Electronic System for Reactive Power Compensation of Sampled Resistive Load

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Abstract – **Although the mechanism of electric energy flow for nonsinusoidal conditions is well described today, so fare is not yet available a generalized power theory, and theoretical calculations for the design of such devices as active filters or dynamic compensators. Therefore the task of designing compensators for optimize energy transmission with nonlinear time-varying loads in nonsinusoidal regimes is, far from clear. In this paper it will show that even if the voltage is sinusoidal and load is pure resistive but sampled, the load current contains reactive part which can be compensated. The compensation is based on the dissipative systems theory which can be used for calculation of compensation elements (capacitors, inductors) for reactive power compensation. The electronic system for automatic reactive power compensation is also described.**

Keywords- active power; apparent power; cyclodissipativity; compensation; dissipative systems; nonlinear load; reactive power

I. INTRODUCTION

One of the most significant current discussions in electrical engineering is the definition of the reactive power (RP) under nonsinusoidal conditions in nonlinear electric systems. New definitions of RP have been discussed in the last years. Although the mechanism of electric energy flow for nonsinusoidal conditions is well described today, so fare is not yet available a generalized power theory, and theoretical calculations for the design of such devices as active filters or dynamic compensators. Therefore the task of designing compensators for optimize energy transmission with nonlinear time-varying loads in nonsinusoidal regimes is, far from clear. Moreover, in the last time more and more the switching power supplies are used. In this paper it will shown that if the sinusoidal voltage and sampled pure resistive load are used, the load current can contain reactive part which can be compensated by means of simple compensating elements - capacitor or inductor [1 -3].

The simplified electrical circuit with sampled resistance load is displayed in Fig. 1. Suppose that

equation of harmonic source is

$$v_S(t) = \sqrt{2} \cdot A_1 \cdot sin(\omega_0 t) = \sqrt{2} \cdot 220 \cdot sin(100\pi t) \quad [V] \quad (1)$$

and load resistor R=10 [Ω]. Switch is sampled (switched ON/OFF) by signal $S(t)$

$$S(t) = \frac{1 + sign\left[sin\left(k_S \cdot 100 \cdot \pi \cdot t - \alpha \right) \right]}{2} \quad (2)$$

synchronously with signal v_S but with k_S time's higher frequency, with phase shift α and width=0.5. The example is shown in Fig. 2, where α=π and k_S=2.
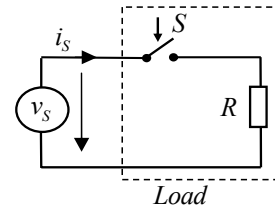

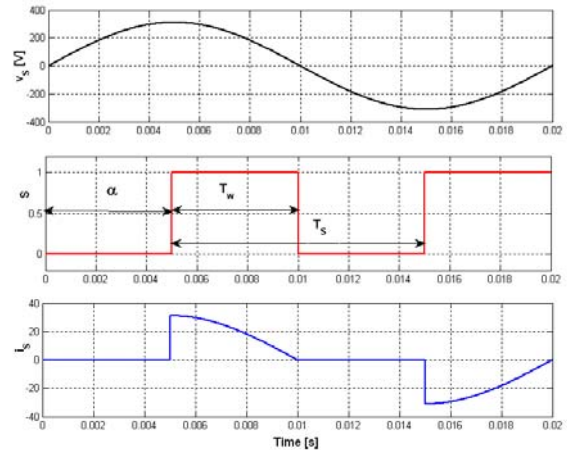Figure 1. Simplified circuit diagram of sampled resistive load

Figure 2. Voltage source, switching signal and current in switched resistive load. Source v_S, (top), switching signal S (middle), current from source i_S (bottom). T_S is sampling period, T_W is width (T_W=T_S/2) and α is phase shift (T_D is T_S/2 for this example and k_S=2).

