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Hydrodynamically lubricated contact between axial rings of the pinion and the wheel of a high-speed gearbox: Contribution to numerical assessment

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1. Introduction

Main application field for high-speed gearboxes is the power sector, particularly gas and steam turbine drives, turbo compressors and auxiliary starting drives. A present-day requirement for such gearboxes is their mechanical efficiency to achieve at least to 99%. This topicality is emphasized by the fact that transmitted power reaches to tens of MW. The high efficiency mentioned above is achieved by optimizing gear design comprising also the reduction of friction loss in bearings. At the same time, low-speed shaft bearings prove the lesser loss than the high-speed ones. In this case, therefore, the axial bearing of a pinion shaft is replaced by axial rings for transmission of axial forces from the high-speed shaft to a low-speed one. Both the gear force and an outside force, e.g. from a turbine or a compressor, are involved. Nevertheless, it is necessary to consider friction loss even between axial rings. Actually, in addition to rolling, slipping of rings occurs due to the shift of their contact centre out of the pitch circles (see Fig. 1).

Nomenclature

F	the contact axial force of rings
h	film thickness, $h = h_a + h_b$
h_a	distance between a pinion ring point and the reference plane
h_b	distance between a wheel ring point and the reference plane
p	pressure
x,y,z	coordinates in global coordinate system
x	direction in the plane of shaft axes
z	the pinion shaft axial direction in global coordinate system
ξ, χ, ψ	the Cartesian coordinates relative to the contact reference plane
ξ	direction perpendicular to the contact reference plane
χ	direction of the intersection of shaft axes with the contact reference plane
u	fluid velocity in the χ direction
v	fluid velocity in the ψ direction
w	fluid velocity in the ξ direction
η	oil dynamic viscosity
μ_0	oil kinematic viscosity for atmospheric pressure
μ	oil kinematic viscosity
α	parametr in the Barus equation
ρ	oil density
$\pi/2 - \varphi$	bevel of the pinion ring
$\pi/2 - \phi$	bevel of the wheel ring
ω	the angular frequency of the pinion shaft
Ω	the angular frequency of the wheel shaft

Subscripts

a	surface of the pinion
b	surface of the wheel

It seems, no convincing research paper dealing with behaviour of axial rings has been published yet. That is why design offices use the over-simplifying computations of the oil film load capacity between the rings. For example, their conicalness and other geometric and operational parameters are not included. Similarly, an assessment based on the Hertz dry contact theory is acceptable only for the case of gearbox run-up. The insufficient knowledge entails a stumbling block in next rings developments and in the increasing of gearbox transmitted power keeping high efficiency.

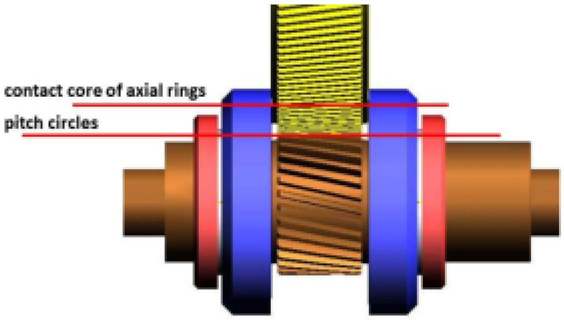

Fig. 1. Pinion and the part of wheel with axial rings

In this study, the hydrodynamically lubricated contact between axial rings considering their steady state operating mode is studied. Perfect geometry of rings and isothermal oil condition are next simplifying assumptions. The assumption that the contact of rings is hydrodynamically lubricated is suitable as the ratio between the minimal oil film thickness, $h_{\min}$, and roughness, σ, fulfils the precondition $h_{\min}/\sigma \gg 3$ (see e.g. [2]) for the operating mode. The general Reynolds equation for 2D situation is then convenient to use for the determination of the oil pressure field and of the oil flow rate. It is remarked in the third section. Before it, in the second section, our focus is payed to the geometry of film thickness, i.e. to the gap between rings. Results for an example with a geometry of rings and with an operating mode are presented in the fourth section. Important dependence between the nominal film thickness and the value of transmitted axial force is brought out here for the considered lubricated contact. An attention is also devoted to friction losses.

2. Clearance between cones

Fig. 2 shows the overall situation with parameters of axial rings depicted in the plane of both parallel shaft axes. The directions ξ and χ of the global coordinate system considered lie in the plane. Further, the reference plane of the lubricated contact of rings, α, is defined so that it is orthogonal to the direction ξ and it touches the pinion cone surface. Points of the reference plane, just as perpendicular projections of points of cone surfaces into the plane, are then uniquely determined by the coordinates χ and ψ. The clearance $h = h(\chi, \psi)$ is the function of these coordinates and is measured in the direction of ξ. Isolines of the film thickness h are depicted in Fig. 3 for the considered parameters.

