COMPUTATIONAL MECHANICS 2019
35th conference with international participation
Srní
November 4 - 6, 2019

Lagrangian tracking of a cavitation bubble

P. Rudolf[a], A. Bossio[b]

[a] Viktor Kaplan Dept. of Fluid Engineering, Brno University of Technology, Technická 2, 616 99 Brno, Czech Republic
[b] Institute of Mathematics, Brno University of Technology, Technická 2, 616 99 Brno, Czech Republic

Cavitation is usually defined as the generation of vapor bubbles in a liquid flow due to a pressure drop under vapor pressure at corresponding liquid temperature [1]. It is a widely studied phenomenon in fluid mechanics, mainly because when the bubbles are exposed again to high pressures, condensation occurs and bubbles experience a violent compression, which releases a large amount of energy. For a very short time period temperature within bubble reaches thousands of Kelvins and dissociation of water molecules results in production of hydroxyl radicals. The effects of this collapse are well known by engineers, especially those involved with hydraulic devices, where a cavitating flow may cause losses in efficiency, high levels of noise and vibration and severe erosion of internal solid surfaces. However this phenomenon can also be exploited positively for elimination of pathogenic microorganisms or reduction of dangerous chemical residuals contained in water (volatile organic compounds, pharmaceuticals etc.). To design efficient devices for removal of undesired biological or chemical contamination it is necessary to understand the process of cavitation bubble collapse in flowing liquid, which is a combination of advection by the liquid stream and bubble dynamics induced by variable pressure field.

Dynamics of an isolated cavitation bubble submerged in a steady flow is studied numerically in present contribution. An Eulerian-Lagrangian approach is considered in which properties of the fluid are computed first, using a two-phase homogeneous mixture model, by means of a Eulerian method within commercial CFD code, and then the trajectory of the bubble is computed in a Lagrangian fashion, i.e., the bubble is considered as a small particle moving relative to the fluid, due to the effect of several forces depending on fluid's pressure and velocity fields previously computed. Important ingredient is change of the bubble's radius imposed by surrounding pressure field, which is modeled by Rayleigh-Plesset equation. In the end energy released by the successive collapses of the bubble is evaluated to estimate the energy available for damage of material surface or rupture of the cell membranes.

Newton's law of motion can be used to describe bubble's trajectory

$$m_b \frac{d\boldsymbol{u_b}}{dt} = \boldsymbol{F_D} + \boldsymbol{F_L} + \boldsymbol{F_P} + \boldsymbol{F_A} + \boldsymbol{F_g}, \tag{1}$$

where m_B is the mass of the bubble, $\boldsymbol{u_B}$ is the absolute velocity of the bubble. The terms on the R.H.S. represent respectively the drag force, the lift force, the force due to pressure gradient, added mass force and buoyancy/gravity effects. In addition, bold fonts imply vector quantities. Obviously the bubble/bubble and bubble/wall interactions are not included. The drag force $\boldsymbol{F_D}$ over a sphere is usually taken as

$$\boldsymbol{F_D} = -\frac{1}{2} C_D \pi R^2 \rho_l |\boldsymbol{u_s}| \boldsymbol{u_s}, \tag{2}$$

where R stands for the radius of the bubble, ρ_f is the surrounding fluid's density, $\boldsymbol{u_s}$ is the slip velocity defined as $\boldsymbol{u_s} = \boldsymbol{u_b} - \boldsymbol{u_f}$, with $\boldsymbol{u_f}$ being the absolute velocity of a fluid's particle located at bubble's center, and C_D is the drag coefficient, depending mostly on the Reynolds number of the flow, Re $= \frac{2R\rho u_s}{\mu}$. Non-linear relation between drag coefficient C_D and Reynolds number is assumed according to formula proposed by Yang et al. [3].

The force $\boldsymbol{F_L}$ accounts for the lift force, a force usually represented by

$$\boldsymbol{F_L} = -C_L \rho_l V_b \boldsymbol{u_s} \times (\boldsymbol{\nabla} \times \boldsymbol{u_l}), \tag{3}$$

$\boldsymbol{F_P}$ is the force due to pressure gradient defined as

$$\boldsymbol{F_P} = V_b \boldsymbol{\nabla} p, \tag{4}$$

$\boldsymbol{F_A}$ is the so called added mass or virtual mass effect, and represents the additional inertia added to the system due to the need of accelerating the surrounding fluid in order to occupy a new position. It is usually implemented as

$$\begin{aligned} \boldsymbol{F_A} &= -C_A \rho_l \frac{D}{Dt} \left(V_b (\boldsymbol{u_b} - \boldsymbol{u_l}) \right) \\ &= -C_A \rho_l V_b \left(\frac{d\boldsymbol{u_b}}{dt} - \frac{D\boldsymbol{u_l}}{Dt} \right) - C_A \rho_l \frac{dV_b}{dt} \boldsymbol{u_s}. \end{aligned} \tag{5}$$

Finally, $\boldsymbol{F_g}$ represents a combined gravitational and buoyancy force and is therefore implemented as

$$\boldsymbol{F_g} = (m_b - \rho_l V_b)\boldsymbol{g}, \tag{6}$$

where $\boldsymbol{g}$ is the acceleration due to gravity. Since bubble is changing its diameter along its trajectory, it is clear that the change also has impact on magnitude of the respective forces (especially drag force, buoyancy force and added mass force). Bubble dynamics of a spherical isolated cavitation bubble was first described by Lord Rayleigh [2]. More accurate equations, which are extension of the original formulation appeared later (Rayleigh-Plesset, Gilmore, Herring, Keller, ...). Due to some stability problems basic formulation by Rayleigh and Plesset [1] is employed in present contribution

$$R\ddot{R} + \frac{3}{2}\dot{R}^2 = \frac{p_v - p_\infty}{\rho} + \frac{p_{g_0}}{\rho} \left(\frac{R_0}{R} \right)^{3k} - \frac{2S}{\rho R} - \frac{4\mu}{\rho R} \dot{R}, \tag{7}$$

where p_v is vapor pressure at liquid's temperature, p_∞ is the pressure in the liquid far from the bubble, p_{g_0} is the partial pressure of the air inside the bubble at a reference bubble size R_0, k is the polytropic coefficient, S is the surface tension and μ is the viscosity of the liquid. This equation assumes spherical shape of the bubble, incompressibility of the surrounding fluid and zero heat and mass transfer between the bubble and the surrounding fluid.

