Simple Nonlinear Circuits and Applications to Chaotic Sensors and Chaos Generators

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***Abstract* –Several simple circuits with rather complex nonlinearities are presented, essentially based on diodes and varistors. These circuits are believed to extend the family of chaotic circuits based on diodes, as illustrated by the literature. Potential applications to chaotic sensors are briefly discussed.**

Keywords-chaos, nonlinear circuit, tent map, sensor

I. INTRODUCTION

The purpose of this article is to introduce simple circuits that exhibit complex nonlinear characteristics in view of their use in sensors based on the variation of the nonlinear dynamics in response to the measured parameter [1-3]. In essence, we suggest the use of several variations of piecewise circuits based on diodes, slightly expanding the ones in the literature, moreover, circuits based on varistors, and circuits using a combination of varistors and diodes in the feedback loops.

There is a vast literature on simple circuits able to develop chaotic dynamics under specified conditions. Most of these circuits are used in continuous time and are thus described by differential nonlinear equations. Sprott et al. [4-6] have listed several such circuits, essentially based on diodes and amplifiers. Essentially, many of these circuits are based on piecewise linear circuits, whose realization with operational amplifiers (OAs) and theory is known since the 1970s, see [7], while in the simpler variant based on diodes and resistors these circuits are much older. Heemels et al. [8] analyzed in detail the dynamics of the piecewise linear networks. Various other schemes for chaos generation have been proposed, for example [9], who proposed modified negative impedance converter op amp oscillators with symmetrical and anti-symmetrical nonlinearities, [9], and Colpits-like chaotic oscillators [10]. Hernandez et al [11] studied the possibility of circuits for arbitrary one-dimensional maps, while tent maps with exponential slopes were studied in [12] and many other varieties of chaos generators form a vast literature, e.g., [13-19]. Several other circuits have been proposed in relation with specific applications, mainly secure communications and data security [20].

II. EXTENDING THE CLASS OF DIODE-BASED PIECEWISE LINEAR CIRCUITS

A basic scheme of piecewise circuit is analyzed in the classic textbook [7], and is recalled here as it is a starting point for the development of the suggested circuits. The basic scheme is shown in Fig. 1 and consists in an OA with two loops of negative feedback in parallel, one of them including a diode polarized by an external voltage source. See the graph of the characteristic of the circuit in Fig. 11.22(b) in [7]. The circuit operation can be seen as an inverting amplifier with gain $A \approx (R_2||(R_f + R_{eq-D})/R_1$, with $R_{eq-D} = \infty$ when the diode is inversely polarized, and $R_{eq-D} = 0$ for direct polarization. Therefore the characteristic has two slopes, R_2/R_1 and $R_2||R_f/R_1$, i.e., it is piecewise. For the circuit in Fig. 1(a), $V_B - \left(V_B + V_{in}\frac{R_2}{R_1}\right)\frac{R_4}{R_3+R_4} < 0$, $V_B\left(1 - \frac{R_4}{R_3+R_4}\right) < V_{in}\frac{R_2}{R_1}\frac{R_4}{R_3+R_4}$ diode opens and slope becomes $-\frac{R_2||R_3}{R_1}$. For more segments in the characteristic, several feedback loops with diodes can be added in parallel. But this recalls and has a direct, if not obvious connection with the elementary AND diode circuit, shown in Fig. 1 (b) and even more with the resistor-diode elementary piecewise linear circuit with diodes, known much before the OAs, Fig. 1 (c).

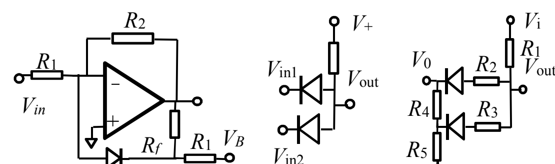

Figure 1. Piecewise circuits with diodes the ‘modern’ one using OAs, and the simpler, older RD circuits.

