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Sensitivity analysis for an almost periodic unsteady flow problem; application to turbo-machinery modelling

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1. Introduction

The shape optimization in both the compressible and incompressible flow problems still belongs to areas of intensive research due to its high applicability on one hand and the complicated mathematical structure of the problem on the other hand. The present contribution is a continuation of our previous work [2], where the shape optimization of a turbine blade was reported for the incompressible 2D flows with the aim to minimize the dissipation power. There the steady incompressible Navier-Stokes (N-S) system of equations was discretized using the finite volume method and the continuous version of the adjoint system method was employed to compute total shape gradients of the objective function.

In the present work, we are concerned with compressible 2D flows in channels whose the geometries vary periodically with time. Such a situation arises in the stator-rotor configuration of turbines where the mutual position of the blades commutes periodically. Obviously, the steady motion the rotor blades which are arranged periodically induces the unsteady flows which can be considered as quasi-periodic. Our study relies on this hypothesis, we assume the existence of T-periodic solutions. The time period $T = H/U$ is given by the circumferential geometric period H and the circumferential speed U associated with a given radius of the rectification plane where the 2D flow problem is defined.

As the new contribution, for a general shape optimization problem we introduce the state problem with an implicitly stated T-periodic condition and propose an iterative algorithm of the sensitivity analysis. In contrast with the standard evolutionary problems, the sensitivity analysis (SA) of a time periodic problem leads to a more complicated computational scheme. For efficiency of the SA computations, we propose an iterative algorithm which is constructed for the discretized state problem. We developed two versions of the SA algorithm according to the two commonly used approaches, the Direct Differentiation and the Adjoint System methods [3, 4]. Numerical solutions of the state problem are obtained using the in-house developed CFD software FlowPro [1] based on the Discontinuous Galerkin Method whereby the first-order approximation is used, thus, actually yielding the discretization scheme of the Finite Volume Method.

2. The flow problem

The flow in the stator-rotor channels is represented by a reduced two-dimensional model. Its geometrical configuration reflects the periodic arrangement of the blades, see Fig. 1. The problem is imposed in a domain constituted by two subdomains associated with the stator and rotor

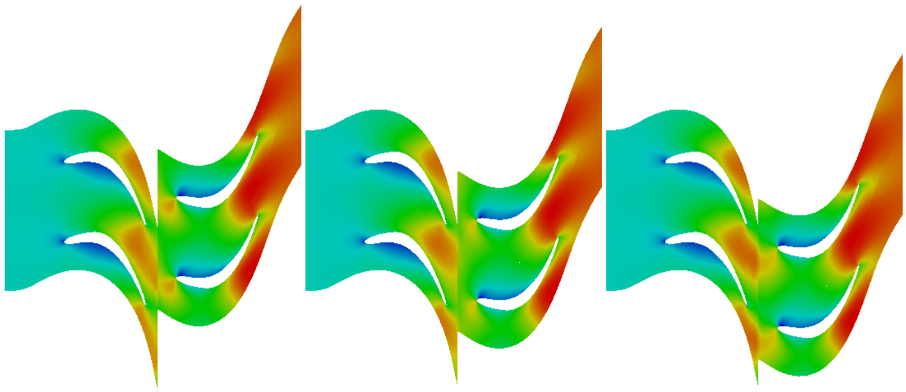

Fig. 1. Mach distribution in the flow domain at various time steps of the simulation with $\alpha = 0$ (attack angle), $p^{out} = 0.75$ (out pressure) and $U = 0.05$ (rotor velocity), the Mach number values range the interval [0.025, 0.41]

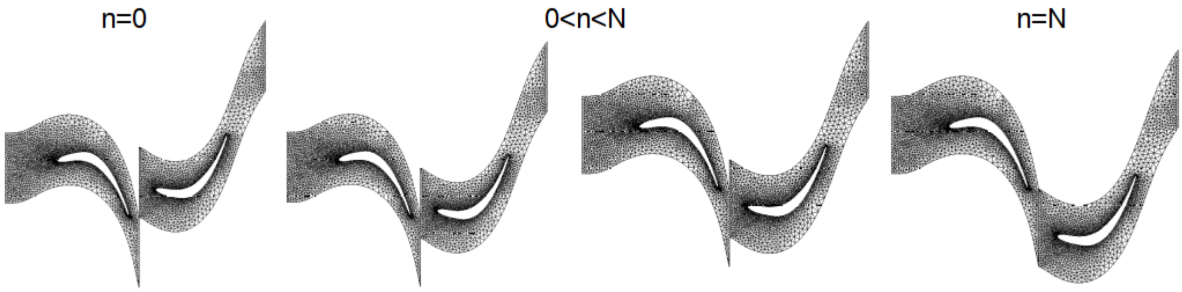

Fig. 2. An illustration of the time sequence of the stator-rotor geometric configurations. For $n = 0$, the meshes of the two subdomains are joint at one node, while at $n = N$ the meshes share all nodes on the discretized stator-rotor interface. Obviously, the configuration $n = 0$ can be released due to the assumed quasi-T-periodicity of the state problem solutions.

