

# Cardiac Mathematical Models for Exercise Testing on Treadmill Ergometer

Milan Stork

Department of Applied Electronics and Telecommunications/RICE  
University of West Bohemia  
Plzen, Czech Republic  
stork@kae.zcu.cz

Jaroslav Novak

Department of Sports Medicine  
Medical Faculty in Plzen  
Charles University in Prague  
Plzen, Czech Republic  
novakj@lfp.cuni.cz

Pavel Broz

Department of Clinical Biochemistry  
Medical Faculty in Plzen  
Charles University in Prague  
Plzen, Czech Republic  
broz@fnplzen.cz

**Abstract** – Cardiac output (CO) is a key parameter in the assessment of cardiac function, and its measurement is fundamental to the diagnosis, treatment, and prognostic evaluation of all heart diseases. In this paper the linear and nonlinear models for noninvasive estimation of CO are presented. The estimation is based from short examination where parameters of subject were measured up to maximal load and from these parameters the CO and stroke volume (SV) were estimated for evaluation of cardiovascular performance of the subject. This approach was used for group of twenty marathon runners. Linear and nonlinear mathematical models and samples of measuring results are also presented.

**Keywords**- cardiac output; linear model; cubic model; non-invasive estimation; oxygen uptake; heart rate; stroke volume

## I. INTRODUCTION

Cardiac output (CO) is a measure of the amount of blood pumped by either ventricle. Until recently, cardiac output determination during exercise had been only possible through invasive methods, which were not practical in the clinical setting. In steady state, the outputs of both ventricles are the same. In a healthy adult male, cardiac output is approximately 5 l/min [1]. CO can vary, however, according to the body's physiological needs; for example, a well-trained athlete, while exercising, can increase CO to up to 30 l/min to increase the rate of transport of oxygen, nutrients, and wastes [2]. Abnormally low levels of cardiac output can also be an indication of pathology. CO is one of the most important hemodynamic signals to measure in patients with compromised cardiovascular performance. There are many methods of monitoring the hemodynamic status of patients, both invasive and non-invasive, the most popular of which is thermodilution [3, 4, 5]. The one noninvasive method is based on monitor consists of a carbon dioxide sensor, a disposable air flow sensor

and a pulse oxymeter. The next method is thoracic electrical bioimpedance. The first derivative of the impedance waveform is related linearly to aortic blood flow. Changes in impedance correlate with stroke volume. The estimation of CO via pulse contour analysis is an indirect method, since CO is not measured directly, as with an electromagnetic flow probe, but is computed from a pressure pulsation on basis of a criterion or model [6, 7, 8, 9, 10]. All of the methods have their advantages and disadvantages, but thermodilution is the golden standard for critical patients, although it does entail many risks. The ideal system for cardiac output monitoring would be non-invasive, easy to use, reliable and possible used at rest and also during physical activity. The estimation based on heart rate (HR) should be good, because HR is easy and reliable measured also during the subject physical activity.

Because oxygen uptake ( $VO_2$ ) is cardiac output times arteriovenous content difference, evaluation of cardiac output is usually included in its measurement. Measurement of CO and  $VO_2$  should correlate well in healthy subjects at rest as well as during exercise. Because both HR and volume of  $VO_2$  can be easily measured during standard incremental cardiopulmonary exercise testing (see Fig. 2) both CO and SV could be accurately quantified. For non-invasive CO estimation, exercise tests were performed on an electronically braked cycle ergometer or motor driven treadmill controlled by computer. Subjects were familiarized with the apparatus and performed a continuous incremental test with step vice increased workload up to the exhaustion for determination of  $VO_{2max}$ ,  $HR_{max}$ , blood pressure, anaerobic threshold etc. The expired gas samples were analyzed by  $O_2$ - $CO_2$  gas analyzer. All electrical signals from HR and volume sensors and from gas analyzer were processed in personal computer. From the measured values the CO was estimated according formula [11, 12]:

$$CO = \frac{100 * VO_2}{\left[ 5.721 + 0.1047 \frac{100 * VO_2}{VO_{2MAX}} \right]} \quad [\text{l/min}] \quad (1)$$

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The function  $CO=f(VO_2)$  according eq. (1) for  $VO_2 \in [0.3 \ 6]$  is shown in Fig. 1.