The operation conditions result from $V_{in} < V_0\frac{R_5}{R_4+R_5} + V_D \rightarrow V_{out} = V_{in}$, $V_0\frac{R_5}{R_4+R_5} + V_D < V_{in} < V_0 + V_D \rightarrow V_{out} = V_{in}\frac{R_3+R_5}{R_1+R_3+R_5} + V_D$.

Notice that (i) the elementary piecewise linear circuit with diodes is a development of the diode AND circuit, and (b) combinations of the elementary piecewise linear circuit and an amplifier, starting from the basic scheme in Fig. 1, produce various versatile piecewise linear circuits. One step further, one can expand the combination to both positive and negative feedback, as in Fig. 2, which shows a relatively straightforward generalization where a diode participates in both the negative and positive feedback.

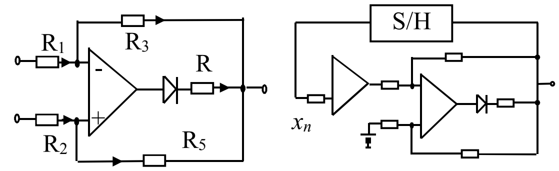

Figure 2. The electronic circuit of the nonlinear block and its use in a chaos generator circuit. The sample and hold (S/H) circuit alows the use of the circuit in a chaos generator.

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The simulation results for the circuit with no resistor in series with the diode are shown in Fig. 3, for the case D1 =1N914 diode, R1 = 1kΩ, R2 = 3kΩ, R3 = 1k, R4 = 1k, OA is U1 = LT1001.

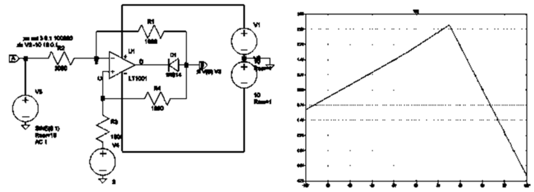

Figure 3. Basic OA-diode circuit with positive and negative feedback loops (LTSpice simulation schematic, left) and example of characteristic (right).

The resistor in the positive feedback loop, R_4, controls the slope of the descending line segment when all other resistors are fixed, see in Fig. 4 the effect when D1 - 1N914 diode, R1 = 1k, R2 = 1k, R3 = 1k, R4 = 1k, U1 Linear Technology LT1001 (left panel), compared to the case when R4 is changed to 4k (right panel). Suitable choices of the resistors produce characteristics with several line segments, see Fig. 5.

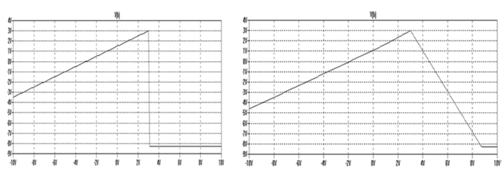

Figure 4. R4 adjusts the slope of the decreasing (descending) line. With R4=3k, right side.

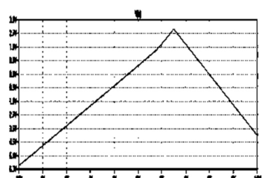

Figure 5. Three-line segment characteristic for R1 = 500, R2 = 1K, R3 = 1K, R4 = 3K. The ascending "line" is in fact not a line.

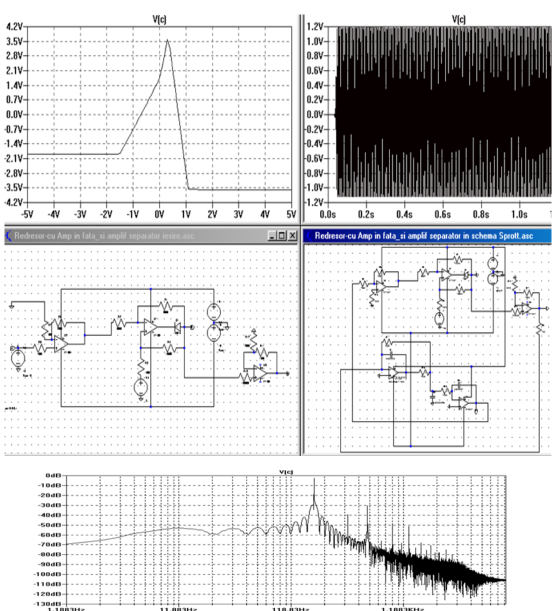

Figure 6. Use of the nonlinear circuit in Fig. 3 in the feedback loop of a typical circuit; spectrum of the generated signal.

