done. For some combinations of the sampling frequency and noise level the polynomial of degree two gives better results and for another combinations is better polynomial of degree four (see figure 3). The results of the experiment are summarised in table I.

Effect of signal filtering has been evaluated using low-pass filter with cut-off frequency $2 \cdot f_p$ (for definition of f_p see chapter II). It was found that filtration has no effect on errors.

C. Signal time shift computation using Fourier transform

For this method evaluation of the influence of the windowing function during Fourier transform computation was carried out. The influence was found to be negligible and thus it can be omitted, see figure 4.

[1] We are using zero crossings detection with averaging and hysteresis.

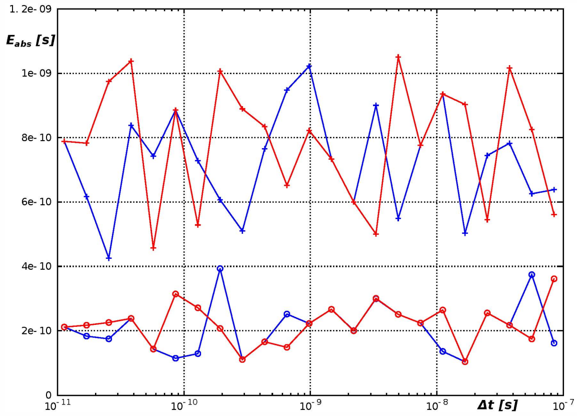

Fig. 2. Effect of filtration after interpolation.

Absolute error for method of interpolation followed by correlation. Filtration after interpolation, $f_p = 1\,\text{MHz}$, pass-band $0.5 \cdot f_p$ to $2 \cdot f_p$, $\text{SNR} = 20\,\text{dB}$, $f_s = 50\,\text{Msample/s}$. Without filtration in blue, with filtration in red.
o …mean error, + …maximal error

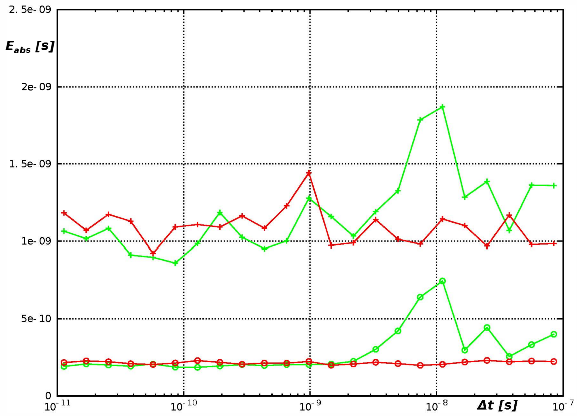

Fig. 3. Absolute error — approximation of correlation function.

Absolute error for method of approximation of correlation function, $\text{SNR} = 20\,\text{dB}$, $f_s = 50\,\text{Msample/s}$.
Polynomial of degree two in green, of degree four in red.
o …mean error, + …maximal error

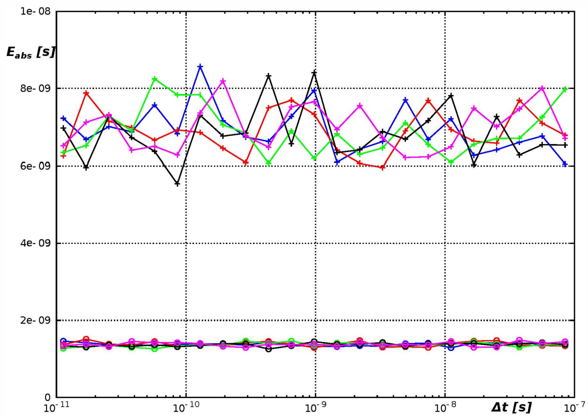

Fig. 4. Time shift computation using Fourier transform - influence of the window.

Absolute error for time shi t computation using Fourier transform. $\text{SNR} = 20\,\text{dB}$, $f_s = 5\,\text{Msample/s}$.
Windowing function: rectangular in blue, triangular in red, hamming in green, hann in black and kaiser in magenta.
o …mean error, + …maximal error.

TABLE I. APPROPRIATE DEGREE OF APPROXIMATION POLYNOMIAL.

Appropriate degree of approximation polynomial for method of approximation of correlation function for various sampling frequencies and noise levels.
$f_p = 1\,MHz$

	50 Ms/s	10 Ms/s	5 Ms/s
Without noise	4	4	2
SNR = 20 dB	4	4	2
SNR = 10 dB	2	2	2
SNR = 0 dB	2	x	x

2 or 4 …appropriate degree of polynomial is 2 or 4;
x …differences between degrees 2 and 4 are insignificant.

D. Signal approximation using least squares method and statistical evaluation

This method was evaluated in versions using polynomial of degree two or four as approximation function $\widehat{x_{ik}}$. For quadratic function the time shift $\widehat{\Delta t_k}$ was computed as distance of vertexes in t axis. Local extrema were found for polynomials of degree four. Time shifts (in t axis) for extrema laying in particular interval were computed. These time shifts were used as $\widehat{\Delta t_k}$.

