# Optimization Methods for Switched Capacitor Circuits

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***Abstract*—The paper investigates the performance of the Differential Evolution (DE) and Particle Swarm Optimization (PSO) algorithm for SC filter Optimization. In order to improve their performance the three algorithm based on their combination are proposed. The performance is investigated on the design of switched capacitor biquadratic filters and the resulting performance and limitations are discussed.**

***Keywords - optimization, DE; PSO; filter; SC circuit.***

## I. Introduction – State of the Art

A lot of digital devices in the modern technology still need to be pre/processed by analogue circuits. One of the most technique used for implementation is switched capacitor (SC) circuits. It offers interesting advantages [1].

Unfortunately, the nonidealities and parasitic properties of used component (e.g. switch resistances, frequency response of OpAmp, offsets, ...) affect the resulting circuit characteristic (e.g. transfer characteristic). The effects have been already studied [2], [3] but their suppression by an analytical method is difficult. Numerical method to suppress the effects has become more used as the computer performance increase. The methods are usually tested on a set of functions [4] and used on a simple analogue circuits, which analysis takes short time [5], [6]).

In contrast to OpAmp design [5], [6] the analyses of switched capacitor circuit take a lot of time, because transient analysis and DFT have to be used for frequency response calculation. Therefore the designers prefer only to adjust their old designs than to design and optimize a new SC circuit.

Used optimization algorithms are inspired by the nature. The most popular methods are Differential Evolution algorithm (DE) [7] and Particle Swarm Optimization algorithm (PSO) [8].

This paper deals with the application of these algorithms to SC filter design. The performance of the method is investigated on Fleischer-Laker SC biquad [1] design in 0.35 μm CMOS technology. Based on the results the paper discuss possible algorithm improvement to increase their speed for SC design.

The optimization method was performed using Maple™ program linked to a WinSpice simulator [9].

## II. Biquad design

To compare the optimization algorithms performance for SC circuits, the design of $4^{th}$ order maximally decimated two channel filter bank was chosen (see its design in [10]). The filter bank consists of two $4^{th}$ order SC filters (with the low-pass filter characteristic $H_{LP}$ and with the high-pass filter characteristic $H_{HP}$). For better implementation the functions were factorized into the biquadratic functions ($H_{LP1}$, $H_{LP2}$, $H_{HP1}$, $H_{HP2}$) [10], where the implemented pole and zero lie as far from each other as possible (because of the sensitivity to capacitors inaccuracy).

Since the transfer functions contain all of the powers of $z$, the Fleischer-Laker biquad topology [1] was chosen for implementation. This biquad contains only two OpAmps which were implemented in 0.35 um CMOS technology [10].

Due to complexity of used transistors models, the biquads had to be simulated by transient analysis and the frequency response were subsequently obtained by means of DFT [11]. Unfortunately, the one biquad simulation took about 2 minutes. Therefore, the biquads were simulated only with the models of switches (simulation took about 20 seconds). The switch model consisted of the on-state resistance, off-state resistance and the overlapping capacitor, which simulated charge injection.

## III. Optimization methods

There exists a big amount of different optimization methods. Each method is more or less suitable to different optimization tasks. Therefore, it is very difficult to choose the best method. As usual, the criteria to compare optimization method performance is the number of algorithm iterations needed to find a global extreme of mathematical functions. These functions usually have a lot of local extremes but only one global extreme [4].

Differential Evolution algorithm (DE) [7], Particle Swarm Optimization algorithm (PSO) [8] and their combinations are compared in this work. The combination of DE and PSO are proposed in order to find faster and more robust algorithm for SC circuits optimization.

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### A. *Differential Evolution Algorithm*

The DE algorithm is a "population based" algorithm where the new population of vectors is generated from the previous population by adding of weighted difference of two different vectors to the third vector.

Storn and Price suggested two scheme of generating of new vectors. $2^{nd}$ scheme ($DE_2$) is used in this work where the weighted difference is added on the line between the best found vector and current vector [7].

Although the DE method is quite robust, the DE results are very dependent on number of used vectors. If small number of agents is used, the DE method has low number of combinations to use and the algorithm has a difficulties to find better vector.

