Chirps for radar based on reciprocal time, essential discontinuities and chaotic generators

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Abstract – **We introduce and analyze a set of essentially discontinuous functions, based on multiplicative inverse of the time, which are usable as chirp signals. An advantage of these chirps is that they can be chosen from a large class to allow multiple radar operators to use each a specific waveform in automotive applications. When the new chirps are derived from chaotic signals, they are also less predictable for an observer, which is an advantage in some radar and sonar applications.**

Keywords- chirp modulation; essential discontinuity; chaos; radar; autonomous vehicles; spread spectrum.

I. INTRODUCTION

Modern cars have several radar modules, devoted to security and driver assistance (Advanced driver-assistance systems, ADAS). The tasks of these radars include collision danger detection and collision avoidance, assistance with danger detection in blind spots and rear, and assistance in drivers maneuvers such as lane change and parking. With the advent of autonomous vehicles (AVs), radar object detectors tend to clutter the roads. While strong attenuation at high frequencies (24, 77, 79,…,125 GHz, see [1], [2] for details) reduce the risk of interferences at larger distances, still on busy lanes there may be hundreds of car radars at interference distances. Narrower bandwidths and specific, 'personalized' waveforms are needed to allow numerous radars interference-free. Numerous studies have dealt with improving the waveform for automotive radars, for example [3], [4], [5], [6], [7], [8]. Frequency modulated continuous wave (FMCW) or chirp pulse radars may have advantages for this type of application.

The use of chirp signals for improving radar and sonar detection with good localization capability is well known. However, chirps based a specified, fixed modulation function are relatively easily perturbed by wide bandwidth noise and jamming. Therefore, making the modulation function more adjustable and less predictable is of interest in various fields.

In radar and sonar applications, for obtaining a precise localization (high resolution) an ideally infinitely-short pulse is needed, which is not obtainable because of too large (infinite) instantaneous power requirement for long-range detection. To alleviate this limit, a chirped pulse is emitted, and the receiving device uses a chirp-like spectrum.

The use of chaotic processes to produce chirps was established decades ago. Carroll [9] "*used a chaotic map whose parameters may be optimized…, producing different chaotic signals that … optimized for different situations."* Other authors also proposed the use of chaotic generators for radar modulators, see [10].

We introduce a set of signals that resemble chirp signals, but are obtained in a different manner, using reciprocal (multiplicative inverse) time; we name these signals 'essential discontinuity-based (pseudo-)chirps', shortly EDB chirps. The discussion is centered mostly on chirps because of easy o description, but the same principle is directly extended to FMCW radars and sonars. The proposed method may also have advantages for high resolution and small distances radars on AV, where pulses may be as short as 1 ns for achieving a location precision of 3 cm in a 20 m range (see [11]).

Section 2 recalls essential discontinuities; Section 3 shows examples; the final Section is conclusive.

II. FUNCTIONS WITH ESSENTIAL DISCONTINUITIES AND CHIRP GENERATION

A. Typical chirp impulse and FMCW signals

Because impulse chirps and FMCW are similar in concept, except that FMCW has no break between the frequency modulated impulses (FMCW is basically a succession of concatenated chirps), we discuss them together. The FMCW typical signal, in its basic form, is described by (see equ. (1) in [12]),

$$s(t) = A \cdot e^{j2\pi\frac{\Delta F}{2T}t^2} \cdot e^{j[2\pi(f_0+\Delta f)t+\varphi_0]} \qquad (1)$$

where t is time, ΔF is the bandwidth of the modulation, T is the pulse width, f_0 is the reference frequency, Δf is the reference frequency displacement, and φ_0 is the signal phase. More general, the chirp signal has the form

$$s(t) = A \cdot e^{j2\pi\frac{\Delta F}{2T}\psi(t)} \cdot e^{j[2\pi(f_0+\Delta f)t+\varphi_0]} \qquad (2)$$

where $\psi(t)$ is some strictly monotonic function of t.