The circuit discussed above can be made more flexible by adding an input and an output OA; these OAs allow the scaling and translation of the input and output ranges. The above nonlinear circuit can be used as a feedback circuit in conjunction with other basic circuits. For example, using it as feedback to one of Sprott's circuit, one obtains the results in Fig. 6. Notice on the spectrum that the circuit behaves as an oscillator with a large spectrum of components decaying about as $1/f$.

Adding a resistor in series to the diode, as in Fig. 7, allows us a larger variability of the characteristic function of the circuit.

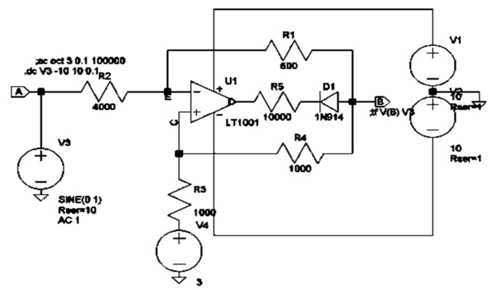

Figure 7. Circuit with double feedback loop with diode and series resistance – simulation schematic.

Using in the above circuit the values R1=500 Ω (negative feedback), R2=4kΩ, R3=1k, R4=1kΩ (negative feedback), R5=10kΩ (series with the diode), one obtains an *N* characteristic, see Fig. 7. Reducing the value of the resistor in series with the diode to 7kΩ, it becomes apparent that the characteristic function has four linear segments, not three, see Fig. 7.

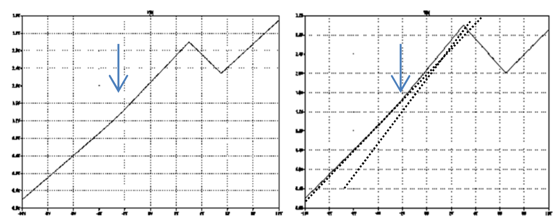

Figure 8. Four segment piecewise linear characteristics

The multiple feedback loops with different resistors values create multiple line segments even with a single diode, which cannot occur in the standard diode piecewise linear circuit in Fig. 1(a). Notice that with such circuits, using multiple (more than two) feedback loops with different polarization voltages, one can achieve the so called "stroboscopic maps" discussed in [18].

III. A Class of Varistor-based Piecewise Linear Circuits

Varistors have not been proposed at our knowledge in chaos generating circuits. Possibly, this is because these devices had nonlinearities only for high voltages (typically >100V) until recently. However, progresses in varistor technologies allow nowadays the manufacturing of varistors with thresholds as low as 4.5 V [21], [22] compatible with AOs.

It is well known that a varistor behaves largely as a diode for each polarity, in the sense that its current starts increasing very fast after some threshold. Therefore, it can be represented equivalently as in Fig. 9(a), as two diodes in anti-parallel, with a resistor in series and one in parallel to the diodes. The approximate U(I) characteristic of varistors is shown in Fig. 10, after [23]. The I(U) characteristic (built based on data in [23] for a varistor is illustrated in Fig. 10, right panel.

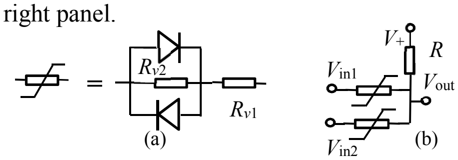

Figure 9. (a) Suggested approximately equivalent scheme of the varistor. (b) Derived "*bipolar AND*" based on varistors.