### B. *Particle Swarm Optimization Algorithm*

The Particle Swarm Optimization (PSO) method is based on keeping agent history in memory and sharing of this information among agents. Agent are moving in optimization space with various speed and direction during optimization process. Each algorithm iteration the agent speed and direction of agent moving is changed by the best position which was found by particular agent in previous iterations and by the best found position of whole swarm (population) [8].

The PSO algorithm is very useful to find the optimum roughly in few iterations. Unfortunately, when the agents get close to the optimum, the algorithm speed decrease and agents circle in spiral around expected minimum.

## IV. Proposed algorithms

The SC circuit optimization time is affected by time consuming transient analysis, which needs to be used together with optimization procedure. Thus the designers have to choose the method which needs as small number of circuit analyses as possible (e.g. lower number of used vectors in population, lower number of algorithm iterations...)

To solve slow convergence of PSO method and sensitivity of $DE_2$ to low number of vectors in population, the method were combined into the next three variants:

### A. *DE-PSO1 algorithm*

An DE-PSO1 method is primary based on the DE algorithm. Compared with the standard DE method which must be restarted if vectors stuck in a local optimum, the DE-PSO1 method "restart" the DE algorithm partially during the optimization process. To be more precise, the algorithm shift the agent which did not change its position for X iteration to another position. In our case X = 3 was used. For the vector shift is used PSO method as this method guarantee shift in limited area itself. The flow chart of the DE-PSO1 algorithm is depicted in Fig. 1.

### B. *DE-PSO2 algorithm*

DE-PSO2 algorithm is an variant of DE-PSO1 algorithm. The DE-PSO2 algorithm also run the PSO

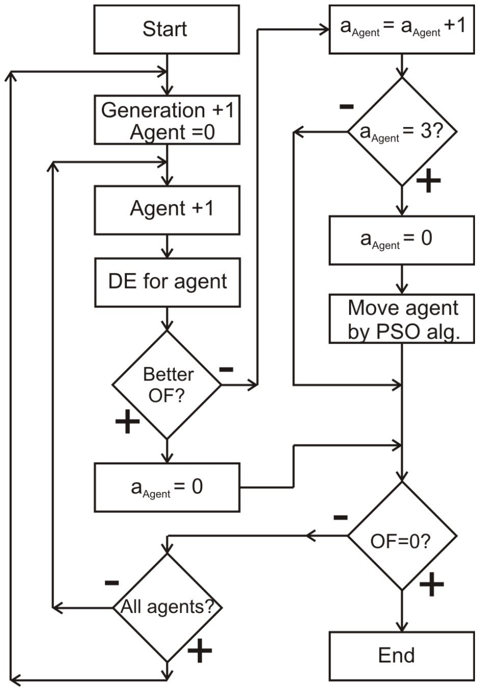


Figure 1. DE-PSO1 method flow chart.

method for the agent, which haven't changed its position in last X iteration. In contrast to DE-PSO1 method, the DE-PSO2 algorithm uses the remembered results of PSO in computation of DE algorithm. If the remembered results of PSO are better than current agent position, the remembered results of PSO method is used for DE computation (the PSO algorithm save the agent best position history as is usual).

This DE-PSO1 modification helps in situation, when PSO is activated more times in row and find better solution. Then the agent position for DE algorithm is fixed and agents aren't so quickly pushed further to space.

### C. *PSO-DEm algorithm*

PSO-DEm algorithm is primary based on PSO algorithm, which is assisted by DE algorithm.

The main idea of this algorithm is to perform the DE method only for a few chosen agents to increase the speed of PSO method, when the agents circling around the expected optimum.

The algorithm can be described in four steps:

- An iteration of PSO algorithm is performed for whole population.
- $Y\,mod\,X$ agents are chosen, where $Y$ is number of agent in population and $X$ is number of agent parameters (e.g. capacitors values needed to be optimized in our case).
- DE method is performed for chosen agents, where the best results from PSO are used.
- If the DE method find better agent position this position is only written to memory of best results (DE does not change current agent position).