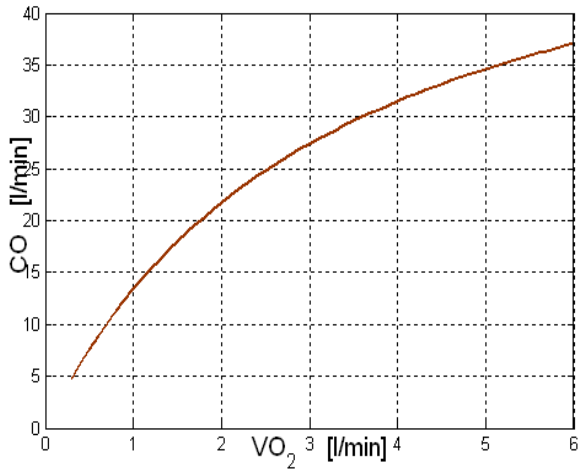


Figure 1. The graph of function  $CO$  versus  $VO_2$  for  $VO_2 \in (0.3 \ 6)$ . The  $CO$  is estimated from measured  $VO_2$ , according eq. (1). The graph is calculated for  $VO_{2MAX} = 6$  [l/min]



Figure 2. Photo of the subject running on treadmill

Measurements of  $CO$  and filling pressure provide information for “early diagnosis, monitoring of disease progression, and titration of therapy in heart failure, shock of any type, sepsis, and during cardiac surgery”. If cardiac output could be controlled at more frequent intervals, or even continuously, clinicians could detect abnormalities in the cardiorespiratory system and provide appropriate interventions sooner. In this paper the non-invasive estimation method for  $CO$  and other parameters was used in group of twenty male marathon runners. Studies were done with the help of measuring system (including  $O_2$ - $CO_2$  gas analyzer) connected to personal computer and Lode treadmill ergometer (photo, see Fig. 2). A non-rebreathing valve was connected to a mouthpiece to prevent mixing of inspired and expired air [13 - 21]. The electrical signals from sensors were connected to microcontroller based measuring system.

## II. MATERIAL AND METHODS

On the beginning for all 20 people, a short exercise test was first performed (Increased incremental load up to maximum, length of test approx. 16 min) on treadmill. Example of the time evolution of measured and calculated parameters versus speed are shown in Fig. 3. From Fig. 3 is shown that measured and calculated parameters have almost same dynamics and therefore it is possible use simple mathematical model based on linear or nonlinear regression approach. Example of such model is shown in Fig. 4 where  $CO$  versus  $HR$  is derived as linear and cubic function. The same process was used also for other subjects. The examples of mathematical models for some subjects are presented in Tab. 1.

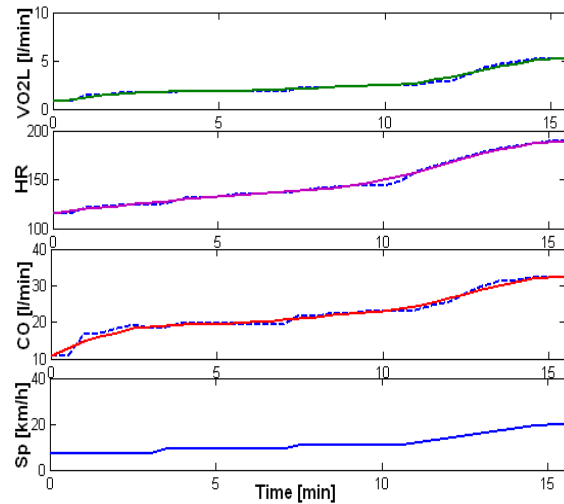


Figure 3. Time evolution of short exercise test. From top to bottom:  $VO_2$ ,  $HR$ ,  $CO$ , speed of treadmill. Dash lines - measured, solid lines - smooth. Subject M1

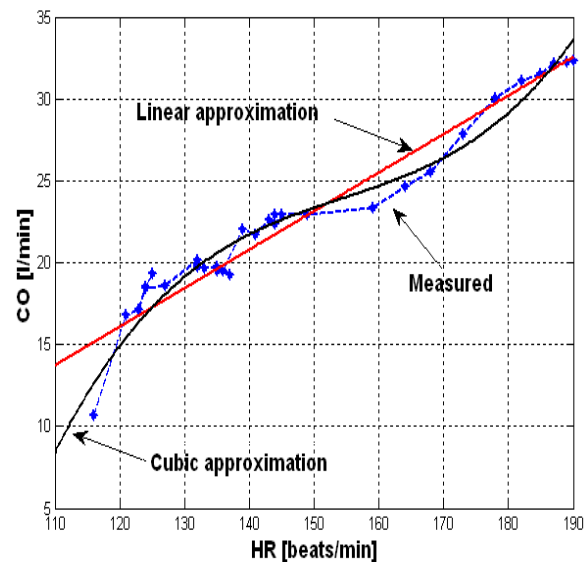


Figure 4. The relation  $CO$  versus  $HR$  from short exercise test on treadmill (16 min test), linear and cubic regression. Subject M1. Blue dash curve - measured, red - linear model, black - cubic model

The two models, linear and cubic are used for estimation of  $CO$  from  $HR$ . Linear estimation  $CO_{E1}$  is

$$CO_{E1} = k_1 HR + k_2 \quad [\text{l/min, beats/min}] \quad (2)$$

where  $k_1$  and  $k_2$  are individual coefficients of subject.