Time evolution of signals for several values of α is presented in Fig. 3. The simplified electrical circuit with sampled resistance load and compensation admittance Y_C is shown in Fig. 4. The current from power source is i_S, compensation current i_{CO} and load current i. Principle of RP compensation is described in next part [4].
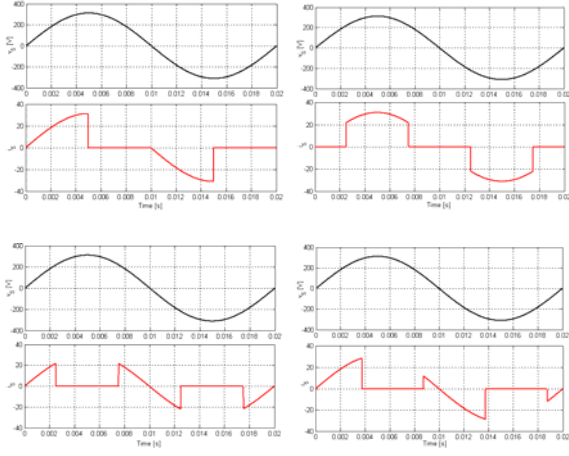
Figure 3. Examples of time evolution of voltage source v_S and current i_S for different phase shifting of $S(t)$. Left top α=0, right top α=π/2, left bottom α=3π/2, right bottom α=7π/4.
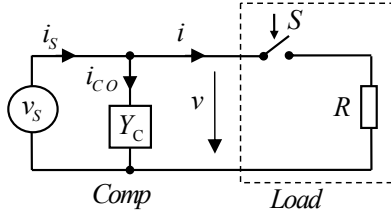

Figure 4. Simplified circuit diagram of sampled resistive load with reactive power compensation by means of compensator Y_C.

II. REACTIVE POWER COMPENSATION PRINCIPLE

The RP compensation of sampling resistive load can be proved by means of spectral decomposition of current i_S. Suppose time evolution of signals according Fig. 2 (sampling with α=π and k_S=2). For non-harmonic current i_S the Fourier decomposition was used. The first harmonics of i_S (designated as ^1i_S) and source voltage ($v_S/10$) is shown in Fig. 5. From this figure it is clear that phase shift exist between voltage and first spectral component of current, therefore it can be compensated.
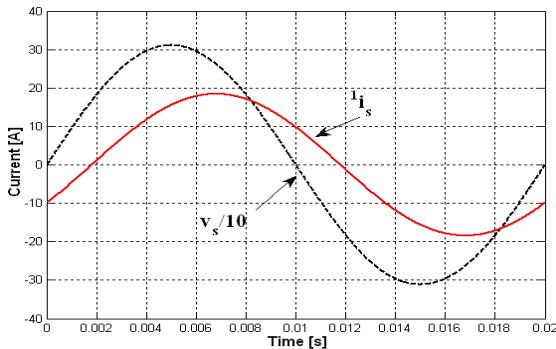

Figure 5. Phase shift between the first spectral component of non-harmonic current i_S (according Fig. 2, α=π and k_S=2) and voltage source $v_S/10$.

The RP compensation is possible by means of capacitor C_C, used instead of compensator Y_C in Fig. 4. The optimal value of capacitor can be find by optimization, see Fig. 6. From this figure it can be seen, that for optimal compensator the effective

current source decreased from 15.5 [A] (without compensation) to 13.9 [A] (with optimal compensation). Voltage and currents after compensation are plots in Fig. 7.
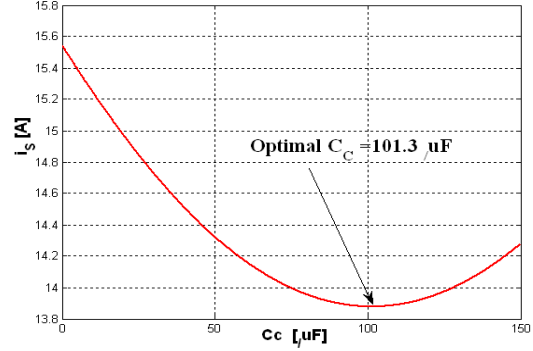

Figure 6. Effective current from source i_S versus value of compensating capacitor (for example according Fig. 2).
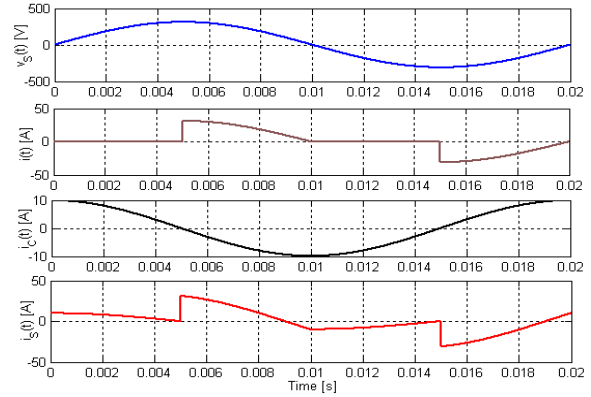