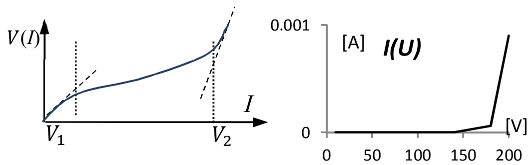

Figure 10. Left: Approximation of U(I) curve of a varistor with one and with three line segments (piecewise linear), on a given interval, based on [23]. Right: Approximate $I(U)$ characteristic of a varistor (for positive voltages).

For low voltages on the varistor, $|V| < V_{1\theta}$, the operation of a varistor is almost linear, $V(I) = R_{v1}I$. For the interval of middle values of the voltage, which is the region of normal operation, $|V| \in [V_{1\theta}, V_{2\theta}]$, with $V_{2\theta}$ the main threshold voltage of the varistor (see Fig. 10), the operation is described [23] by a power law, $V(I) = C \cdot I^{\beta}\ ;\ I = \frac{1}{\beta} \cdot \ln V - \alpha, \alpha = \ln C / \beta$.

Unlike the diodes, the varistors are non-polarized devices: the characteristic of the varistors is symmetric with respect to the null voltage (identical response to positive and negative voltages), see Fig. 9. Therefore, we can consider only one polarity in the discussion, see Fig. 10. We discuss only cases involving $V_{in1} \geq 0$, $V_{in2} \geq 0$. Several cases should be considered. In the first place, when V_+ is lower than the first threshold of both varistors, the varistor circuit behaves as a resistive circuit, with both varistors equivalent with Ohmic resistors with very high resistances. When V_+ is below the second threshold of both varistors, at least one of the varistors, for low values of the corresponding V_{in} will behave in the power law region. In a linear approximation, the varistor essentially behaves as a non-ideal diode, with $I(U) = 0$ for $U < V_{\theta 1}$ and $I(U) = U/R_{v2}$ for $V_{\theta 1} < U < V_{\theta 2}$, with R_{v2} very small. If the two varistors have the same characteristic (are identical), then for $V_{\theta 1} < V_+ < V_{\theta 2}$ the circuit behaves as the diode AND.

(i) $V_+ - V_{in1} < V_{11\theta}$ and $V_+ - V_{in1} < V_{21\theta}$, that is, both varistors are in the pre-threshold region. They can be considered as simple (Ohmic) resistors, with very high resistance, $R_{v1}, R_{v2} \gg R$. Then, $V_{out} \approx V_+$.

(ii) Assuming that $V_o - V_{in1} \in [V_{11\theta}, V_{12\theta}], V_o - V_{in2} \in [V_{21\theta}, V_{22\theta}]$, that is, $V_o - V_{in1}$ and $V_o - V_{in2}$ are the normal operation regions of the two varistors, $V_o - V_{in1} = C_1 I_1^{\beta_1}$, $V_o - V_{in2} = C_2 I_2^{\beta_2}$, $V_+ - V_o = R(I_1 + I_2)$. Thus, $V_+ - R(I_1 + I_2) - V_{in1} = C_1 I_1^{\beta_1}$, $V_+ - R(I_1 + I_2) - V_{in2} = C_2 I_2^{\beta_2}$. Then $V_{in1} - V_{in2} = C_2 I_2^{\beta_2} - C_1 I_1^{\beta_1}$ and with identical varistors, $V_{in1} - V_{in2} = C(I_2^{\beta} - I_1^{\beta})$. For larger than threshold operation, $V(I) \approx C \cdot I^{\beta} + I \cdot R_{v2}$, $|V| > V_{2\theta}$, with R_{v2} the equivalent resistance of the "active" varistor, with R_{v2} very low.