Cubic estimation  $CO_{E2}$  is given as

$$CO_{E2} = c_1 HR^3 + c_2 HR^2 + c_3 HR + c_4 \quad [\text{l/min, beats/min}] \quad (3)$$

where  $c_1, c_2, c_3$  and  $c_4$  are individual coefficients of subject.. The coefficient values of some subject are presented in Tab. I.  $SV$  is given as:

$$SV = 1000 \cdot \frac{CO}{HR} \quad [\text{ml/beat, l/min, beats/min}] \quad (4)$$

TABLE I. LINEAR AND CUBIC MATHEMATICAL MODELS CO AS FUNCTION OF HR FOR SOME SUBJECTS CALCULATED FROM SHORT EXERCISE

Subject	Coefficients
M1	$k_1=0.24; k_2=-12$
M1	$c_1=0.000111; c_2=-0.0515; c_3=8.1; c_4=-405$
M3	$k_1=0.15; k_2=-5$
M3	$c_1=6.89e-005; c_2=-0.0338; c_3=5.64; c_4=-301$
M6	$k_1=0.21; k_2=-8.6$
M6	$c_1=0.000231; c_2=-0.0916; c_3=12.2; c_4=-527$
M10	$k_1=0.22; k_2=-4.7$
M10	$c_1=0.000112; c_2=-0.05; c_3=7.52; c_4=-353$

TABLE II. SUBJECT CHARACTERISTIC (N=20). SUB – SUBJECT, M\_CO – MEAN VALUE OF CARDIAC OUTPUT, M\_SV – MEAN VALUE OF STROKE VOLUME (MEASURED DURING MARATHON RUN)

Sub	Age	Height	Weight	BMI	M_CO	M_SV
M1	32	183.5	75.4	22.4	23.7	147.7
M2	26	186	75.2	21.7	16.2	96.7
M3	28	180	74	22.8	18.2	109.7
M4	37	174.5	78.4	25.7	19.4	135.6
M5	39	186	90.4	26.1	20.1	133.8
M6	48	193	97.4	26.1	16.5	133.5
M7	24	183	68	20.3	22.8	134.1
M8	71	171.5	63	21.4	15.6	111.7
M9	27	179	74.60	23.3	21.7	136.4
M10	38	179	94	29.3	20.2	125.8
M11	41	187	85.8	24.5	24.2	171.0
M12	35	197	79.3	20.4	21.2	125.9
M13	35	179	77.4	24.2	18.9	106.8
M14	37	184	86.6	25.6	23.4	150.6
M15	44	179	78.6	24.5	19.3	133.2
M16	33	181.5	81.4	24.7	19.8	136.6
M17	38	175	73	23.8	16.1	101.7
M18	42	183.5	74.2	22.0	15.5	99.1
M19	37	184.5	84.3	24.8	22.1	128.1
M20	39	180	70	21.6	16.0	127.0
Mean	37.6	182.3	79.1	23.8	19.5	127.2
SD	10.0	6.0	8.7	2.2	2.9	18.7

### III. RESULTS

After a short period of time from the first short examination (approx. until 14 days), a long-term 2-hour test was performed. The basic parameters of all 20 marathon runners are displayed in Tab. 2 including mean values and standard deviations. The one example of time evolution of parameters during marathon exercise is shown in Fig. 5. The short peak in the chart is caused by stopping for snacks (drink) or taking blood sample. Evolution of  $SV$  and linear (regression) model is displayed in Fig. 6.

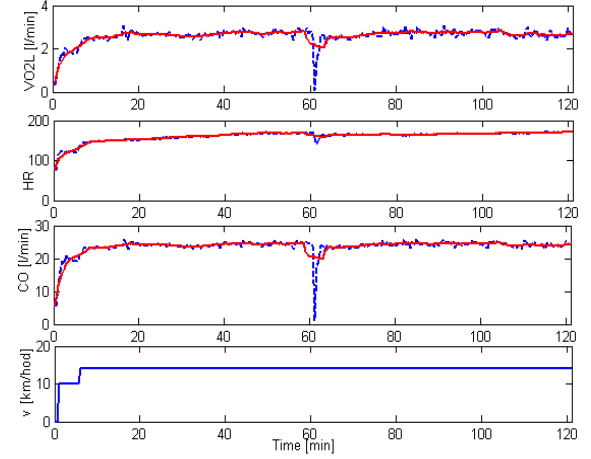


Figure 5. Subject M1, marathon exercise. From top to bottom: Time evolution of  $VO_2$ ,  $HR$ ,  $CO$  and speed of the treadmill. Blue (dash) – raw measured values, red (solid) – filtered values