To increase the diversity of the method the each agent remember its last 3 best results and these results are used in DE method for the chosen agents (PSO

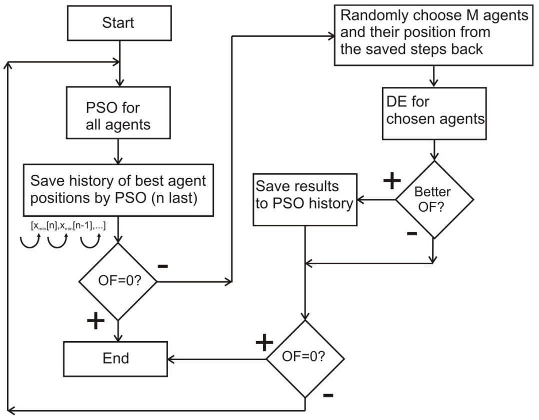


Figure 2. $PSO - DEm$ algorithm principle.

algorithm still uses only the very last best results). Then the DE algorithm part can be written in formula:

$$\overrightarrow{X_{new?,G+1}} = \overrightarrow{X_{curr,G}} + \lambda(\overrightarrow{X_{Glob.best,G}} - \overrightarrow{X_{curr,G}}) + \\ + F(\overrightarrow{X_{r2,r2Opt[rand(1..3)]}} - \overrightarrow{X_{r3,r3Opt[rand(1..3)]}}) \quad (1)$$

where $\overrightarrow{X_{r2,r2Opt[rand(1..3)]}}$ and $\overrightarrow{X_{r3,r3Opt[rand(1..3)]}}$ are randomly chosen best positions of agents $r2$ and $r3$ from their last 3 best position, $\overrightarrow{X_{curr,G}}$ is the very last best position of chosen agent, $\overrightarrow{X_{new?,G+1}}$ is possible new best position of the agent $\overrightarrow{X_{curr,G}}$ and F and $\lambda$ are chosen constants (usually in range $\langle 0, 1 \rangle$). The algorithm flow chart can be seen in Fig. 2.

## V. SC Filter Optimization

Each biquad frequency response should be as close to ideal one as possible (especially in case of cascade structure). The goal of optimization process is to find the new capacitor values to minimize the difference between the ideal and real circuit magnitude frequency response with the design tolerance of 10 mdB which leads to filter bank design error under 0.5 % (without considering of capacitor accuracy). The tolerance should be fulfilled in whole frequency range (0 - 8 kHz), especially near the zeros and poles.

The required accuracy is checked by Objective function OF which express a distance (difference) of biquad magnitude frequency response from the band defined by design tolerance. The distance is checked in 250 equidistant frequencies. If the whole biquad magnitude frequency response lie in the tolerance the OF value is zero.

OF progress, its value in iteration of 100 and used computer time have been used to compare performance of different methods.

The algorithm test should be independent on the initial population which is tested, too.

## VI. Results

$DE_2$, PSO and their combination was used for optimization of all biquads. Fig. 3 shows the OF value during optimization process powered by $DE_2$ algorithms. The different settings of method (parameters F and $\lambda$) and different initial population was used to exclude their influence on optimization performance. Unfortunately, the $DE_2$ method was inefficient for $H_{LP2}$ and $H_{HP1}$ biquads for all of the tried settings.

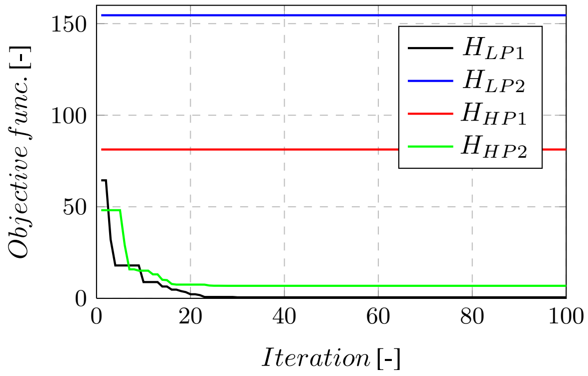


Figure 3. $DE_2$ method performance for all biquad.

