Applicative Appraisal of the Temperature Dependence of the Noise in Preamplifiers

The Very Low Frequency Case

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Abstract – **Several matters related to noise design for preamplifiers are presented for very low frequencies, also pointing to issues raised by some circuits in the literature, including those with noise cancelling circuits. The changes in excess noise with temperature are of interest in applications such as wearable, where large variations of temperature can occur.**

Keywords- excess noise, very low frequencies, 1/f noise, Hooge' law, temperature variation, amplifier design

I. Introduction

Low noise amplifiers (LNAs) are used in virtually all precision measurements, including medical equipment. Bioinstrumentation amplifiers work in a frequency range from about 0.01 Hz to a few hundred Hz (higher for fiber EMG). The lowest frequency limit used in ECG is 0.05 Hz; 0.1 to 0.5 Hz is typical for the lower limit in EEG ([1] p. 9), while [2] recommends 'full-band EEG', FbEEG, with a much larger bandwidth because "*physiological and pathological EEG activities range from* 0.01 *Hz to several hundred Hz*". The challenge in medical applications – actually in numerous other fields where VLF signals occur– is that the signals to amplify have a $1/f$ power spectral distribution, the same type as the so-called excess noise in all electronic devices. For example, EEG, MEG, neural activity and even cognition processes exhibit this power spectral density (psd) [3, 4]. Therefore, achieving a high signal to noise ratio (SNR) at very low frequencies (VLFs) is imperative, but hard.

The purpose of this paper – a review as much as an original study – is to put down facts that should be considered in the design and operation of the low noise instrumentation amplifiers (LNIAs) for VLFs, in the range 0.001-100 Hz, as used in equipment for seismology and bioinstrumentation, among others. The focus is on VLFs lower than 1 Hz. Current LNIAs may have circuits for offset compensations, input current nulling and noise cancellation. Only high quality LNIAs, typically including at least the offset and bias current compensations, are briefly discussed. The paper is complemented by [5].

The temperature variation will be discussed in terms of temperature coefficient (TC). The use of the TC is of limited use in assessing the temperature variation of the noise on larger intervals because of the nonlinear (exponential) nature of variation of several terms in the noise equation. Therefore, the TC may be misleading in the design for applications where operation under large temperature ranges is required, such as biopotential amplifiers for wearable devices and for sensors used in the natural environment.

II. Instrumentation Amplifier Noises

A. Basics

Consider the noise in the typical topology of instrumentation amplifiers (IAs) – the three operational amplifiers (OAs), symmetric topology, as sketched in Fig. 1(c), where the impedances and signal sources in front of the amplifier are modeling the sources of perturbations. The gain of the IA is $G_{IA} = 1 + 2R_f/2r_0$, where we denoted $2r_0$ the single resistor "seen" by both OAs in the input stage. In the equivalent noise scheme, pictured for a single OA in Fig. 1(a), we used the convenient and widespread (but not always correct) convention of merging the two input noise generators in a single one. We will use in computations the scheme with two independent noise current sources, as in Fig. 1(b). The current sources will be denoted by $i_+^2(f) = i_-^2(f) = i^2(f)$ drawing the attention that the schemes in Fig. 1(a) and 1(b) are not equivalent, that the correct one is the one in Fig. 1(b) despite the fact that many application notes and textbooks use the circuit in Fig. 1(a), and that the value of $i^2(f)$ in Fig. 1(b), when seen by the generator resistance, is half the value of $i^2(f)$ in Fig. 1(a). The symmetry of the IA is assumed perfect, that is the two feedback resistances R_f are identical and the same for the ones of the summing stage, R_S.

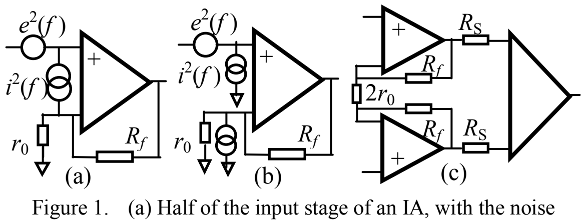

Figure 1. (a) Half of the input stage of an IA, with the noise generators at the input and the current generators merged. (b) With two current generators. (c) The 3-OAs topology for IAs

The OA's noise voltage generator, denoted by $e_a^2(f)$, has a power spectral distribution with a white noise component and an $1/f$ component, that is, $e_a^2(f) = a + bf^{\alpha_1}$, where $\alpha_1 = 1$ for ideal 'pink' noise. Similarly, the power spectral distribution, $i_a^2(f)$, of the OA noise current generator is $i_a^2(f) = A + Bf^{\alpha_2} = 2i^2(f)$, with $i_+^2(f) = i_-^2(f) = i^2(f)$. Notice in Fig. 1(b) that the current generator $i_-^2(f)$ 'sees' r_0 in parallel with R_f. It produces on r_0 a noise voltage seen at the input, $\left(r_0 \frac{R_f}{r_0+R_f}\right)^2 i^2(f)$, see Fig. 2(a).