The comparison of both versions of method can be found in table II. In the table there can be seen that polynomial of degree of two is better then the one of degree of four with exception of noiseless signal sampled at low frequency. Dependence of error on sampling frequency can be seen in the figure 5.

For statistical evaluation of the set of time shifts $\left\{\widehat{\Delta t_1}, \widehat{\Delta t_2}, \ldots, \widehat{\Delta t_n}\right\}$ arithmetic mean and median were used. Absolute error is many times $(10^2 - 10^{10})$ higher when using arithmetic mean than in case of usage of median.

TABLE II. APPROPRIATE DEGREE OF APPROXIMATION POLYNOMIAL.

Appropriate degree of approximation polynomial for method using approximation and statistical evaluation for various sampling frequencies and noise levels.
$f_p = 1\,\text{MHz}$

	50 Ms/s	10 Ms/s	5 Ms/s
Without noise	2	4	4
SNR = 20 dB	2	2	2
SNR = 10 dB	2	2	2
SNR = 0 dB	2	2	2

2 or 4 …polynomial of degree two or four gives smaller error.

V. METHODS COMPARISON

Various modifications of methods were compared in the previous chapter (chapter IV) and for each method was found the best variant. In this chapter, there are compared above mentioned methods one another for various combinations of noise level and sampling frequency. The results are summarized in the table III. In the cases of interpolation followed by correlation being the best also the next best method has been chosen because computation complexity of interpolation followed by correlation is much higher than computation complexity of the other methods.

In the table III there can be seen that the choice of method for particular application depends on signal to noise ratio and is independent on the sampling frequency. The error magnitude depends on sampling frequency.[2]

Error magnitudes for various combinations of noise level and sampling frequency can be found in the table IV. Methods used for computations are summarized in the table III. All computation were carried out in double precision floating point arithmetic.

TABLE III. COMPARISON OF METHODS.

This table summarizes the best methods for various combinations of noise level and sampling frequency. $f_p = 1\,\mathrm{MHz}$.

	$50\,\mathrm{Ms/s}$	$10\,\mathrm{Ms/s}$	$5\,\mathrm{Ms/s}$
Without noise	FP	FP	FP
$\mathrm{SNR} = 20\,\mathrm{dB}$	IK (KC4)[a]	KC4	KC2
$\mathrm{SNR} = 10\,\mathrm{dB}$	KC2	KC2	KC2
$\mathrm{SNR} = 0\,\mathrm{dB}$	PS	PS	PS

[a]This method has much higher computation complexity than the other methods. The next best method is KC4.

IK …Interpolation followed by correlation,
FP …time shift computation using Fourier transform,
PS …signal approximation using least squares and statistical evaluation,
KC2 or KC4 …Approximation of correlation function using polynomial of degree two or four.

VI. CONCLUSION

Various methods for measurement of time shift of fast analogue signals were proposed in the paper. Error of each method was evaluated and the results were compared one another. It was found that for time shift measurement of signals characterized in chapter II without noise the best choice is the method based on Fourier transform. For signals with high signal to noise ratio (i.e. weak noise, $\mathrm{SNR} > 10\,\mathrm{dB}$) the method using approximation of correlation function gives the best results. The error is less than 3% of the sampling period f_s^{-1} for $\mathrm{SNR} = 10\mathrm{dB}$ and less than 1% for $\mathrm{SNR} = 20\mathrm{dB}$. For signals with strong noise the best choice is the method of signal approximation using least squares method and statistical evaluation with quadratic approximation polynomial. The error in this case is about 20% of the sampling period f_s^{-1}.

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[2] In cases of very low sampling frequency (near Nyquist's frequency) the appropriate method must be chosen, for example computation of time shift using nonuniform Fourier transform.

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TABLE IV. MAGNITUDE OF ERROR.

In this table, there are summarized magnitudes of errors for methods from the table III. Computations were carried out in double precision floating point arithmetic. $f_p = 1\,\mathrm{MHz}$.

	$50\,\mathrm{Ms/s}$	$10\,\mathrm{Ms/s}$	$5\,\mathrm{Ms/s}$	Figure number
Without noise	0.1 %	0.38 %	0.35 %	6
$\mathrm{SNR} = 20\,\mathrm{dB}$	170 ps / 220 ps [a b]	450 ps / 400 ps [a]	700 ps	7
$\mathrm{SNR} = 10\,\mathrm{dB}$	0.5 ns	1.5 ns	2 ns	8
$\mathrm{SNR} = 0\,\mathrm{dB}$	4 ns	20 ns	37 ns	5

[a]Error magnitudes for methods KC4 / KC2.
[b]Error magnitude for interpolation followed by correlation is (IK) is 200 ps. (Interpolation using zero–valued samples insertion and filtration. Interpolation rate 1:100.) Computation time for this method is much higher than for other methods.