Figure 7. Signals after compensation. From top to bottom: voltage source $v_S(t)$, current through the load $i(t)$, current through the compensation capacitor $i_{CO}(t)$, current from the source $i_S(t)$.

It must be pointed, that is possible calculate optimal value of compensation by means of mathematical approach [5 - 8]. For any given periodic signal $x(t)$ and $y(t)$ with period T the inner product is defined as

$$\langle x,y \rangle := \frac{1}{T}\int_0^T x(t)y(t)dt \qquad (3)$$

and norm of variable $x(t)$ (X is rms – root mean square) is

$$\|x\| = \sqrt{\langle x,x \rangle} := \sqrt{\frac{1}{T}\int_0^T x^2(t)dt} = X \qquad (4)$$

Let's $x(t)$ and $y(t)$ be differentiable periodic functions with period T. Then

$$\left\langle \frac{d}{dt}x,y \right\rangle = -\left\langle x,\frac{d}{dt}y \right\rangle \qquad (5)$$

and also

$$\left\langle \int x,y \right\rangle = -\left\langle x,\int y \right\rangle \qquad (6)$$

The active power delivered by the source is defined as

$$P := \langle v_S, i_S \rangle = \frac{1}{T} \int_0^T v_S^\mathsf{T}(t) i_S(t) dt \qquad (7)$$

where $\langle . \rangle$ denotes the inner product. From (7) and the Cauchy–Schwarz inequality it follows

$$P \le \|v_S\| \cdot \|i_S\| =: S \qquad (8)$$

where the apparent power S is defined. The compensator Y_C is placed in parallel. Also, to avoid power dissipation, Y_C is restricted to be lossless, that is

$$\langle v, i_{CO} \rangle = 0 \qquad (9)$$

where i_{CO} is compensator current and notice that $v_S = v$ (see Fig.4). From the previous can be derived that the *RP* compensation problem is mathematically equivalent to the problem of minimization of $\|i_S\|$ subject to the constraint (9), according

$$\|i_S\|^2 = \|i\|^2 + \|i_C\|^2 + 2\langle i_C, i \rangle \qquad (10)$$

For circuit compensated by capacitor

$$\|i_S\|^2 = \|i + i_{CO}\|^2 = \|i\|^2 + \|i_{CO}\|^2 + 2\langle i, i_{CO} \rangle \quad (11)$$

where current i_{C0} for compensation capacitor is

$$i_{CO}(t) = C_{CO} \frac{dv_S(t)}{dt} \qquad (12)$$

after substitution and some manipulations

$$\|i_S\|^2 = \|i\|^2 + C_{CO}^2 \left\|\frac{dv_S}{dt}\right\|^2 + 2 C_{CO} \left\langle i, \frac{dv_S}{dt} \right\rangle \qquad (13)$$

Minimal value of i_S (or minimum of apparent power S) can be found for

$$\frac{d}{dC_{CO}}\left(\|i\|^2 + C_{CO}^2 \left\|\frac{dv_S}{dt}\right\|^2 + 2 C_{CO} \left\langle i, \frac{dv_S}{dt} \right\rangle \right) = 0 \quad (14)$$

