A NOVEL MULTI-ELEMENT, FOUR-PARAMETER WINDKESSEL MODEL OF THE ARTERIAL TREE

Zdravko VIRAG[1], Fabijan LULIĆ[2], Ivan KORADE[1]

[1] University of Zagreb, Faculty of Mechanical Engineering and Naval Architecture, Ivana Lučića 5,
Zagreb, Croatia, E-mail: zdravko.virag@fsb.hr; ivan.korade@fsb.hr;

[2] Pulmonary Disease Clinic, University of Zagreb School of Medicine, Jordanovac 104, Zagreb,
Croatia, E-mail: fabijan.lulic@kbc-zagreb.hr;

1. Introduction

Windkessel models of the arterial tree play an important role in clinical practice as well as in prescribing the boundary conditions of one and three-dimensional simulations of blood flow. Such models should describe the main features of the arterial tree (compliance, resistance, inertance and energy dissipation) realistically by using the smallest possible number of parameters. All existing windkessel models consist of a few basic elements modeling inertance, compliance and resistance of the arterial tree, and they can describe the arterial input impedance realistically, but they can model neither the pulse pressure nor flow wave attenuation from the arterial inlet to the capillary outlet.

The goal of this work is to define a novel tree-like structure windkessel model, with only four free parameters which can also model the pressure and flow wave attenuation toward the arterial periphery.

2. Materials and methods

Fig. 1 shows the analog electrical scheme of the proposed windkessel model. Capacitance C_0 and resistance η_0 represent a viscoelastic chamber modeling large arteries. Inertance and resistance in large arteries are modeled by inductance L_0 and resistance r_0. The rest of the arterial tree is modeled by N generations of bifurcating blood vessels. Each generation of blood vessels contains 2^n identical vessels modeled by L_n and r_n while each bifurcation point is a viscoelastic chamber defined by C_n and η_n, where n is the generation number ($n = 1, N$). It is assumed:

$$r_n = f_r r_{n-1},\quad C_n = f_C C_{n-1},\quad L_n = f_L L_{n-1},\quad \eta_n = f_\eta \eta_{n-1} \qquad (1)$$

where $f_r = f_\eta = \sqrt[3]{16}$, $f_C = f_L = 1/f_r$. The total peripheral resistance is defined as:

$$R = r_0 + \sum_{n=1}^{N} r_n / 2^n = r_0 \left[1 + \sum_{n=1}^{N} \left(\frac{f_r}{2} \right)^n \right] \qquad (2)$$

The total arterial compliance is defined as:

$$C_{\mathrm{tot}} = C_0 + \sum_{n=1}^{N} 2^{n-1} C_n \qquad (3)$$

It is assumed that the compliance of large arteries ($C_0 + C_1$) is 2/3 of the C_{tot}. Thus, for a given C_0 and taking into account Eq. (1), all other C_n are uniquely defined.

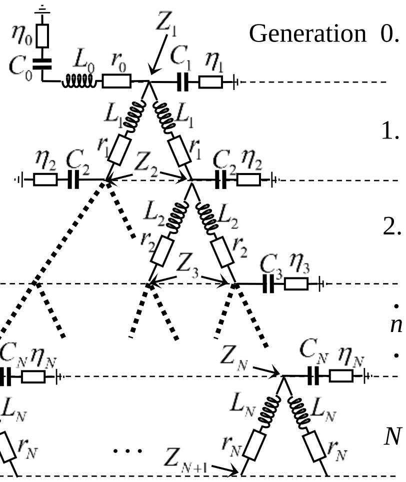

Fig. 1. Electrical analog scheme of the proposed windkessel model.

The impedance of the n-th generation is defined in the frequency domain by:

$$Z_n(\omega) = \left[\frac{sC_n}{1 + sC_n\eta_n} + \frac{2}{sL_n + r_n + Z_{n+1}} \right]^{-1} \qquad (4)$$

where $s = \sqrt{-1}\omega$, and ω is the circular frequency.

Eq. (4) is applied from $n=N$ to $n=1$, and the input impedance is

$$Z_{\text{in}}(\omega)=\frac{\hat{p}_{\text{in}}(\omega)}{\hat{Q}_{\text{in}}(\omega)}=\left[\frac{sC_0}{1+sC_0\eta_0}+\frac{1}{sL_0+r_0+Z_1}\right]^{-1} \quad (5)$$

where $\hat{p}_{\text{in}}$ and $\hat{Q}_{\text{in}}$ are harmonic phasors of the Fourier series of input pressure and flow, respectively. Free parameters in the model are C_0, η_0, L_0, and r_0.

