ON THE POSTERIOR PREDICTIVE IN BAYESIAN FAILURE ANALYSIS

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1. Introduction

This paper presents one way of data processing for estimating the remaining life of a component or system. To achieve this failure distribution is needed. Every distribution is a function of parameter(s). Methods such as probability paper or Maximum Likelihood Estimator (MLE) are used to determine distribution parameters. For both of these methods, it is desirable to have a large number of data, which is not always the case. The data for parameter estimation can be obtained from the generic databases. The most famous example of such a base is OREDA Reliability Data Handbook [1]. The data in every used generic database may deviate from the actual values. The reason is the difference between data gathering conditions and exploration conditions. Bayesian analysis offer unique framework, which can take into account the manufacturer's or database data, combines them with an expert opinion and update data based on evidence. The final result of Bayesian analysis is distribution, i.e. posterior. If the whole posterior distribution is included in prediction instead of posterior expected value, then it gets posterior predictive distribution. The latter is the main scope of this paper.

2. Theoretical background of the Bayesian analysis

Bayesian analysis is based on a combination of prior distribution and the likelihood function as it is represented by Eq. (1)

$$\pi_1(\varphi|x) = \frac{f(x|\varphi)\pi_0(\varphi)}{\int_\Omega f(x|\varphi)\pi_0(\varphi)d\varphi} \qquad (1)$$

where $\pi_1(\varphi|x)$ is posterior distribution, $f(x|\varphi)$ likelihood function, and $\pi_0(\varphi)$ is prior distribution. For certain choices of the prior, the posterior has the same algebraic form, and such a choice is called a conjugate prior [2]. In this paper Gamma exponential conjugate prior were used. Future failures can be predicted by posterior predictive distribution by using Eq. (2).

$$f(x|T) = \int_\Omega f(x|\varphi)\pi_1(\varphi|T)d\varphi \qquad (2)$$

where $\pi_1(\varphi|T)$ is posterior distribution and $f(x|\varphi)$ is model, i.e. some distribution of interest. The posterior distribution of the Gamma exponential model [3] is :

$$f(\lambda|T) = \frac{\kappa_n{}^{\alpha_n}}{\Gamma(\alpha_n)}\lambda^{\alpha_n-1}e^{(-\kappa_n\lambda)} \qquad (3)$$

with hyperparameters $\alpha_n = \alpha_0 + n$ and $\kappa_n = \kappa_0 + \sum_{i=1}^{n} T_i$. The exponential distribution is represented by the Eq. (4).

$$f(t|\lambda) = \lambda e^{(-\lambda t)} \qquad (4)$$

Combining Eq. (3) and Eq. (4) with Eq. (2) it can be shown that the posterior predictive distribution takes form:

$$f(t|T) = \frac{\alpha_n\kappa_n{}^{\alpha_n}}{(t+\kappa_n)^{\alpha_n+1}} \qquad (5)$$

3. Discussion and analysis

Due to the complexity of Bayesian analysis, only one component was analyzed. It is a safety valve block noted as VB1 in functional scheme in Fig. 1. The failure rate of the safety valve block VB1 is estimated from the data in NPRD (Nonelectronic Parts Reliability Data) database [4]. The value is 0.22 failures per year. It is important to note that in all databases of this kind, the failure rate is given as a constant value, in other words, the model of failure is an exponential distribution. In the absence of actual data, five random failures were generated with mean value 0.22. The samples will be implemented in gamma-exponential model.