ISBN 978-80-261-0386-8, © University of West Bohemia, 2015

The resistor r_0 also has a thermal (Johnson) noise with power density $4kTr_0$, which sums with the voltage produced by $i_-(f)$. Not shown in Fig. 1 is the signal generator, with internal resistance R_g, half of which is distributed to each amplifier. This resistance produces an input noise $4kTR_g/2$ at the input of each OA. There is also the thermal noise of the feedback resistance R_f, $4kTR_f$, see Fig. 2(b). This generator is seen at the inverting input as $4kTR_f \times \left(\frac{r_0}{r_0+R_f}\right)^2$. Finally, r_0 produces $4kTr_0$, which must be added.

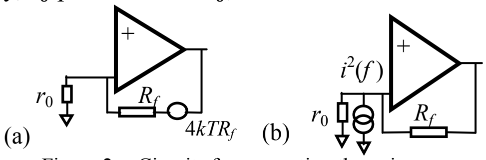

Figure 2. Circuits for computing the noises

Summing all noise contributions referred at the input, and including the noise of the generator, $4kTR_g$, one obtains the total noise density at the input as

$$2e_a^2(f) + 2\left(r_0 \frac{R_f}{r_0+R_f}\right)^2 i_-^2(f) + \left(i_+^2(f) + i_-^2(f)\right)R_g^2 + 4kTR_f \cdot \left(\frac{r_0}{r_0+R_f}\right)^2 + 4kT(2r_0 + R_g)$$

Considering $i_+^2(f) = i_-^2(f) = i^2(f)$, using the amplification of a single OA as $G = 1 + R_f/r_0$, and using the expressions of $e_a^2(f)$ and $i^2(f)$, one derives that the power spectral density of the total input noise of the input stage of the IA is given by (1),

$$e_t^2(f) = 2(a + bf^{\alpha_1}) + 2(R_g^2 + R_f^2/G^2)(A + Bf^{\alpha_2}) + 4kT\left(R_g + 2r_0 + 2R_f/G^2\right) \quad (1)$$

thus,

$$e_t^2(f) = 2\left(a + A(R_g^2 + R_f^2/G^2)\right) + 4kT\left(R_g + 2r_0 + 2R_f/G^2\right) + 2\left[\left(R_g^2 + \frac{R_f^2}{G^2}\right)Bf^{\alpha_2} + bf^{\alpha_1}\right] \quad (2)$$

In (2), the excess noise of the external resistors is not included. When $\alpha_1 = \alpha_2 = -1$, in a frequency band $B_f = [f_L, f_H]$, the total input noise $n[f_L, f_H]$ is obtained by integration of (2) as

$$n^2[f_L, f_H] = 2\left(a + A\left(R_g^2 + \frac{R_f^2}{G^2}\right)\right)(f_H - f_L) + 4kT\left(R_g + 2r_0 + 2R_f/G^2\right)(f_H - f_L) + 2\left(b + B(R_g^2 + R_f^2/G^2)\right)\ln(f_H/f_L) \quad (3)$$

In the next subsections we are interested in the excess noise of the external resistors, r_0 and R_g.

III. Excess noise in resistors: 1/f or 'Hooge's law'?

A. Hooge's law and approximation formulas

One source of confusion in some data sheets and textbooks is the use of approximated formulas for the computation of the 1/f noise of the resistors in a frequency band. This subsection clarifies the issue and suggests other approximations. The excess noise of resistors and other devices is a noise that exceeds the thermal noise and that appears only when a voltage is applied to the device [6]. Because it appears only when a current passes through, the excess noise is also named *current noise* [6], [7]. 'Hooge's empirical law' [8] essentially says that the energy per bandwidth Δf around a frequency f and for specified operation conditions has the form $ct.\frac{\Delta f}{f}$ [8]. Probably the next remark is known, but still worth mentioning it.