channels, respectively, each embedding a single turbine blade profile, as shown in Fig. 2. These subdomains have a common interface on which the coupling conditions are prescribed according to the actual instantaneous position of the rotor. The viscous compressible fluid obeys the Navier-Stokes equations which can be expressed in a compact dimensionless form, as follows

$$\frac{\partial \boldsymbol{w}}{\partial t} + \nabla \cdot \boldsymbol{F} = 0 \ , \quad \text{i.e.} \ \frac{\partial w_j}{\partial t} + \sum_i \frac{\partial F_{ji}}{\partial x_i} = 0 \ , \tag{1}$$

$$\boldsymbol{w} = \begin{pmatrix} \rho \\ \rho v_1 \\ \rho v_2 \\ \rho e \end{pmatrix} , \quad \boldsymbol{f}_i^E = \begin{pmatrix} \rho v_i \\ \rho v_1 v_i + p\delta_{1i} \\ \rho v_2 v_i + p\delta_{2i} \\ v_i(\rho e + p) \end{pmatrix} , \quad \boldsymbol{f}_i^V = \frac{1}{Re} \begin{pmatrix} 0 \\ \tau_{1i} \\ \tau_{2i} \\ \sum(\tau_{ij} v_j) - \frac{\gamma}{Pr}\nabla(e - \frac{\boldsymbol{v}^2}{2}) \end{pmatrix} ,$$

where $\boldsymbol{f}_i^E$ and $\boldsymbol{f}_i^V$, are the columns of $\boldsymbol{F}^E$ and $\boldsymbol{F}^V$, respectively, and $\boldsymbol{F} = \boldsymbol{F}^E(\boldsymbol{w}) - \boldsymbol{F}^V(\boldsymbol{w}, \nabla \boldsymbol{w})$. The state variable $\boldsymbol{w}$ involves the density ρ, velocity (v_1, v_2) and the internal energy e. The viscous part of the stress is denoted by τ_{ij}. We consider laminar subsonic flows and the boundary conditions are classically defined by the flow direction at the inlet (the angle of attack α) and a pressure at the outlet p^{out}, while the value 1 is assigned to the inlet stagnation density and pressure. The initial conditions can be chosen almost arbitrarily, since the state problem solution is defined as a steadily periodic solution. Although an exact periodic solution is not enforced and, thus not guaranteed, we assume the existence of time t_0 such that the solutions $\boldsymbol{w}(t)$ of the initial value problem satisfy an approximate periodicity condition

$$\|\boldsymbol{w}(t) - \boldsymbol{w}(t+T)| < \epsilon \,, \quad \text{for all } t \geq t_0 \,, \tag{2}$$

where ϵ is a given precision and the norm $\|\boldsymbol{w}\|$ is represented by the Euclidean norm of the space-discretized state vector $\boldsymbol{w}$. Adhering to the assumption (2), we consider only the time steps $n \in [1, N]$ such that $t^n = t^0 + n\Delta t$. Thus, the quasi-T-periodic solutions of the state problem are represented by the N-tuples $\{\boldsymbol{w}^n\}_{n=1}^N$, whereby $\boldsymbol{w}^0 \approx \boldsymbol{w}^N$ in the sense of (2)

To capture the motion of the rotor part domain, we consider a decomposition of the velocity field $\boldsymbol{v}$ involved in (1) into the mesh velocity $\boldsymbol{v}^{msh}$, and the relative velocity $\boldsymbol{v}^{rel}$, so that $\boldsymbol{v} = \boldsymbol{v}^{rel} + \boldsymbol{v}^{msh}$. For the stator-rotor system, $\boldsymbol{v}^{msh}$ is defined piecewise constant; while it vanishes in the stator subdomain, in the rotor subdomain, $\boldsymbol{v}^{msh}$ equals a constant vector $\boldsymbol{U}$ which describes the revolution speed $U = |\boldsymbol{U}|$ of the rotor. Therefore, $\boldsymbol{U}$ is involved in the convective acceleration terms of the flow equations in the rotor domain only. The discontinuity of $\boldsymbol{v}^{msh}$ on the stator-rotor interface requires updating the position of the mesh nodes of the rotor mesh. Consequently, pairs of the homologous nodes on the interface commute with subsequent time levels. One time period T is subdivided into N steps $\Delta t = U\delta d$ according to the interface discretization δd which defines the shift of the rotor mesh position with respect to the stator ones, see Fig. 2, where the sequence of N relevant geometric configurations is displayed. Thus, for a given time level, this shift provides a particular coupling scheme for the nodal values of the discretized quantities along the matching interfaces of the subdomains.