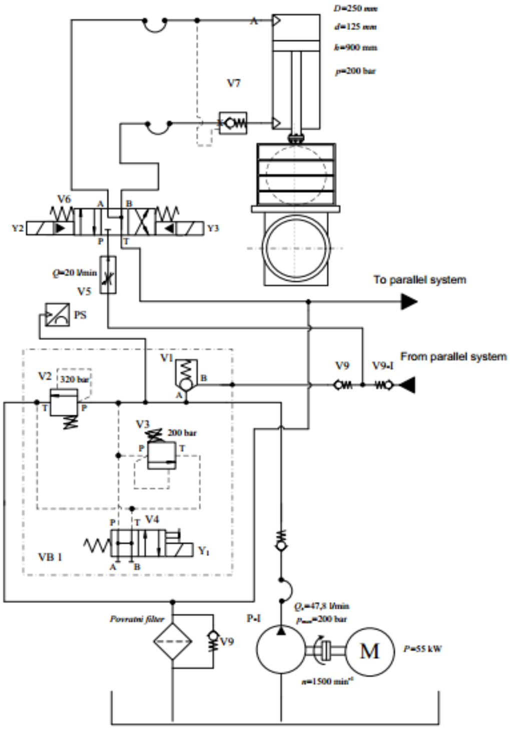

Fig. 1. Functional scheme [5].

Generated random failures are presented in Table 1. The prior parameters are $\kappa_0 = 15$ and $\alpha_0 = 2$.

Table 1. Generated random failures.

	T1	T2	T3	T3	T5
Time	6.2	2.2	3.4	4.7	0.8

Results of Bayesian analysis are presented in Fig. 2. The top plot is prior, middle plot is posterior after just 1 failure, and bottom plot is posterior distribution after all 5 failures. It is worth to note that target value is λ = 0.22 year^{-1}.

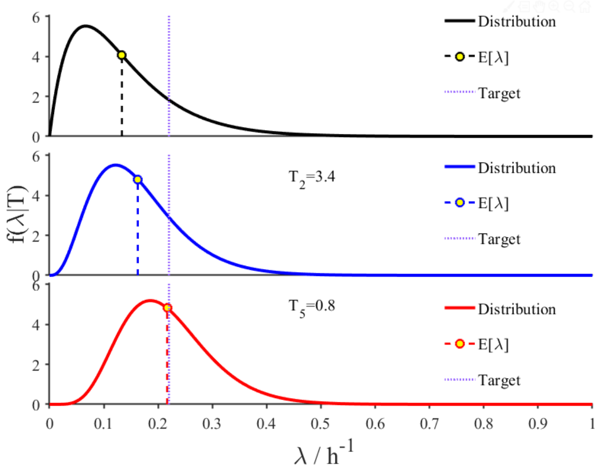

Fig. 2. Convergence of posterior distributions.

Instead of using one point estimation like previously in a Bayesian framework it is possible to use the whole posterior distribution, i.e. posterior predictive. The Fig. 3 shows the posterior predictive distribution (Eq. (5)). By calculating the mean time to failure for every single distribution mean residual life can be estimated.

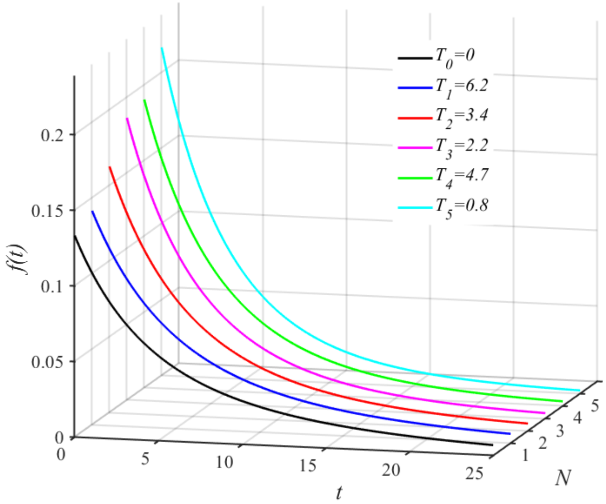

Fig. 3. Posterior predictive distribution of the Gamma-exponential model.

4. Conclusion

Since there is no large amount of data (failures) when reliability is analyzed, Bayesian analysis represents a good choice. It can combine several different sources and update belief when data occur. Generally, databases represent the most common form of data source. Since the failure rate occurs as constant value it is reasonable to use a Gamma-exponential model. It has been shown how to include whole posterior distribution in failure analysis via posterior predictive. Also, it has been shown that it is possible to avoid numerical integration, especially time consuming MCMC method via using conjugate priors

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