Remark 1. For narrow bandwidths, the so-called 'Hooge's empirical law', determined for resistors, can be applied for the excess noise approximation. Indeed, applying Taylor series at f_1, $\ln\frac{f_2}{f_1} = \ln f_2 - \ln f_1 = \ln f_1 + \frac{f_2-f_1}{f_1} + \cdots - \ln f_1 \approx \frac{\Delta f}{f_1}$, where $\ln f_2 \approx \ln f_1 + \left.\frac{d\ln f}{df}\right|_{f_1} \times (f_2 - f_1) + \cdots$. However, [6], [8] use this law for Δf of a decade of frequencies, for which the error becomes consistent and should be included in the constant if $f_1 \geq 1$. For example, for $f_H = 1$ and $f_L = 0.1$, $\ln 1/0.1 = \ln 10 \approx 2.3$, while, even taking the center of the interval 1…10 instead of f_L, $\frac{\Delta f}{f} = \frac{9}{5.5} = 1.63$. However, for $f_L < f_H$ and both close to 1, the approximation of $\ln f \approx f - 1$ is good, hence the approximation of $\ln f_H - \ln f_L \approx f_H - f_L$ is also good. This approximation is poor for values significantly lower than 1. It must be emphasized that Hooge worked mostly with the general law 1/*f*, see [9], [19].

Various other approximations can be derived from the logarithm approximations, for example:

Remark 2. For frequencies $\ll 1$, the power per decade of frequency of the 1/*f* noise is approximately $C\left(\frac{(10f)^{10f}-1}{10f} - \frac{f^f-1}{f}\right) = C\frac{(10f)^{10f}-10f^f+9}{10f}$, because a good approximation of the logarithm is $\ln x \approx \frac{x^x-1}{x}$. Also, using the equality $\ln x = 2\, artanh\, \frac{x-1}{x+1}$ and the approximation $artanh\, \frac{x-1}{x+1} \approx \frac{x-1}{x+1}$, good around $x = 1$, the excess noise per decade is $\ln f_H - \ln f_L = 2\left(\frac{f_H-1}{f_H+1} - \frac{f_L-1}{f_L+1}\right)$ which, for $f_H \approx f_L \approx 1$, $f_H - f_L \ll 1$ produces $f_H + 1 \approx f_L + 1$, so $\ln f_H - \ln f_L$ can be further approximated as $2(f_H - f_L)/2\frac{f_H+f_L}{2} = \Delta f/f$, as for Hooge' law. As the approximations leading to Hooge's law are too restrictive for large bandwidths, we suggest the use of the 1/*f* law every time in design.

B. Issues related to the excess noise in resistors – resistance dependence

The excess noise of resistors is typically given in datasheets as a factor, specific to that type and value of resistor, $F\ [\frac{\mu V}{V}]$. The definition of this factor is 'excess noise produced by a resistor in *a frequency decade* when a voltage $U = 1V$ is applied to it and the definition corresponds to 'Hooge's empirical law'. Thus, noise factor is computed based on F as $e^2[f_L, 10f_L](\mu V)^2 = F^2U^2 = \int_{f_L}^{10f_L} e^2(f)df$. As the definition has to remain valid for any frequency decade, $\int e^2(f)df = \phi(f)$ should satisfy the functional equation $\phi(mf_L) - \phi(f_L) = \psi(m) \quad \forall f_L$. This is non-trivially true for $\phi(f) = \psi(m)\ln f$, hence this is a solution of the functional equation.

Notice that the 'definition relation' $e^2[f_L, 10f_L](\mu V)^2 = F^2U^2$ is in fact a hypothesis that seems to be verified in all experiments, but in this hypothesis the value of the resistance is not present, implying either that F is a constant for all values of the resistor R and depends solely on the technology and material, or that in fact $F = F(R)$. Several manufacturers, e.g. [6], and several authors [9], [11-18] imply in their datasheets and papers that, at least for a large range of not too high values ($R < 1-10\ k\Omega$), $F(R)$ is a constant. Let us see if this hypothesis makes sense physically. Apply to the resistor R a mental experiment, breaking it in two equal parts of resistance $R/2$ united by a perfect (zero resistance) contact. Consider first the Johnson noise. The two imaginary equal, series resistances have each a noise $e_{1,2}(f) = \sqrt{2kTR}$, which, when summed and squared, produce $(e_1 + e_2)^2 = 4kTR$, *assuming that the two noises are uncorrelated*, which is true for Johnson noise. Let us assume now that the initial resistor has factor F and that the two parts preserve the same F value (F independent of R). Then, each will produce a noise voltage in a bandwidth B_w of value $e_{1,2}(B_w) = F\frac{U}{2}$ and the total noise is $e^2(B_w) = (e_1 + e_2)^2$. Assuming no correlation between the two (imagined) parts of the initial resistor,