B. Basic concepts of function discontinuity

We recall a few basic concepts. A real valued function f defined on **R** is Hölder α-continuous at right of some point x_0 if there is some constant K such that, for any pair x_1, x_2, $x_0 < x_1 < x_2$, the condition $|f(x_1) - f(x_2)| < K|x_1 - x_2|^\alpha$. When $\alpha = 1$, Hölder continuity stands for standard continuity. Essential discontinuities are defined by exclusion: they are discontinuities that are not removable or 'jump' discontinuities. We will call a Hölder essential discontinuity one that remains an essential discontinuity for whatever value of $\alpha < \infty$. Subsequently, we consider only essential

discontinuities that are finite, that is, non-asymptotic (function not tending to infinity). For details on the discontinuity concepts, see [13], [14].

A simple example of essentially discontinuous signal is $\sin(1/t)$. There are numerous sub-classes of (finite) essential discontinuities. For example, one can make a difference between the following classes that are introduced by the study of the sequence or function for the inverse of the variable, $1/x$. If $f(1/x)$ is a harmonic function, then the function $f(x)$ has a "harmonic" essential discontinuity at 0. Similarly, zero is a point of α-essential discontinuity at 0 if $f(1/x^\alpha)$ is a harmonic function. When $f(1/x)$ is not a harmonic function, one can say that $f(x)$ has an "anharmonic" essential discontinuity at 0. The last category includes the cases $f(1/x)$ toroidal, chaotic, and random.

The following properties are also useful in imagining essential discontinuities usable in chirp generation: If $f(t)$ is periodical, $f(1/t)$ has no limit at 0. If f has derivatives up to order n and if f is periodical, all its derivatives are periodical. Therefore, if f has derivatives up to order n and if f is periodical, all the derivatives of $f(1/t)$ tend to ∞ at 0.

If $f(t)$ is periodical, derivable, and has limit 0 at 0, then $f(t)f(1/t)$ also tends to 0 at 0, but $f'(t)f(1/t)$ may have no limit at 0. Moreover, $f(t)f'(1/t)$ may have no limit at 0. Example: $f(t) = \sin t$, $\sin t \cdot \sin\frac{1}{t} \to 0$ but $\cos t \cdot \sin\frac{1}{t}$ $not \to 0$ (has no limit).

A function is said oscillatory bounded if (i) it is bounded on **R** and (ii) it has no limit at ∞. For an oscillatory bounded function, the condition (ii) in the definition is equivalent with: (iii) $f(1/x)$ has no limit at zero (has an essential discontinuity at $x = 0$). If f has derivatives, all its derivatives are unbounded at 0 in reciprocal time, $1/t$. If f is oscillatory, bounded everywhere, and has a derivative of order n, its derivative is oscillatory and its derivative with reciprocal time $1/t$ is unbounded at zero.

If a system is not asymptotically stable at ∞ and is bounded everywhere, there is a linear function in one or several squared variables of the system such that it is oscillatory. The derivatives of that function in reciprocal time, when they exist, are unbounded and limitless in reciprocal time, at 0.

C. Essential discontinuity-based chirps

What we seek is to show that chirp functions, which have the general form

$$s(t) = A\cos(a + bt + c\psi(t)) \qquad (3)$$

where $\psi(t)$ is a (typically strictly) monotonic function in t and $t \in [t_k + t_1, t_k + t_2]$, $t_2 > t_1$, have as counterpart functions with essential discontinuities at the moments t_k. In this paper we are concerned with chirp-equivalent functions

$$s(t) = A \cdot \cos\left(a + \frac{b}{t_k-(t_k-\psi(t))}\right) \qquad (4)$$

where $\lim_{t\to 0} \psi(t) = 0$, as well as with functions

$$s(t) = A \cdot g\left(\frac{1}{t_k-\psi(t)}\right). \qquad (5)$$

The simplest case of chirp based on reciprocal time, according to (3), is shown in Fig. 1, along with its spectrum. We are not aware of previous proposals of chirps based on reciprocal time.