3. Shape sensitivity of an objective function

The shape sensitivity analysis is needed to allow for using gradient based optimization methods. In this context, we consider a general objective functional

$$\Phi = \int_{t^0}^{t^0+T} \varphi(\boldsymbol{w}(t), t) \mathrm{d}\, t \,, \tag{3}$$

where φ evaluates a criterion of the flow optimality at time t. For instance φ can express the loss of stagnation pressure relative to the inlet kinetic energy. In general, we assume Φ is bounded below over the set of all admissible solutions of the State Problem (SP) given by (1) – (2). Since the sensitivity has been derived for the fully discretized SP, we introduce the shape Optimization Problem (OP) in terms of the discretized SP; by $\boldsymbol{W}^n$ we refer to the discretized state $\boldsymbol{w}^n$. The shape of the blades profiles is described the non-uniform rational basis spline functions involving m^P control points P^i. By $\boldsymbol{Y}$ we denote the vector of $m = 2m^P$ coordinates of the control points P^i; in Fig. 3, locations of P^i in the reference configuration is depicted. Below we consider the set $\mathcal{Y}$ of all admissible blades designs restricting the shape geometry with respect to criteria independent of the state $\boldsymbol{W}$.

We consider the general OP: Find $\boldsymbol{Y}^* \in \mathcal{Y}$ and the state $\boldsymbol{W}^* = \boldsymbol{W}(\boldsymbol{Y}^*)$, such that

$$\begin{aligned} &\Phi(\boldsymbol{Y}^*, \boldsymbol{W}(\boldsymbol{Y}^*)) \leq \Phi(\boldsymbol{Y}, \boldsymbol{W}(\boldsymbol{Y})) \quad \forall \boldsymbol{Y} \in \mathcal{Y} \,, \text{ where } \quad \Phi := \sum_{n=1}^{N} \phi^n(\boldsymbol{Y}, \boldsymbol{W}^n) \,, \\ &\text{subject to: } \boldsymbol{\mathcal{F}}^n(\boldsymbol{Y}, \boldsymbol{W}^n, \boldsymbol{W}^{n-1}) = 0 \,, n = 1, \ldots, N \,, \quad \text{and } \boldsymbol{W}^N \approx \boldsymbol{W}^0 \,, \end{aligned} \tag{4}$$

where $\boldsymbol{\mathcal{F}}^n = \boldsymbol{0}$ represents the space-time discretized approximation of Problem (1) – (2), and $\boldsymbol{W} = \{\boldsymbol{W}^n\}_{n=1}^N$ involves the state vectors at all relevant time levels. The admissible states $\boldsymbol{W}(\boldsymbol{Y})$ are defined as an implicit function constituted by the constraint $(4)_2$.

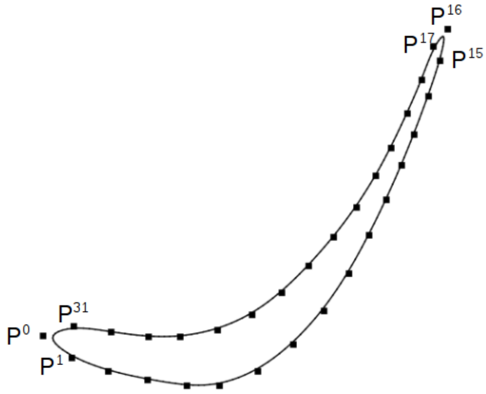

Fig. 3. The rotor blade profile and positions of 32 control points in the reference configuration.

Within the design-state space of any $(\boldsymbol{Y}.\boldsymbol{W})$, the sensitivity analysis provides the total design gradient $\mathrm{d}^{\mathrm{tot}}\,\Phi(\boldsymbol{Y},\boldsymbol{W}(\boldsymbol{Y}))/\mathrm{d}\,\boldsymbol{Y}$ on a manifold of admissible states $\boldsymbol{W}(\boldsymbol{Y})$,

$$\frac{\mathrm{d}^{\mathrm{tot}}\,\Phi}{\mathrm{d}\,\boldsymbol{Y}} = \sum_{n=1}^{N}\left(\frac{\partial\phi^n}{\partial\boldsymbol{Y}} + \frac{\partial\phi^n}{\partial\boldsymbol{W}^n}\boldsymbol{G}^n\right)\,, \tag{5}$$

where the matrix $\boldsymbol{G}^n = \partial\boldsymbol{W}^n/\partial\boldsymbol{Y}$ is the design gradient of the state $\boldsymbol{W}^n$. To evaluate total design gradient, we pursue the Direct Differentiation (DDM) and the Adjoint System (ASM) methods, the latter one allows to avoid computing matrices $\boldsymbol{G}^n$, however, the adjoint vectors $\boldsymbol{\lambda}^n$ must be solved for. In contrast with the standard transient problems, the initial condition $\boldsymbol{W}^0$ is not known, as explained above. Instead, the time periodicity conditions must be handled. Computing $\boldsymbol{\lambda}^n$ rigorously would require solving a set of coupled linear equations for all $\{\boldsymbol{\lambda}^n\}_{n=1}^{N}$. In the case of the DDM, a similar difficulty is encountered. Therefore, we propose iterative DDM and ASM algorithms. The dimension of $\boldsymbol{\lambda}^n$ is independent of the number of design variables. Consequently, when the number of the design variables is large, the ASM is preferred to the DDM, cf. [3]. Both these methods have been implemented to our application. Numerical tests and verifications have been performed.

Acknowledgments

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