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Numerical approximation of fluid flow problems by discontinuous Galerkin method

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The discontinuous Galerkin methods (DGM) are gaining popularity in solving partial differential equations arising from modeling scientific and engineering problems, see e.g. [1]. Amongst other, the DGM was successfully applied for the numerical solution of incompressible fluid flows, i.e., Navier-Stokes equations, see e.g. [2, 3, 5]. This contribution focus on the verification of the high order discontinuous Galerkin method implementation for the solution of the Navier-Stokes equations in two dimensions. The numerical method was implemented within the Julia programming language.

The fluid flow of an incompressible viscous fluid in the domain Ω is described by the system of the Navier-Stokes equations (see e.g. [3]), i.e.,

$$\frac{\partial \boldsymbol{u}}{\partial t} + \boldsymbol{\nabla}\cdot(\boldsymbol{u}\otimes\boldsymbol{u}) = -\boldsymbol{\nabla} p + \nu\,\boldsymbol{\nabla}^2\boldsymbol{u}, \quad \boldsymbol{\nabla}\cdot\boldsymbol{u} = 0, \tag{1}$$

where $\boldsymbol{u}$ is the fluid velocity vector, p is the pressure divided by the constant fluid density ϱ and ν is the constant kinematic viscosity of the fluid. The boundary of the computational domain Ω, see Fig. 1 (left), is divided into three distinct parts, i.e., $\partial\Omega = \Gamma_I \cup \Gamma_W \cup \Gamma_O$, where Γ_I and Γ_O are inlet, and outlet parts of the computational domain, respectively and Γ_W denotes the impenetrable wall. The system (1) is supplemented by suitable initial conditions $\boldsymbol{u}(\boldsymbol{x},0) = \boldsymbol{u}_0(\boldsymbol{x})$, $p(\boldsymbol{x},0) = p_0(\boldsymbol{x})$, and boundary conditions $\boldsymbol{u} = \boldsymbol{u}_I$ on Γ_I, $\boldsymbol{u} = \boldsymbol{0}$ on Γ_W and $p = p_O$ on Γ_O.

In order to solve the problem (1) the computational domain $\Omega_h(\approx \Omega)$ is discretized using a triangulation $\mathcal{T}_h$, where the higher order isoparametric elements were used on curved boundary, see e.g. [3]. The numerical solution is represented by piecewise polynomials of degree $N \geq 1$

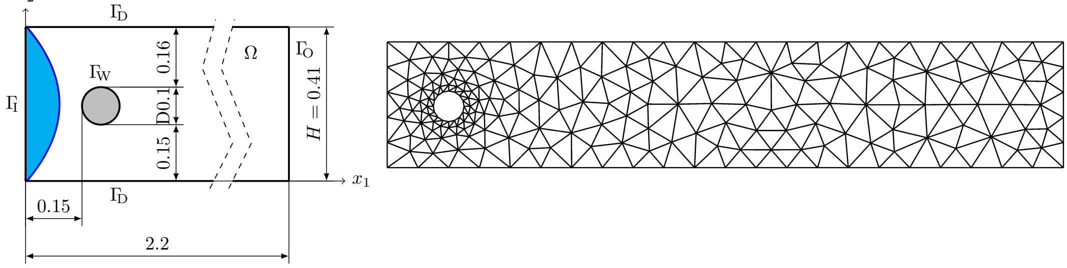

Fig. 1. (*Left*) Sketch of the computational domain. (*Right*) Computational grid consisting of 356 elements.

on each triangle $k \in \mathcal{T}_h$. The time integration is done by splitting scheme with three stages which takes into account the mixed nature of equations (1), see [3].

The implemented numerical method was tested on the benchmarks of laminar flow around a cylinder [4], so-called DFG benchmarks. Simulations were done on the very coarse grid consisting of 356 elements (see Fig. 1 (right)) and the different degrees of polynomials N were considered. Table 1 shows obtained reference values. One can see that values obtained by DGM corresponds well for higher orders of polynomials both for $\mathrm{Re} = 20$ (steady) and $\mathrm{Re} = 100$ (periodic) with DFG data. Fig. 2 shows vorticity contours for $N = 5$ and $N = 9$. Fig. 3 shows lift coefficient c_l during the computation.

Table 1. Overview of obtained reference values for the cases $\mathrm{Re} = 20,\ 100$; $*$ values from [4]

	$\mathrm{Re} = 20$			$\mathrm{Re} = 100$		
N	c_d	c_l	Δp	$\max(c_d)$	$\max(c_l)$	$\Delta p(t = 8)$
1	6.511912	0.816498	0.159512	7.811225	2.939393	0.140196
5	5.579752	0.012096	0.117506	2.958082	0.475532	0.110243
9	5.579598	0.010619	0.117532	2.943942	0.477448	0.111623
$*$	5.579535	0.010619	0.117520	2.943764	0.477488	0.111541

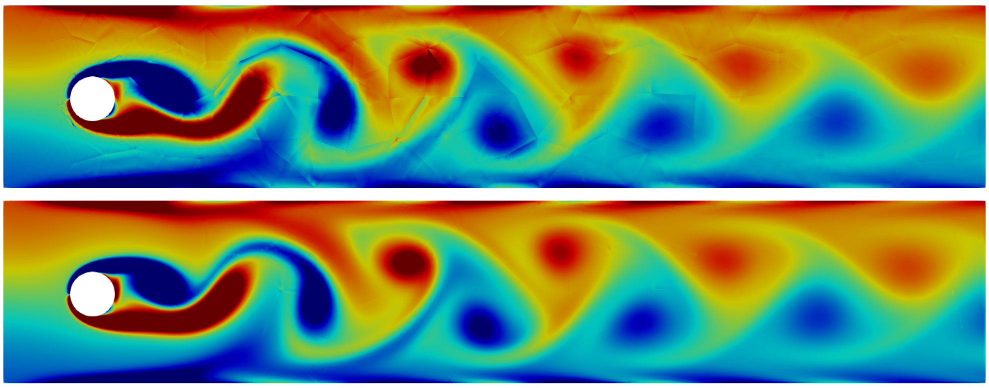

Fig. 2. Contours of vorticity for different N, $\mathrm{Re} = 100$ at 8 s: (*top*) $N = 5$, (*bottom*) $N = 9$

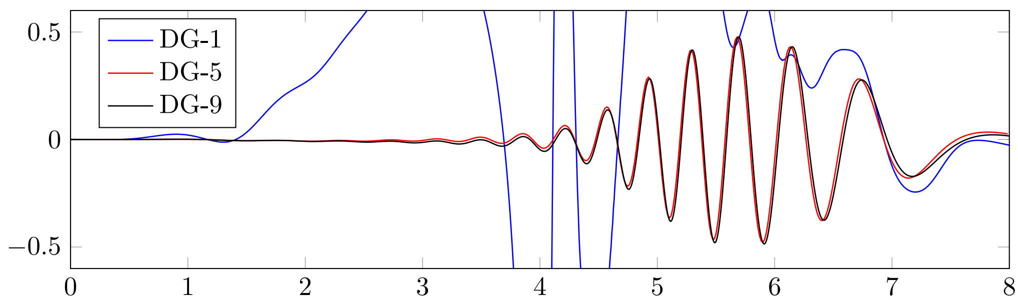

Fig. 3. Lift coefficient $c_l(t)$, $\mathrm{Re} = 100$

Conclusions. In this contribution the in-house implementation of the high-order discontinuous Galerkin method is used to compute flow past the cylinder. Obtain results correspond very well with DFG benchmarks reference values.

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