$$e^2(B_w) = 2 \times F^2U^2/4 = F^2U^2/2 \neq F^2U^2.$$

Thus, either the two noises are (totally) correlated, or F should be a function of R, with $F\left(\frac{R}{2}\right) = \sqrt{2} \times F(R)$. Similarly, if we broke the resistor in p equal pieces, the factor should satisfy $F\left(\frac{R}{p}\right) = \sqrt{p}F(R)$, or, again, the noises should be totally correlated. One can repeat the reasoning by breaking the resistor in p equal resistors in parallel.) The functional equation $F\left(\frac{x}{p}\right) = \sqrt{p}F(x)$ is satisfied by $F(x) = 1/\sqrt{x}$; so, one can infer that $e^2[f_L, 10f_L](\mu V)^2 = F^2U^2$ implies that $F = C/\sqrt{R}$. Taking into account that $\ln F = \ln C/\sqrt{R} = \ln C - \frac{1}{2}\ln R$ and comparing with the curves in [6, p. 45], there is no way to achieve agreement, because the experimental slope is positive. On the other hand, on Au films, [9] found (see Fig. 4 in [9] and notice that the horizontal axis is in fact logarithmic, not linear as stated) a dependence $\ln F = ct. \times \ln R$, with slope 1, not agreeing with the -0.5 slope as above. However, because these authors misinterpret their log-log graph their conclusions based on that graph are wrong. For a detailed analysis of the several models proposed successively by Hooge, see [10], [19].

Consider that the $1/f$ noise is given by $i^2(f) = \frac{C\,I^2}{f}$, $e^2(f) = R^2\frac{C\,I^2}{f} = \frac{C\,U^2}{f}$, where I is the current through the resistor. Imagine that the resistor is split in p equal resistors along its length such that there are p resistors equal with pR in parallel. Assume the current is kept constant. Through the resistors, a current I/p passes. It produces a noise density $i_k^2(f) = CI^2/p^2f$ through the k^{th} resistor. Summing over all resistors, assuming total lack of correlations between the currents $I_k, I_j, k \neq j$, the total noise current is $i^2(f) = CI^2/pf \neq CI^2/f$. To correct this impossibility one must admit either a total correlation between the elementary currents, with unclear meaning, or that $C = C(1/I)$. Because C has no physical unit (is a scalar), one must admit $C = C(I_0/I)$, where I_0 is a constant of unknown nature. Writing $C(I_0/I) = C\left(\frac{U/R_0}{U/R}\right) = C\left(\frac{R}{R_0}\right)$, where R_0 is a constant (for all resistors) value of unknown for yet nature, it results that the current noise density increases with the value of the resistor, which is consistent with experimental findings. The dependence of $C(R)$ will change when there is a spatial correlation between the currents in various parts of the resistor. The nature of R_0 remains to be established. The above discussion may be useful in guiding designers: increasing the values of resistors on the chip will increase the excess noises as R^{χ}, when keeping the voltage constant on them; using external resistors is submitted to the same rule; using larger supply voltages that translate by larger voltages on the internal and external resistors will increase quadratically the excess noises.

Little is known about the change of the excess noise with temperature; on gold films, [9] found no variation of the excess noise with temperature. Other authors, report some variations. Detailed information about the issue is given in [19]. Consequently, we will not deal with this variation, assuming that it is null.

C. *Noise factors for composed resistors*

Consider two resistors, R_1 and R_2 having the noise factors F_1 and F_2. Then, for the series connection, $R_{ser} = R_1 + R_2$, under voltage $1\ V$, using $e^2(f; R) = CU^2/f$, the total excess noise constant, C_{ser}, is $(C_1R_1^2 + C_2R_2^2)/(R_1 + R_2)^2$, while for the parallel connection, the noise constant is $C_p = \left(\frac{C_1}{R_1^2} + \frac{C_2}{R_2^2}\right)\frac{R_1^2R_2^2}{(R_1+R_2)^2}$. An interesting case is when one of the serried resistors is in fact the contact resistance of the resistor. While for the contacts the resistance is low, the literature suggests that contacts may have much higher F, which corroborated with the expression of C_{ser} may explain why low value resistors (under $1\ k\Omega$) seem to have F independent of the resistance value, which does not happen for larger resistances.

