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Local tilings informed synthesis of micro-mechanical fields by means of Wang tiles

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In this paper, we present our work based on Wang tiles method and its utilisation on synthesis of micro-mechanical fields extended by local tilings information.

The very basic concept of the Wang tiles method [5] can be compared to jigsaw puzzle or classic board game domino. The fundamental pieces are positioned next to each other according to corresponding edge information like fitting edge shape, number of dots or in case of Wang tiles for example colours, patterns and others. Tiles are modelled visually as squares and by rotating or mirroring existing tile a new tile is created.

Group of tiles is called set and specially designed tile set in combination with particular tiling algorithm can construct planar domain of arbitrary size.

In contrast with other methods modelling heterogeneous materials like periodic unit cell method (PUC) or statistically equivalent periodic unit cell method (SEPUC) (used for example for homogenisation) the main advantage of Wang tiles method is the ability to preserve stochastic layout of original micro-structure. This is achieved by the tiling algorithm that in addition requires specifically defined tile set and tile design.

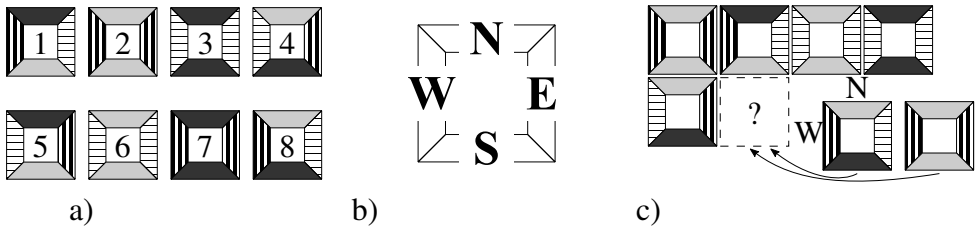

Fig. 1. a) Wang tiles set W8/2-2 (eight tiles with two different codes on vertical and horizontal edges), b) Tile edge labels, c) North-West corner position

As a tiling algorithm, we use algorithm presented in [1] (further called CSHD) which specifies one simple rule. Every time the tile is positioned in the *NW* corner position (Fig. 1c) the set must comprise at least two valid tiles. From these tiles one is randomly selected and the process is repeated. Algorithm can be also modified to prevent occurrence of group of same tile in the tiling by allowing to repeat the tile selection n-times. The minimal Wang tile set that fulfils the requirements of CSHD algorithm is presented in Fig. 1a.

Tiles can be designed with various methods. One method is similar to generating SEPUC where the original micro-structure is analysed and the tiles are optimised in such way the resulting micro-structure holds the same spatial characteristics. This approach is very robust but also extremely slow.

Next method to create tiles is based on samples from the original micro-structure where the number of extracted samples is equal to number of different edge information in set. For each tile the respective samples are arranged in to rhombus shape with overlap and stitched together by means of image quilting algorithm [1]. This method gives almost instant results, but struggles with fine details of micro-structures. Then other methods based on particle dynamics [4] and level set [2] can be also used, but these methods have limited control over spatial statistics.

To obtain micro-mechanical fields (e.g. stresses, strains or displacements) the whole micro-structural domain can be discretized by very fine FEM mesh and evaluated. Unfortunately according to domain size and fineness of the mesh the computational requirements and evaluation time can be very demanding. These can be reduced by utilising the Wang tiles method as the process of micro-structure reconstruction can be applied to micro-mechanical field synthesis.

Each tile is discretised by fine FEM mesh in such way that the meshes are still edge compatible. Then the required mechanical response is evaluated on each tile separately and synthesised back according to underlying map of tiles. However, because of the non-local character of mechanical quantities the results contains discontinuities on tile edges and therefore the underlying grid of tiles is recognisable [3]. That is because of the mechanical response of each tile is affected by a different combination of surrounding tiles.

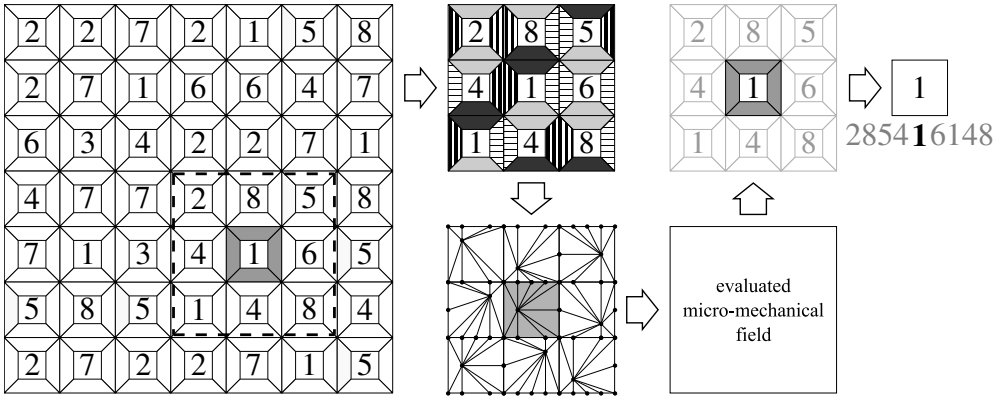

Fig. 2. Local tiling evaluation process

To solve this problem we create so-called local tiling for each tile T_i from underlying tiling map by including surrounding tiles into the evaluation of the mechanical response. This process is demonstrated on the Fig. 2 where on left is part of the reconstructed domain illustrated as grid of tiles. For currently solved tile T_1 (highlighted by grey colour) the small sub-domain of this centre tile and first layer of surrounding tiles is created. This sub-domain is discretised by FEM mesh and required micro–mechanical response is evaluated. Results for tiles in surrounding layer are discarded and only results for centre tile T_1 are saved under label $T_{28541\mathbf{1}6148}$. This process is repeated for every tile in the domain until results for each tile in the domain are obtained. Then the micro-mechanical field for the whole domain can be synthesised or these results can be used as fluctuation fields in Partition of Unity methods.

Acknowledgements

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