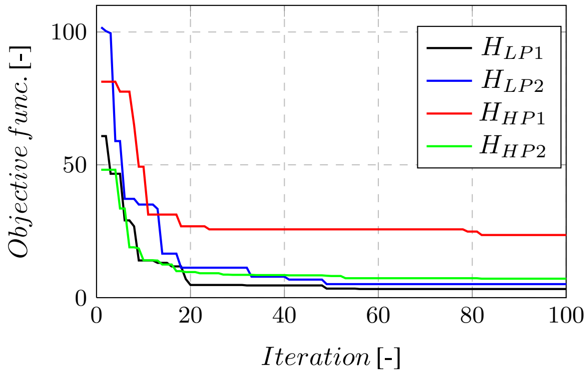


Figure 4. $PSO$ method performance for all biquad.

PSO method was able to optimize all of the biquads. However, the main progress was in first 30 iterations. After $50^{th}$ iteration the method progress technically stopped and agent apparently circle around the optimum (see Fig. 4).

The proposed method based on DE and PSO algorithms (DE-PSOx) were also able to optimize the biquads very well. Moreover, these methods finished for the biquads $H_{LP1}$ and $H_{HP2}$ with the best OF value in iteration of 100. Unfortunately, these methods were also the slowest methods for the optimization of biquads $H_{LP2}$ and $H_{HP1}$ (40 % more then other methods). That was mainly caused be frequent activation of auxiliary PSO method. The progress of the OF value for the DE-PSOx methods can be seen in Fig. 5 and Fig. 6.

The method PSO-DEm was also tested. Although the progress can be seen through whole 100 iterations, the method PSO-DEm offered the best resulting OF value only for $H_{LP2}$ biquad (see Fig. 7). However, the resulting OF for other biquads lie very close to the best results. Moreover the PSO-DEm method were the fastest method from proposed methods. Whereas the other method based on DE method needed 48 hours in average, the PSO-DEm needed only about 16 to 18 hours.

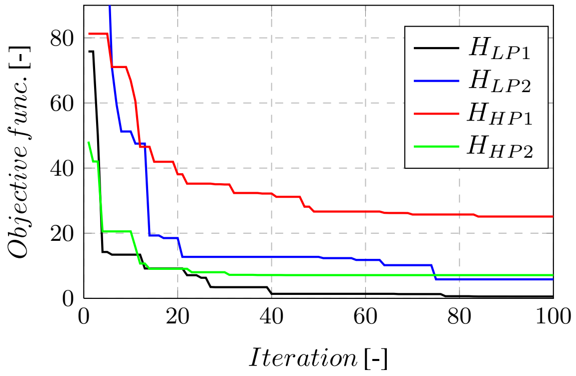


Figure 5. $DE - PSO1$ method performance for all biquad.

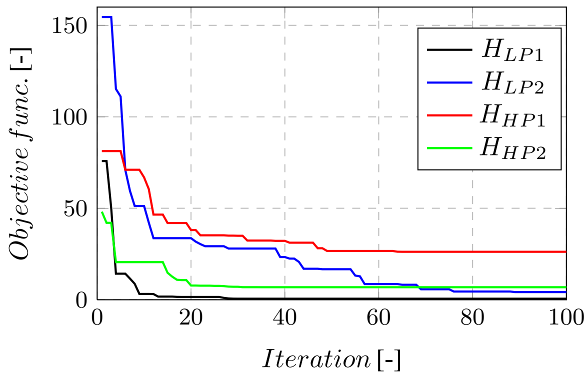


Figure 6. $DE - PSO2$ method performance for all biquad.

## VII. Conclusion

The paper presents a comparison of performance optimization methods suitable for application in SC filters. For this purpose the design of SC biquads in 0.35 um CMOS technology for $4^{th}$ order two-channel filter bank was used. The comparison was performed for $2^{nd}$ scheme of DE method, PSO method and their proposed combinations.

The DE method showed to be inefficient for SC filter optimization. Moreover with requirement of using lower number of agents the $DE_2$ method can stretch agents to one point very quickly, so the method must be restarted. Thus $DE_2$ method can't guarantee the best optimization results in limited time.

Conversely, the proposed DE based methods (DE-PSOx) "restart" DE method during optimization process without complete loss of time. Unfortunately, this repeated start caused that the optimization time increased by about 50-70% compared to other methods. On the other hand, for two biquads ($H_{LP1}$, $H_{HP2}$) these methods offered the best optimization results in iteration of 100.