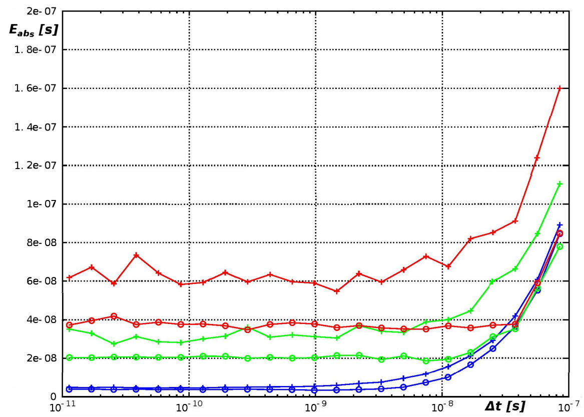

Fig. 5. Signal approximation using least squares method — dependence of error on sampling frequency.

Absolute error for signal approximation and statistical evaluation method. Degree of approximation polynomial is two, $\mathrm{SNR} = 0\,\mathrm{dB}$.
Sampling frequency $f_s = 50\,\mathrm{Msample/s}$ in blue, $f_s = 10\,\mathrm{Msample/s}$ in green and $f_s = 5\,\mathrm{Msample/s}$ in red.
o . . . mean error, + . . . maximal error.

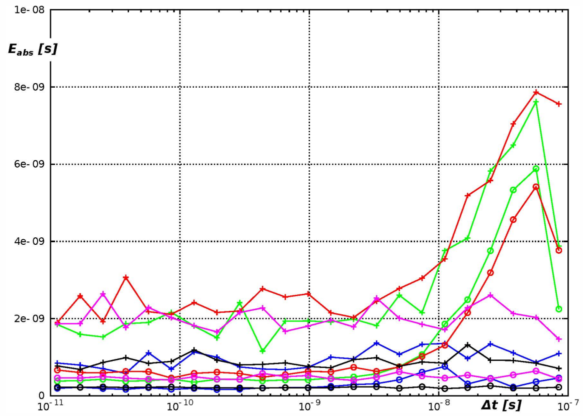

Fig. 7. Absolute error for method of approximation of correlation function — $\mathrm{SNR} = 20\,\mathrm{dB}$.

Absolute error for method of approximation of correlation function. $f_p = 1\,\mathrm{MHz}$, $\mathrm{SNR} = 20\,\mathrm{dB}$.
Approximation polynomial of degree 2: Sampling frequency $f_s = 50\,\mathrm{Msample/s}$ in blue, $f_s = 10\,\mathrm{Msample/s}$ in green, $f_s = 5\,\mathrm{Msample/s}$ in red.
Approximation polynomial of degree 4: Sampling frequency $f_s = 50\,\mathrm{Msample/s}$ in black, $f_s = 10\,\mathrm{Msample/s}$ in magenta.
o . . . mean error, + . . . maximal error.

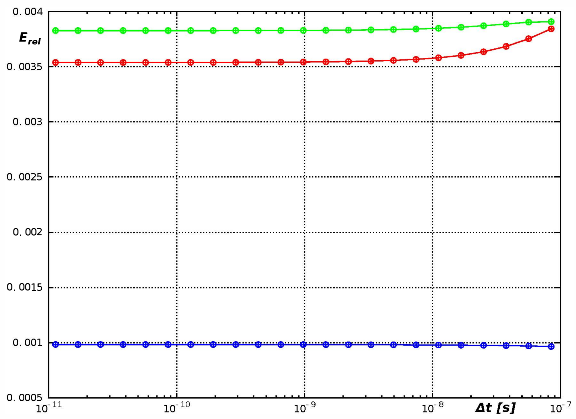

Fig. 6. Relative error for signal time shift computation using Fourier transform — without noise.

Relative error for signal time shift computation using Fourier transform. $f_p = 1\,\mathrm{MHz}$, without noise.
Sampling frequency $f_s = 50\,\mathrm{Msample/s}$ in blue, $f_s = 10\,\mathrm{Msample/s}$ in green, $f_s = 5\,\mathrm{Msample/s}$ in red.

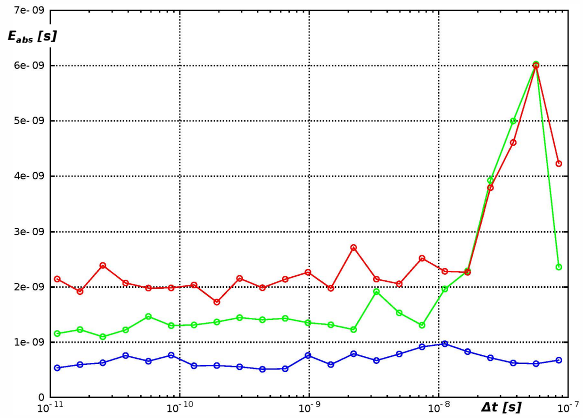

Fig. 8. Absolute error for method of approximation of correlation function — $\mathrm{SNR} = 10\,\mathrm{dB}$.

Absolute error for method of approximation of correlation function. $f_p = 1\,\mathrm{MHz}$, $\mathrm{SNR} = 10\,\mathrm{dB}$.
Approximation polynomial of degree 2. Sampling frequency $f_s = 50\,\mathrm{Msample/s}$ in blue, $f_s = 10\,\mathrm{Msample/s}$ in green, $f_s = 5\,\mathrm{Msample/s}$ in red.