Next, we re-consider the circuit in Fig. 1 (a) where the diode is replaced by a varistor with threshold lower that the maximal output voltage of the amplifier, Fig. 3 (a). The result is shown in Fig. 11. If needed, an output stage can be added to the amplifier to extend the range of output voltages, or a high voltage amplifier should be used (e.g., OPA454 high voltage output, at 100V and high output currents up to 50mA, or LTC6090 with ±70V power supply and output currents up to 50mA are suitable for circuits with varistors with threshold voltages in the range 14 to 40 V). The higher voltage chaotic signals that may be generated with such circuits are useful when the application is related to masking data, such as in [20].

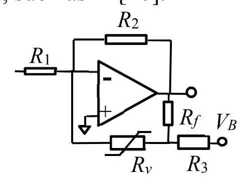

Figure 11. Circuit based on the circuit in Fig. 1(a), using a varistor instead of a diode.

Additional polarization voltage, V_B, can be added to the varistor, to control the piecewise (approximately-) linear characteristic.

IV. Combined Diode-Varistor Circuits

Adding a varistor to the schemes in the previous Section enhances the capabilities of the scheme in generating and controlling piecewise linear characteristics. One possibility of adding the varistor is shown in Fig. 12.

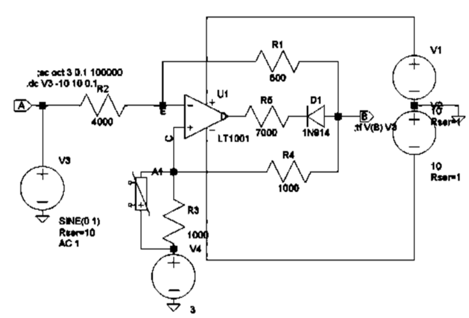

Figure 12. Varistor-diode circuit with piecewise linear characteristic.

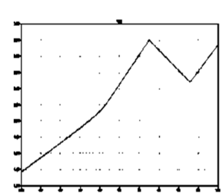

Figure 13. Characteristic obtained with the scheme in Fig. 12, with varistor and for D1 - 1N914, R1=500, R2 = 4K, R3 = 1K, R4 = 1K; R5 = 7K; U1 Linear Technology LT1001 integrated circuit (LTSpice simulation, varistor model from LTSpice)

For the scheme in Fig. 12, the characteristic obtained for D1 1N914, R1=500 (negative feedback), R2 =4K, R3 =1K (in parallel with the varistor), R4 = 1K (positive feedback); R5=7K (in series with the diode), U1 LT1001, the characteristic is in Fig. 13.

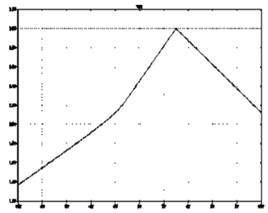

Figure 14. Characteristic obtained with the scheme in Fig. 12, with the same components and values, except R5 = 1K (only value different from Fig. 13).

The same scheme with R5 =1k (resistor in series with the diode) provides the characteristic in Fig. 14.

V. CONCLUSIONS

We have introduced and briefly analyzed several classes of nonlinear circuits suitable for chaos generation that we believe are novel in the literature. Some of these classes of circuits are suitable for generation of chaotic signals with large amplitude values with minimal circuitry. The proposed circuits include an extension of the piecewise circuits based on diodes and OAs, using combinations of positive and negative feedbacks, circuits based on varistors, and circuits based on pairs of transistors, with feedback.

The flexibility of changing the nonlinearity of these circuits makes them a good choice in optimization of the design for chaotic sensors, where a rich range of behaviors is a premise for finding regions of high sensitivity of the sensors [24]. Moreover, the larger number of parameters that change the operation of the proposed circuits allows the designer to use the same circuit in sensors determining at the same time several parameters, e.g., temperature, light level, and pressure.

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The circuits in Figs. 2, 3, and 7, proposed by the author, were discussed with one of his former Ph.D. students, Dr. Ciprian Zamfir, who also made some more simulations. Also stimulating was the collaboration with another former Ph.D. student, Dr. Victor Cojocaru, during his PhD studies.

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