The PSO based algorithm methods showed their ability to scan optimization space roughly in short time with low number of agents. Moreover, the proposed PSO-DEm method offered comparable results to DE-PSOx methods in lower time. Therefore the PSO based method can be recommended for SC filter optimization.

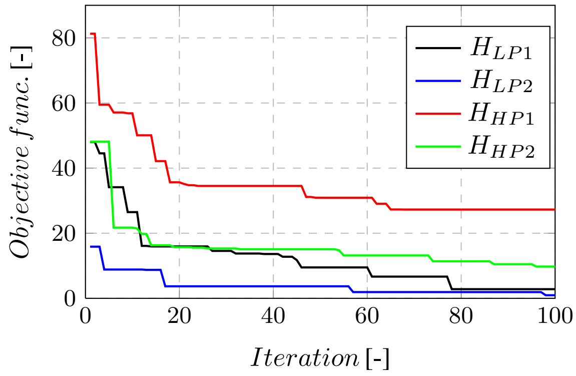


Figure 7. $PSO - DEm$ method performance for all biquad.

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## References


[1] P. V. A. Mohan, V. Ramachandran, M. N. S. Swamy, "Switched capacitor filters - theory, analysis and design," Prentice Hall International, 1995, ISBN 0-13-879818-4.

[2] B. J. Sheu, Hu Chenming , "Switch-induced error voltage on a switched capacitor," Solid-State Circuits, IEEE Journal of, vol.19, no.4, pp.519,525, Aug 1984 doi: 10.1109/JSSC.1984.1052176.

[3] K. Martin, A. S. Sedra, "Effects of the op amp finite gain and bandwidth on the performance of switched-capacitor filters," Circuits and Systems, IEEE Transactions on, vol.28, no.8, pp.822,829, Aug 1981 doi: 10.1109/TCS.1981.1085052.

[4] S. Yuhui, R.C. Eberhart, "Empirical study of particle swarm optimization," Evolutionary Computation , 1999. CEC 99. Proceedings of the 1999 Congress on, vol.3, no., pp.,1950 Vol. 3, 1999 doi: 10.1109/CEC.1999.785511

[5] A. Jafari, S. Sadri, M. Zekri "Design optimization of analog integrated circuits by using artificial neural networks," Soft Computing and Pattern Recognition (SoCPaR), 2010 International Conference of , vol., no., pp.385,388, 7-10 Dec. 2010, doi: 10.1109/SOCPAR.2010.5686736

[6] M. Barari, H. R. Karimi, F. Razaghian, "Analog Circuit Design Optimization Based on Evolutionary Algorithms," Mathematical Problems in Engineering, vol. 2014, Article ID 593684, 12 pages, 2014. doi:10.1155/2014/593684

[7] R. Storn, K. Price, "Differential evolution - a simple and efficient adaptive scheme for global optimization over continuous spaces," Technical report TR-95-012, ICSI, 1995.

[8] J. Kennedy, R. Eberhart, "Particle swarm optimization," Neural Networks, 1995. Proceedings., IEEE International Conference on, vol.4, no., pp.1942,1948 vol.4, Nov/Dec 1995 doi: 10.1109/ICNN.1995.488968

[9] M. Smith, WinSpice User's Manual [online] 2009 [cit. 2012-10-06], URL: http://www.winspice.com, 2012.

[10] J. Nahlik, B. Mullane, J. Hospodka, P. Sovka, D. O'Hare, "Optimized switched capacitor biquads for two-channel quadrature-mirror filter bank," Irish Signals Systems Conference 2014 and 2014 China-Ireland International Conference on Information and Communications Technologies (ISSC 2014/CIICT 2014). 25th IET, pp.412,417, 26-27 June 2013, doi: 10.1049/cp.2014.0724

[11] J. Bicak, J. Hospodka, "Frequency Response of Switched Circuits in SPICE," ECCTD'03 Proceedings of the European Conference on Circuit Theory and Design 2003, Monterey: IEEE Circuits and Systems Society, 2003, vol. 3, p. 333-336. ISBN 83-88309-95-1.