COMPARISON OF METHODS FOR CALCULATION THE DEFLECTION OF COMPOSITE WOUND TUBES

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1. Introduction

The aim of this work is to facilitate methods for design of composite wound tubes. The comparison of several analytical methods and FEA methods is performed. This paper is focused on the application of the known methods of computing the deflection of composite beam deformation and on its experimental validation. The three-point bending experiment was implemented on several composite beams. The results and comparisons are presented in this paper.

2. Methods for the Deflection Calculation

The FEM calculations are performed in Abaqus. The model of the composite beam contains the fixed composite tube loaded by the single force at the free end. The geometry of the beam is the same for all models. (Fig.1.). The material constants are entered as parameters of the model.

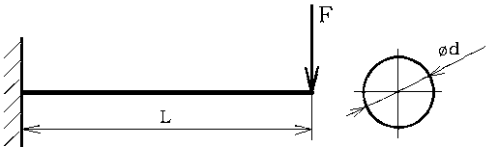

Fig. 1. The geometry of the models.

2.1 Conventional Shell and Continuum Shell

In this model the beam is represented by a shell. The thickness of the shell is specified as a parameter of the model or it is included in ABD matrices which define the stiffness of the beam.

The Continuum Shell is modelled as a solid body. The real thickness of the hollow beam wall is specified as parameter of the model and by the composite layup. The elements are in this model distributed through the whole thickness of the solid body.

The advantage of the Conventional Shell model is the simplicity of the whole model, its preparation and the computational simplicity. The modelled volumes must have the disposition to observing the assumptions for using the shell theory. [1] The Continuum Shell can be stacked. The meshing is more difficult with Continuum Shell because the thickness of the shell must be meshed. Consequently the computation is more difficult too.

2.2 Volume Model

For the calculation using the volume model the full 3-D geometry is specified. Each ply could be created separately as a separate solid body with its own material specification or the composite layup should be assigned to elements by the parameters of the model. This model is most demanding to the preparation and the computation time, but it is more detailed than the other models.

2.3 Analytical methods

The aim to determine the bending stiffness is approached applying the Hooke's law for plane stress state of orthotropic layer under the assumptions of Bernoulli's beam theory. The compliance matrix of orthotropic material is considered

$$C_{xy} = S_{xy}^{-1} = \begin{bmatrix} \frac{1}{E_x} & -\frac{\nu_{yx}}{E_x} & 0 \\ -\frac{\nu_{xy}}{E_y} & \frac{1}{E_y} & 0 \\ 0 & 0 & \frac{1}{G_{xy}} \end{bmatrix} \qquad (1)$$

As the bending stress has the direction of the x-axis the modulus of elasticity E_x from the compliance matrix is used for calculation of the bending stiffness. More details is in [2].

The deflection of the beams is given by Bernoulli's equation

$$w''(x) = -\frac{M_o(x)}{\sum_k E_{x_k} \cdot J_{y_k}(x)} \qquad (2)$$

From this equality it is evident that the main problem is the correct determination of the bending stiffness ($E_x \cdot J_y(x)$) of the composite material.

The other method uses ABD matrices. The ABD matrices are assembled according to the classical laminate theory. The main problem is the determination of the bending stiffness of the whole shell and the related equivalent modulus of elasticity. In this case the equivalent modulus of elasticity of the whole material is obtained from the elements of the tensile stiffness matrix **A** from the equation [3]

$$\begin{bmatrix} \mathbf{N} \\ \cdots \\ \mathbf{M} \end{bmatrix} = \begin{bmatrix} \mathbf{A} & \vdots & \mathbf{B} \\ \cdots & \vdots & \cdots \\ \mathbf{B} & \vdots & \mathbf{D} \end{bmatrix} \begin{bmatrix} \boldsymbol{\varepsilon}^{\circ}{}_{\mathbf{m}} \\ \cdots \\ \mathbf{k} \end{bmatrix} \qquad (3)$$

where **A** is the extensional stiffness matrix, **B** is the bending-extension coupling stiffness matrix and **D** is the bending stiffness matrix. The equivalent Young's modulus E_{eq} is determined from the matrix **A**. Than the equation (2) is used to obtain the deflection as in the previous case.

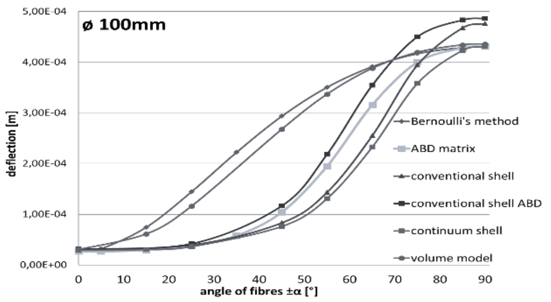

Fig. 2. The comparison of the deflection calculated on the same model by different ways [3].

3. Experiment

The experiment is described in detail in [4]. Two different beams (thick and thin walled) are used. The geometry of the tubes and the material constants are specified. The following experiment contain four types of layup on two diameters of the tubes and the three different materials are used for specimens.

4. Conclusions

The experimental analysis of the three-point bending of the composite tubes was performed. The same problem was modelled by the three different FEM models and two analytical methods for computation of the beam bending stiffness and the deflection. The results of the comparison of all computational methods with the experimental data will be presented. The results are satisfactory in the case of the thin-walled tubes. For the thick-walled case all methods give different values and great deviance compared with the experimental data. This is caused by the composite layup and it is apparent that the used computational methods are not good in predicting the stiffness of thick-walled beams from orthotropic material.

Acknowledgements

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