Notice that the chirp $\sin(at + bt^2)$ has a parabolic phase function $\varphi(t) = at + bt^2$, with f the frequency, $f = \frac{1}{2\pi}\frac{d\varphi}{dt} = \frac{1}{2\pi}2bt$, $\varphi(f) = \frac{a}{b'}f + \frac{b}{b'^2}f^2$, $b' = \frac{1}{\pi}b$, a parabolic function in f. In addition, $\varphi = c\varphi' + d\varphi'^2$ is the differential equation of the chirp. Instead, $\sin(1/t)$ has decreasing phase $\varphi(f) = 1/t$ and *negative* instantaneous frequency $f = \frac{1}{2\pi}\frac{d\varphi}{dt} = -\frac{1}{t^2}$, with a phase function $\varphi(f) = \sqrt{-f}$. When using $\sin\frac{1}{t_0-t}$, with $\varphi(f) = \frac{1}{t_0-t}$, the frequency is $f = \frac{1}{2\pi}\frac{d\varphi}{dt} = \frac{1}{(t_0-t)^2}, \varphi(f) = \sqrt{f}$.
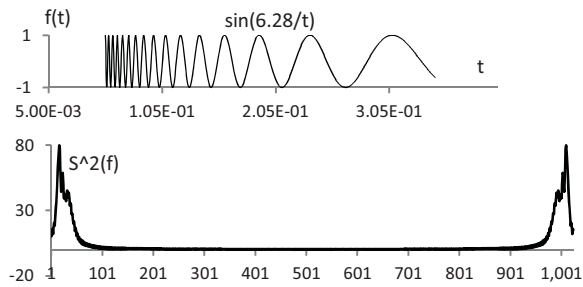

Figure 1. Signal $sin(2\pi/t)$ and spectrum obtained for samples from 336 up to 674, padded with 0 from 1 ro 335 and from 674 to 1024. Series $sin(2\pi/t)$, step of incrementing t, Δt =2.74E-04, starting from t_0 =5.48E-02 (for sample #336). No window applied.

We name chaotic essential discontinuity at x_0 any essential discontinuity of a function g continuous on a left-closed interval (or right-closed) and is produced by a chaotic process $f(t)$ as $g(x) = f(1/t)$.

III. CHAOTIC SIGNALS WITH DISCONTINUITIES

A. Introductory issues

Let $x(n)$ be a chaotic signal, e.g., produced by one of Sprott's circuits [15], or circuits as in [16], [17-18], or a logistic series etc. Define

$$y(u) =$$

$$= \begin{cases} x(n) & for\ \frac{1}{u} = n \in N \\ \left(x\left(\left\lfloor\frac{1}{u}\right\rfloor + 1\right) - x\left(\left\lfloor\frac{1}{u}\right\rfloor\right)\right) \cdot \left(u - \left\lfloor\frac{1}{u}\right\rfloor\right) & for\ \left\lfloor\frac{1}{u}\right\rfloor \notin N \end{cases} \qquad (6)$$

Then, there is no $\lim_{u\to 0} y(u)$. Also, there is no $\lim_{u\to 0} g(y(u))$, where g is some function, e.g., sin().

B. Examples

Recall that the logistic map is $x_{n+1} = ax_n \cdot (1 - x_n)$; also chaotic is $x_{n+1} = ax_n^p \cdot (1 - x_n^p)$, $p \in N$ and suitable a. Fig. 2 shows a chirp based on the logistic map in reciprocal time. Figs. 3 and 4 show chirps based on the logistic map and functions of the reciprocal time, based on (5). Fig. 5 provides another example together with the spectrum of the chirp. Notice that the graphs are drawn using linear interpolation between the generated points; therefore, the signal equation should be written for all intervals

$\left[\frac{1}{n+1}, \frac{1}{n}\right]$ as $x(t) = x(n+1) + \frac{x(n)-x(n+1)}{\frac{1}{n+1}-\frac{1}{n}}\left(t - \frac{1}{n+1}\right)$.
To derive the Fourier spectrum, the sampling of the above should be done with a maximal sampling period of $T_{sampl} \ll 1/N(N-1)$, where N is the maximal value of the index in the series $\{x_n\}_{n=0,...,N}$.
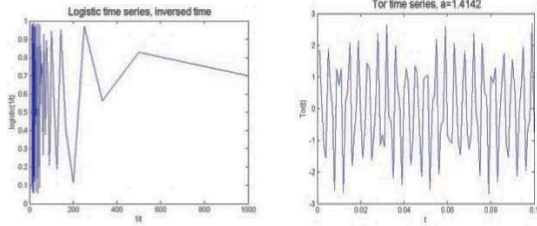

Figure 2.   Generating an essentially discontinuous chaotic chirp signal with the logistic map, with a toroidal regime.
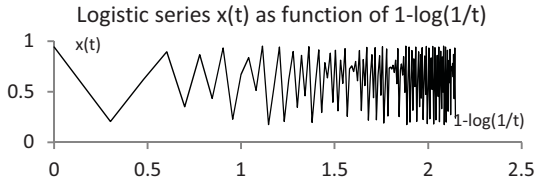