Venturi tube with dimensions according to Fig. 1 was used as an example for the bubble tracking.

Velocity and pressure fields were computed numerically using ANSYS Fluent 19.1, with Reynolds-averaged Navier-Stokes (RANS) equations accompanied by realizable k–ε model, the numerical method is finite volume method (FVM), using segregated approach with SIMPLE algorithm. The geometry and equations are adopted for axisymmetric assumption. Multiphase cavitating flow was implemented by homegeneous mixture approach with simplified Rayleigh-Plesset equation. See distribution of the phases for cavitation number $\sigma = 0.42$ in Fig. 2. These fields were used as an input for MATLAB code and its Adams-Bashforth-Moulton method,

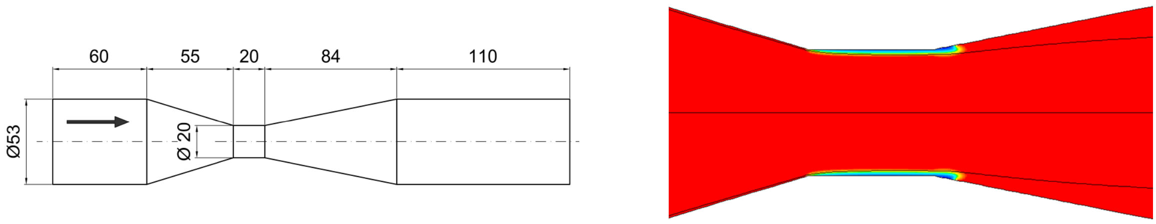

Fig. 1. Geometric description of the simulated Venturi tube (all dimensions in mm) (*left*), phase distribution for the Eulerian approach (*right*)

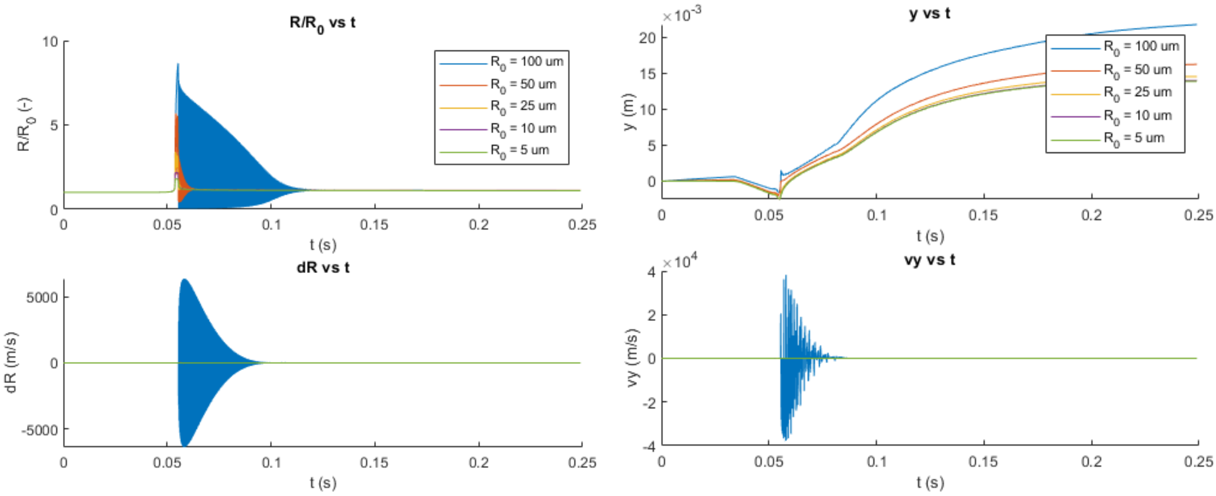

Fig. 2. Bubble dynamics along its trajectory (*left*), bubble position and velocity in the y-direction (*right*)

where the set of equations (1) – (7) was implemented. Results were computed for a range of different initial bubble radii and trajectory of the bubbles was depicted in Fig. 2.

Energy released by bubble collapse is computed according to [4] as work done by the pressure inside the bubble against the ambient pressure

$$W = \int_{R_{min}}^{R_{max}} 4\pi R_B^2 (p_\infty - p_B)\,\mathrm{d}R. \tag{8}$$

The energy is then determined by subtracting the values for two successive maxima of the radii. Generally, the energy released during the i-th collapse is given by

$$E_i = W\Big|_{R_{min_i}}^{R_{max_i}} - W\Big|_{R_{min_{i+1}}}^{R_{max_{i+1}}}. \tag{9}$$

Accompanied by correlation of material (or cell membrane) damage obtained from experimental investigations the presented approach can constitute a basis for cavitation erosion model or cell rupture model.

Acknowledgement

Research was supported by Czech Science Foundation grant project No. 19-10660S "Removal of estrogens from waste water using hydrodynamic cavitation in combination with advanced oxidation processes".

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