Optimal value of compensation capacitor is

$$C_{CO} = -\frac{\left\langle i, \frac{dv_S}{dt} \right\rangle}{\left\|\frac{dv_S}{dt}\right\|^2} \quad \text{or} \quad C_{CO} = \frac{\left\langle \frac{di}{dt}, v_S \right\rangle}{\left\|\frac{dv_S}{dt}\right\|^2} \qquad (15)$$

where property (5) was used. Optimal value of compensating inductor is

$$L_{CO} = \frac{-\left\|\int v_S\right\|^2}{\left\langle i, \int v_S \right\rangle} \quad \text{or} \quad L_{CO} = \frac{\left\|\int v_S\right\|^2}{\left\langle \int i, v_S \right\rangle} \qquad (16)$$

where property (6) was used. For compensating element calculation according (15) or (16) is possible the choice what is "better" for calculation. In present example is less complicated use left part of equation (15) to avoid current derivation. Compensation capacitor calculation, according (15) is

$$C_{CO} = -\frac{\left\langle i, \frac{dv_S}{dt} \right\rangle}{\left\|\frac{dv_S}{dt}\right\|^2} = -\frac{-4.838 \cdot 10^5}{(220 \cdot 100 \cdot \pi)^2} = 101.27 \cdot 10^{-6}$$
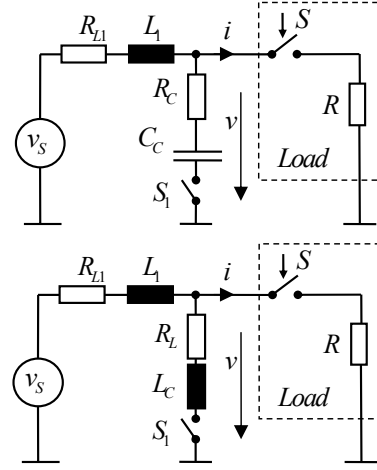

Figure 8. Circuits diagrams for inductive (top) and capacitive (bottom) RP compensation.

III. RESULTS

In this part the results of RP compensation are presented. It must be pointed that calculation of compensation inductor, according (16) is also possible, but result is negative (for $\alpha = \pi$), which means that compensation by inductor is not real. The types of compensation (capacitive, reactive) depends on phase shift α, circuits diagrams are in Fig.8. The results for different values of α are presented in Table 1 (for $k_S = 2$). The power factor (PF) is defined as

$$PF := P / S \qquad [\text{W, V·A}] \qquad (17)$$

TABLE I. COMPENSATION RESULTS FOR DIFFERENT PHASE OF SAMPLING

α - phase shift of sampling signal, *PFn* - power factor for non-compensated circuit, *PFc* - power factor of compensated circuit, *Isn* - effective current from source for non-compensated circuit, *Isc* - effective current from source for compensated circuit, *Cc* - value of compensation capacitor, *Lc* - value of compensation inductor. If the compensation by capacitor is not possible, value in table is 0. If the compensation by inductor is not possible, value in table is ∞

| α	PFn	PFc	Isn	Isc	Cc	Lc
0	0.708	0.793	15.57	13.91	0	0.1
0.314	0.773	0.841	17.02	15.66	0	0.10
0.628	0.829	0.872	18.234	17.33	0	0.12
0.943	0.87	0.891	19.138	18.69	0	0.17
1.257	0.9	0.902	19.712	19.59	0	0.325
1.571	0.904	0.904	19.9	19.89	0	∞
1.885	0.896	0.902	19.72	19.6	3.1E-05	∞
2.199	0.87	0.891	19.14	18.69	5.9E-05	∞
2.513	0.828	0.872	18.23	17.32	8.2E-05	∞
2.827	0.773	0.841	17.02	15.66	9.6E-05	∞
3.142	0.707	0.791	15.54	13.88	1.0E-04	∞
3.456	0.633	0.721	13.93	12.24	9.6E-05	∞
3.77	0.559	0.630	12.31	10.925	8.2E-05	∞
4.084	0.493	0.533	10.85	10.04	5.9E-05	∞
4.398	0.444	0.455	9.77	9.52	3.1E-05	∞
4.712	0.426	0.426	9.38	9.38	0	∞
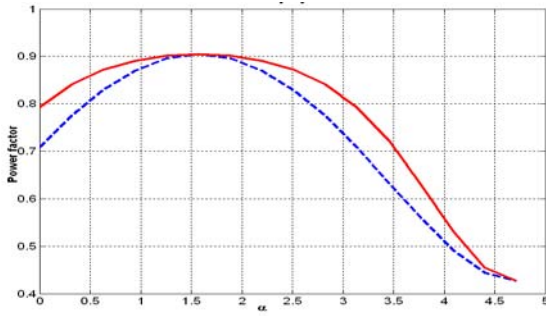
Figure 9. PF for non-compensated circuit (dash-blue) and PF for compensated circuit (solid-red) versus sampling phase α.[rad]