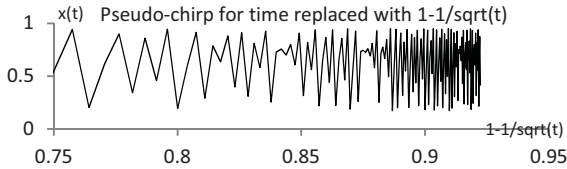Figure 3.   Pseudo chirp obtained using the logistic map as a fucntion of 1-log(1/t).

Figure 4.   Pseudo chirp obtained using the logistic process as a fucntion of 1-log(1/$\sqrt{t}$).
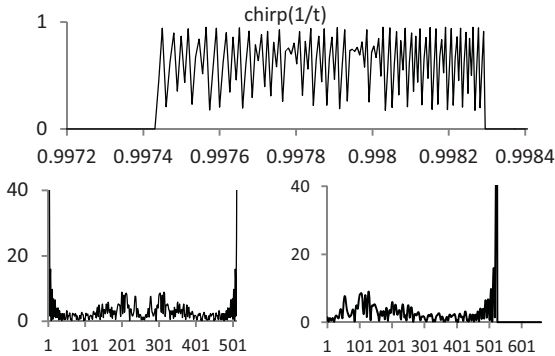

Figure 5.   Logistic chirp in reciprocal time and spectrum (spectrum truncated at amplitude 40 for better perceptibility.) Cases of periods of 512 and 1024 samples, chirp of 128 samples.

Notes: The use of reciprocal time requires supplementary technical means to implement the operation. One way is to record in a LIFO memory the samples generated by a chaotic system and then produce them as radar signal at the corresponding reciprocal time moments. If the reciprocal time is between $\left[\frac{1}{t_1}, \frac{1}{t_N}\right]$, $\frac{1}{t_N} > \frac{1}{t_1}$, then the smallest time interval between samples is $t_N - t_{N-1} = \delta t$. The value of δt is the sampling period for the original signal. The reciprocal time values are then $1/t_{N-k} = 1/(t_N + k\delta t)$.

C. Further theoretical issues

The class of functions that can be used to produce EDB chirps is very large (infinite). The next properties help detailing this statement. In general, when $x(t)$ is bounded but has no limit at infinity, $x(1/t^\alpha)$, $\alpha > 1$ has an essentially discontinuity at (no limit at) zero. In addition, if $P_n(t)$ is a polynomial and x_i are its real roots, $x(1/P_n(t))$ is essentially discontinuous at x_i.

Property. There is an injective mapping from the set of trajectories starting from different initial values of a chaotic time series and the set of sequences with essentially no limit. Also, there is an injective mapping from the set of trajectories starting from different initial values of chaotic functions with specified initial condition $f(t \in [1, \infty), x_0)$ and the set of functions with essential discontinuity (essentially no limit) at 0.

By fixing arbitrarily the initial condition, one obtains the following property:

Property. There is an injective mapping from the set of chaotic functions $f: Time \to \mathbf{R}^n$ with specified initial condition $f(t \in [1, \infty), x_0)$ and the set of functions with essential discontinuity (essentially no limit) at 0. Thus, the study of chaotic time series could be brought to the study of a subset of bounded sequences $x_{1/n}$ with no limit for $1/n \to 0$ (trivial).

Remark. Any *bounded* continuous function $f(t)$ with no asymptotic limit at infinity (divergence at infinity) has the property that it is divergent at zero for reversed time, $f(1/t)$ (essential discontinuity at zero).

All harmonic functions $g \in H$ and thus all finite order trigonometric polynomials with at least one non-null coefficient) are oscillating at infinity and therefore have no limit at zero in reciprocal time; similarly for all chaotic functions. Notice that the density of points $f(1/t) = 0$ in the vicinity of zero, defined as $\frac{\log n([\varepsilon, \varepsilon+\Delta\varepsilon])}{\log \Delta\varepsilon} \sim \varepsilon$.