IV. Noise Temperature Sensitivity in IA

When the temperature changes, the values of the resistors R_g, R_0 and R_f have a variation of about 0.01%/°C, which will be neglected. We also neglect the excess noise on the external resistors and the variation of that noise. The temperature sensitivity of the total noise is then

$$\frac{d}{dT}n^2 = 2\left(\frac{da}{dT} + \frac{dA}{dT}\left(R_g^2 + \frac{R_f^2}{G^2}\right) + 2k\left(R_g + 2r_0 + 2R_f/G^2\right)\right)(f_H - f_L) + 2\left(\frac{db}{dT} + \left(R_g^2 + \frac{R_f^2}{G^2}\right)\frac{dB}{dT}\right)\ln\frac{f_H}{f_L} \quad (4)$$

However, this is a formal representation of limited use. The manufacturers do not provide any indication on the temperature variation of the noise of the amplifiers. Yet, the temperature coefficients in (4) can be estimated, knowing the type of transistors in the input stage. Subsequently, we clarify the meaning of the parameters in (5) for a typical IA. Because the interest is for VLFs, one neglects all capacitors in the equivalent schemes.

Two cases of amplifiers must be analyzed: that with input bias current cancelling, respectively the ones without bias compensation. In the two cases, there are significant differences between the circuit elements contributing to noise and thus to noise variation with temperature. The computation of the noise of the bipolar junction transistor (BJT) and of the noise of the corresponding differential stage with common emitter is well-known and sketched in Fig. 3.

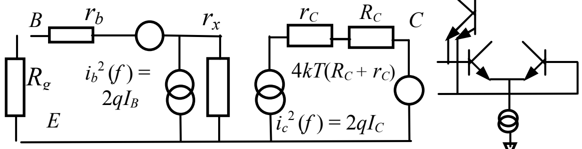

Figure 3. Equivalent noise circuit for the input bipolar transistor (left) and bias compensation circuit (right)

In case of the input stage based on bipolar transistors, the noise term a in (1) is given by the resistances of the bases, $4kTr_b$, by the noise in the collectors of the input stage referred at the input, $4kTr_c/\beta$, where β is the current gain of the input stage, and by the shot noises created by the basis current, $2qI_B$ on the basis resistor, $2qI_B r_b$, moreover by the collector current. So, the sum of Johnson and shot noise referred at the basis is – taking also into account the generator resistance but not its thermal noise: $e_{white}^2(f) = 4kTr_b + 2qI_B\left(r_b + R_g\right)^2 + (4kT(r_c + R_C) + 2qI_C(r_c + R_C)^2)/g_m$.

In medical applications, e.g., $R_g \approx 1\,k\Omega \gg \mathrm{r_b} \approx 10 \ldots 100\ \Omega$, so, r_b can be neglected. Similarly, r_c can be neglected because $r_c \approx 10 \ldots 100\ \Omega \ll R_c{\sim}1\,..10\ k\Omega$. The input noise expression is accordingly simplified. Further, the expressions of the currents are well known, e.g. I_B also has an exponential variation, $I_B = qD_n n_b e^{v_{EB}/V_T}$, with D_n the diffusion constant, n_b the number of carriers. Thus, the temperature coefficients can be determined. Notice that, especially in the input bias current compensated OAs, the temperature change in the input current is a poor indicator of the shot noise increase in the OA. Instead, the total change in the various currents seen at the input should be considered, including currents in the compensating devices, see Fig. 3. Further details are given in [5], [20]-[23].

V. Discussion and Conclusions

The paper clarified and partly unified some concept and design techniques in view of improving the performances of LNIAs for VLF. A special attention was paid to excess (current) noises in resistors and to the implications of the presence of these noises. It was stressed that the study of temperature variation of the $1/f$ noises, at VLFs, in IAs is still in its infancy and that efforts should be made to enhance our understanding on the topic. The study has implications in the design of wearables for health monitoring, among others.

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