PF for non-compensated and compensated circuit versus sampling phase shift α is shown in Fig. 9. The examples of time diagrams of automatic RP compensation are presented in Fig. 10 and 11. The blocks diagrams of electronic systems for RP compensation are displayed in Fig. 12. Implementation of the control part of system is possible by means of a microcontroller.
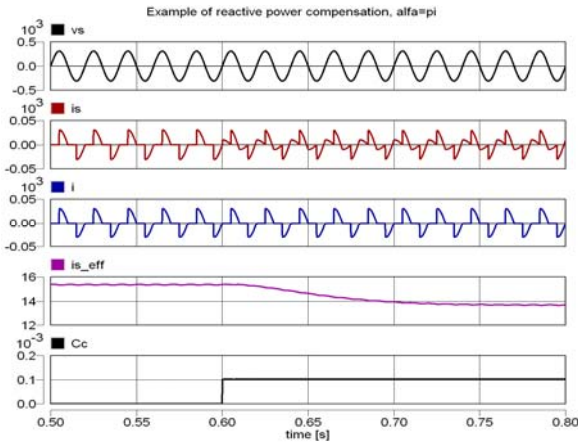

Figure 10. Time diagram - without RP compensation (S_1=OFF) and with RP compensation (S_1=ON) for (α=π and k_S=2). The switch S_1 (see Fig. 8) is switched ON for *time*>0.6. From top to bottom: v_S, i_S, i, is_eff - effective current from source, Cc - compensation capacitor.

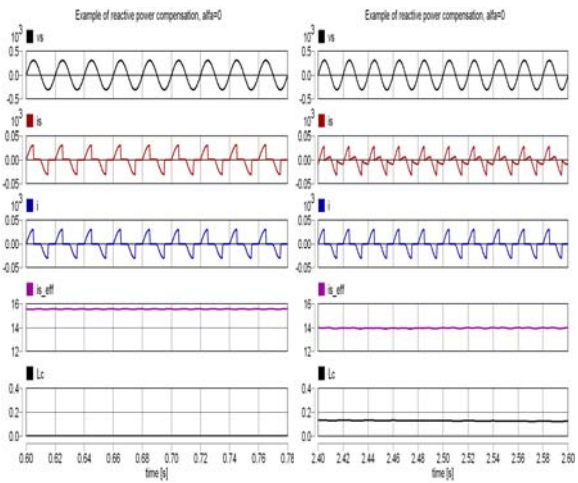Figure 11. Time diagram - without RP compensation (S_1=OFF) and with RP compensation (S_1=ON) for (α=π and k_S=2). The switch S_1 (see Fig. 8) is switched ON for *time*>0.8. From top to bottom: v_S, i_S, i, is_eff - effective current from source, Lc - compensation inductor. The figure is separated on left and right parts because of long transient response after Lc is connected in circuit.
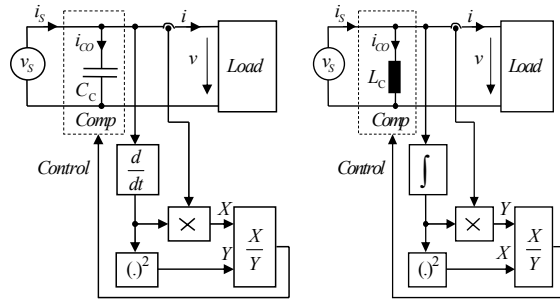

Figure 12. Block diagram of electronic systems for RP compensation - inductive (left) or capacitive (right). This system is suitable for different types of loads (linear, nonlinear or switched load)

IV. CONCLUSION

In this paper the simple system for RP was presented. On the example of harmonic power source and pure resistive load, but sampled was shown, that for some values of sampling phase shift the RP is generated and can be partly suppressed by means of appropriate passive compensation, therefore PF can be increased [9]. It was also shown that generated RP is inductive or capacitive. Active PFC circuits are based on switchmode converter techniques and are designed to compensate for distortion as well as displacement on the input current waveform. They tend to be significantly more complex than passive approaches, but they are so fare limited for power up to kW.

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