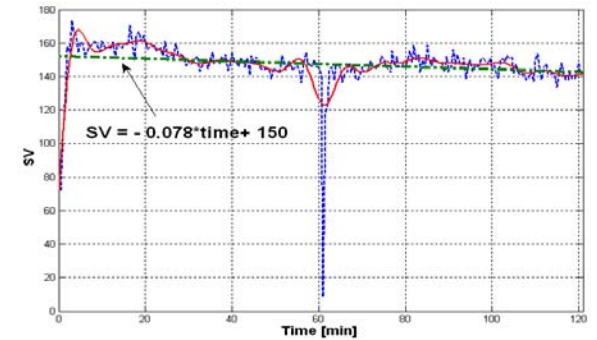


Figure 6. Subject M1, marathon exercise.  $SV$  time evolution (blue, dash line), smoothed  $SV$  (red, solid line) and linear approximation (green, dot-dash line) and equation

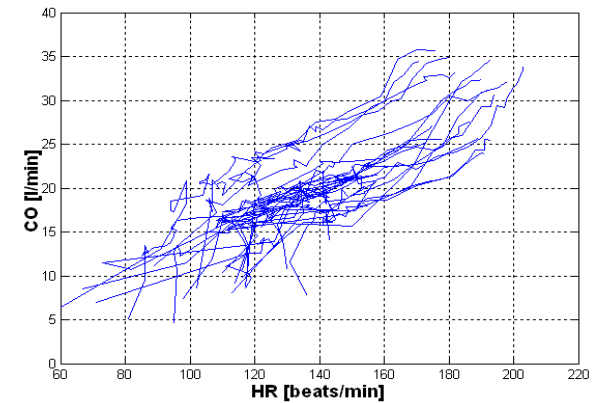


Figure 7. All Subject M1 to M20,  $CO$  versus  $HR$  measured from short examination

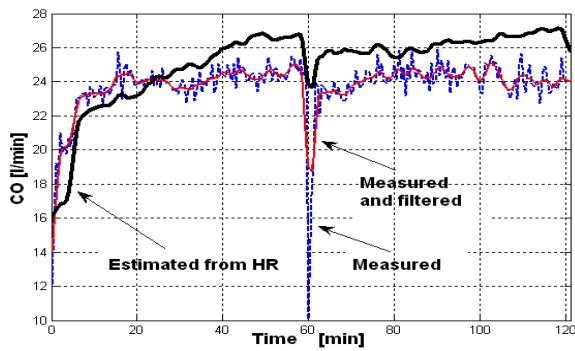


Figure 8. Subject M1, marathon exercise.  $CO$  derived from  $VO_2$  (blue - dash),  $CO$  derived from  $VO_2$ -filtered (red-solid),  $CO$  estimated from  $HR$  (black-solid) ( linear model)

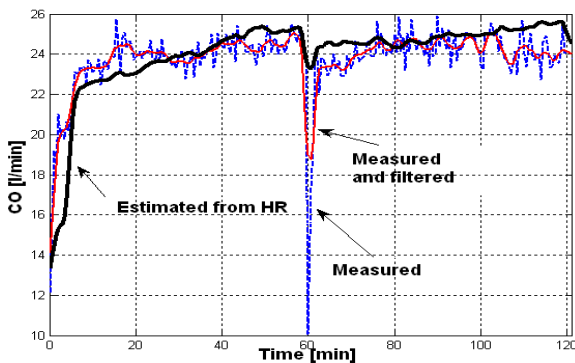


Figure 9. Subject M1, marathon exercise.  $CO$  derived from  $VO_2$  (blue - dash),  $CO$  derived from  $VO_2$ -filtered (red-solid),  $CO$  estimated from  $HR$  (black-solid) (cubic model)

The results of measuring of all 20 subjects ( $CO$  versus  $HR$ ) are presented in Fig. 7. In Fig. 8 the  $CO$  estimation from  $VO_2$ , filtered estimation from  $VO_2$  and estimation  $CO$  based on linear model gained from relation  $CO$  versus  $HR$  according eq. (2) is shown. In Fig. 9 the estimation based on cubic model is displayed. From Fig.8 and 9 can be seen that cubic model used for estimation of  $CO$  is much better then linear model. Therefore  $CO$  for different outdoor activities can be estimated from  $HR$  if the mathematical model from short examination is known.

#### IV. DISCUSSION

In this work the 20 marathon runners were tested and calculation of  $CO$  based on  $VO_2$  measuring was presented. From short examination for increased load up to maximum, the linear and cubic mathematical models  $CO$  versus  $HR$  were derived. This models was used for estimation  $CO$  during marathon from  $HR$ . The result shows good match between measuring and models, but nonlinear - cubic model is better. The main advantage of the estimation  $CO$  from  $HR$  is that if the mathematical model  $CO$  versus  $HR$  was derived the  $CO$  can be estimated for outdoor activities only from  $HR$  measuring.

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