3. Results and discussion

The model was applied to the systemic arterial tree in a middle-aged man and a pig with available data of input flow and pressure. The model parameters were obtained by minimizing the difference (pressure root mean square error – PRMS) between the measured pressure and pressure calculated from the model using Eq. (5). Figs. 2 and 3 show obtained results in the man, and pig, respectively. In both cases $N=20$ generations of blood vessels were used.

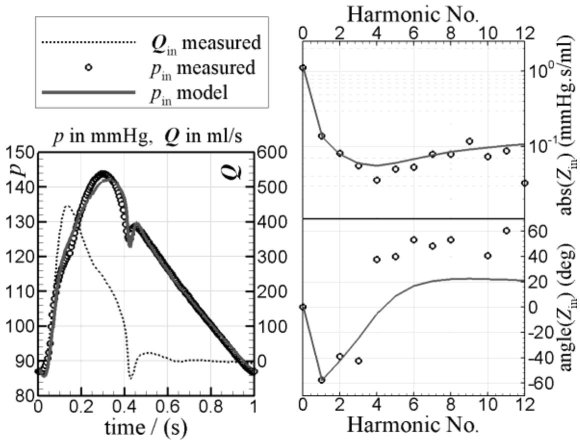

Fig. 2. Results for a middle-aged man, data from [1]. Model parameters: $C_0 = 0.717$ ml/mmHg, ($C_{\text{tot}} = 1.238$ ml/mmHg), $R = 1.15$ mmHg·s/ml, $L_0=2.62\cdot10^{-3}$ mmHg·s^2/ml, $\eta_0 = 0.141$ mmHg·s/ml, achieved PRMS=1.76 mmHg.

In both cases the proposed model describes the arterial input impedance very well. The achieved PRMS is acceptably small. The absolute value of the impedance is minimal at the frequency (or harmonic number) where the angle of the input impedance changes its sign. Westerhof's three element windkessel model (the most frequently used one) can show neither positive values of the angle of Z_{in}, nor local minimum in the absolute value of Z_{in}. This can be seen only in windkessel models with at least five elements (containing at least two capacitors connected by an inductor).

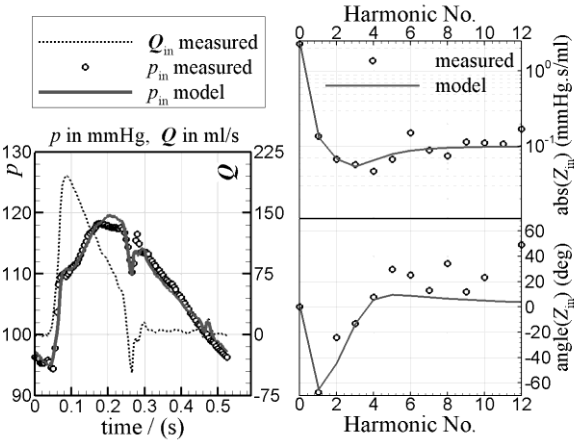

Fig. 3. Results for a pig, data from [2]. Model parameters: $C_0 = 0.374$ ml/mmHg, ($C_{\text{tot}} = 0.646$ ml/mmHg), $R = 2.31$ mmHg·s/ml, $L_0=2.57\cdot10^{-3}$ mmHg·s^2/ml, $\eta_0 = 0.102$ mmHg·s/ml, achieved PRMS=1.25 mmHg.

Fig. 4 shows the fall of the systolic, mean and diastolic pressure in consecutive generations of the pig blood vessels. The generation denoted by zero models the large arteries (let say aorta), and the last generation models arterioles next to the capillaries. A significant reduction in the mean and pulse pressure occurs at the last six generations.

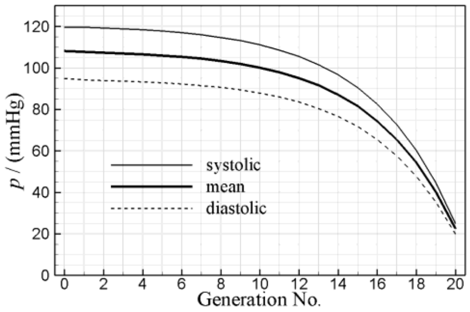

Fig. 4. Attenuation of the pressure pulse.

The proposed model realistically describes the input impedance of the arterial tree as well as the waves attenuation in peripheral small vessels using only four model parameters.

References

[1] Nichols, W.W; O'Rourke, M.F.: McDonald's Blood Flow in Arteries, (5th ed.) Oxford Univ. Press, New York, USA, 2005.

[2] Segers, P., Stergiopulos, N. et al.: Systemic and pulmonary hemodynamics assessed with a lumped-parameter heart-arterial interaction model. J. of Eng. Math., 47, 2003, pp. 185-199.