If f is periodical with period T, then there are $\left|\frac{1}{T}\left(\frac{1}{\varepsilon} - \frac{1}{\varepsilon+\Delta\varepsilon}\right)\right| = \left|\frac{1}{T} \cdot \frac{\Delta\varepsilon}{\varepsilon\cdot(\varepsilon+\Delta\varepsilon)}\right|$ such points in $[\varepsilon, \varepsilon + \Delta\varepsilon]$. For $\Delta\varepsilon = \varepsilon$, one obtains $\left|\frac{1}{T} \cdot \frac{1}{2\varepsilon}\right|$, which tends to infinity for $\varepsilon \to 0$, with the density $\rho(\varepsilon) = \frac{n([\varepsilon, \varepsilon+\Delta\varepsilon])}{\varepsilon} = \frac{1}{\varepsilon} \cdot \left|\frac{1}{T} \cdot \frac{1}{2\varepsilon}\right|$ tending quadratically in $\frac{1}{\varepsilon}$ to infinity. So, any process for which there is no constant value A such that $\rho(\varepsilon) = \frac{A}{\varepsilon^2}$ is not periodical; it may be slower than periodical when the time is stretched, like for $\sin(at + bt^2)$, or it can be faster than periodical when time is compressed, such as for $\sin(\sqrt{t})$ or for $\sin(\ln(t))$.

Assume that $f(t) \in [-1, 1]$. The constant T that minimizes for a chaotic process $f(t)$ the expression

$$T = \arg \min_\tau \int_{-\infty}^{\infty} |1 - \arcsin(f(t/\tau))| dt \quad (7)$$

if the integral and the minimum exist is named the pseudo-period of the chaotic process. The waveform used in the chirp according to the method described in this article should include more than a single pseudo-period, for reasons of robustness of the detection.

Let $y = \sin\left(\frac{1}{t}\right)$. Then, $y' = -\frac{1}{t^2}\cos\left(\frac{1}{t}\right)$, $y'' = \frac{1}{t^4}\sin\left(\frac{1}{t}\right) + \frac{2}{t^3}\cos\left(\frac{1}{t}\right)$, or $y'' = \frac{1}{t^4}\,y - \frac{2}{t}\,y'$. Similar equations can be found that are satisfied by $y = \cos(1/t)$ and other functions.

Consider $g(x)$ of bounded variation for $[1, \infty)$, with no zero (fixed point) and satisfying $\lim_{x \to \infty} g(x) = c$. Let f be of unbounded variation on any interval $[a, \infty)$. Then, $g\big(f(1/x)\big)$ has unbounded variation at 0.

In principle, chaotic chirps have a spectrum unpredictable by observers (anti-active jamming of the radars). In this case, the radar knows what the correct chirp spectrum is and thus the correct chirp-decoding filters to use in the next chirp bursts, while the attacker does not. The next chirp may be generated by the same chaotic process or by one with modified parameters.

IV. DISCUSSION AND CONCLUSIONS

The typical chirp signals in the literature are continuous on **R**, with no limit at infinity. We proposed the use of chirps that (i) have essential discontinuities at the specified time moments $\{t_k\}_{k=1\dots\infty}$; (ii) where t_k are generated pseudo-randomly or chaotically, and (iii) the signals generated at each time moment are functions with essential discontinuities, moreover (iv) the functions are produced as solutions (realizations) of chaotic processes with specified initial conditions. In specific applications, (i)) can be replaced by uniformly distributed time moments.

The wide spectrum of these essential discontinuity-based pseudo-chirps may help improving the detection of multiple targets (resolution) and may be more resistant to jamming.

Carroll [9] suggests that "*chaos-based signals may have advantage in the detection of complex targets*". The same author "*found that one can optimize a chaos-based signal …thus allowing the identification of a complex target.*" We believe that the same advantage may occur for the proposed signals, with the extra advantage of better optimization capabilities.

It is unclear for now what benefits can be obtained using EDB chirps under interferences [19] and under clutter conditions as in [20], but following [9] we believe that benefits are achievable with EDB chirps.

Finally, the use of these shorter chirps may help reduce the radiation dose (Specific absorption rate, SAR) for pedestrians exposed to automotive radars. While radars operate in the range 20 GHz-85 GHz, they produce non-ionizing radiation and their heating effect is limited by standards such as ICNIRP. At the levels for automotive radars, radars are not dangerous, but their field adds up to other radiation sources.

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