3. Theory

Oil is considered as a Newtonian fluid. Its inertial effects can be neglected in our case. With the boundary conditions u_a, v_a, u_b, v_b, prescribed on surfaces of rings, the velocity components are

$$u(\xi,\chi,\psi) = \frac{1}{2\eta}\frac{\partial p}{\partial \chi}\{\xi^2 - \xi(h_b - h_a) - h_a h_b\} + u_b\frac{\xi + h_a}{h} + u_a\frac{h_b - \xi}{h}\ , -h_a \le \xi \le h_b\ ,$$

$$v(\xi,\chi,\psi)=\frac{1}{2\eta}\frac{\partial p}{\partial \psi}\{\xi^2-\xi(h_\mathrm{b}-h_\mathrm{a})-h_\mathrm{a}h_\mathrm{b}\}+v_\mathrm{b}\frac{\xi+h_\mathrm{a}}{h}+v_\mathrm{a}\frac{h_\mathrm{b}-\xi}{h}\ ,-h_\mathrm{a}\le\xi\le h_\mathrm{b}\ ,$$

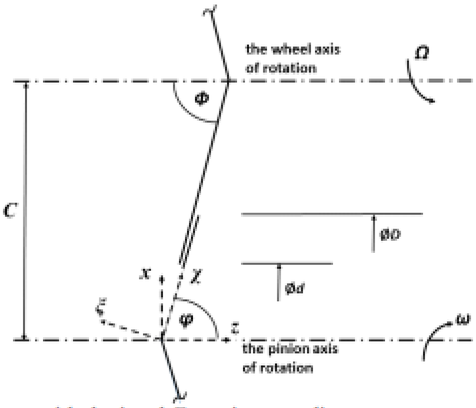

Fig. 2. The plane of shaft axes with depicted Cartesian coordinate systems and with parameters of axial rings

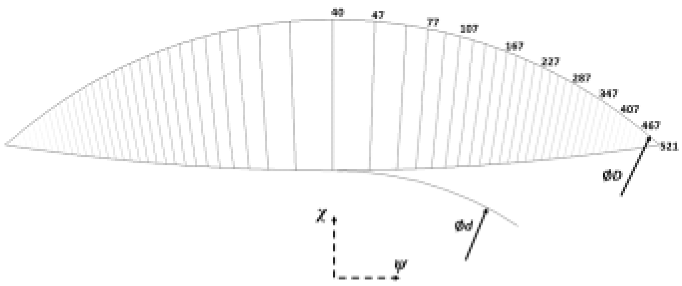

Fig. 3. Film thickness, h, in μm for the nominal thickness 40 μm and parameters d = 125 mm, D = 180 mm, C = 450 mm, $\varphi = \Phi = 89°$

where quantities h, h_a, h_b and p with the boundary velocities are functions dependent on coordinates χ, ψ. Integrating the continuity equation over the thickness h yields, after some algebra near to of [1], the general Reynolds equation

$$\frac{\partial}{\partial\chi}\left(\frac{\rho h^3}{12\eta}\frac{\partial p}{\partial\chi}\right)+\frac{\partial}{\partial\psi}\left(\frac{\rho h^3}{12\eta}\frac{\partial p}{\partial\psi}\right)=\frac{\partial}{\partial\chi}\left(\frac{\rho h(u_\mathrm{a}+u_\mathrm{b})}{2}\right)+\frac{\partial}{\partial\psi}\left(\frac{\rho h(v_\mathrm{a}+v_\mathrm{b})}{2}\right)+$$
$$+\rho(w_\mathrm{a}-w_\mathrm{b})+\rho u_\mathrm{b}\frac{\partial h_\mathrm{b}}{\partial\chi}-\rho u_\mathrm{a}\frac{\partial h_\mathrm{a}}{\partial\chi}+\rho v_\mathrm{b}\frac{\partial h_\mathrm{b}}{\partial\psi}-\rho v_\mathrm{a}\frac{\partial h_\mathrm{a}}{\partial\psi}$$

for unknown pressure, p. The viscosity, η, and the density, ρ, change with oil temperature and pressure, but, for the sake of brevity, only isothermal situation is considered here. Furthermore, the Barus empirical equation $\ln\frac{\mu}{\mu_0}=\alpha p$ (see [3]) is used to express the dependence of viscosity on pressure. Besides, the mathematical model described is necessary to complete by some condition for a cavitation region. Here, the simple approach taken by Kapitza that ignores the negative pressures, i.e. $p\ge 0$, is considered [1].

4. Results

Before starting computational simulations, an in-house program based on the theory described above was created by the first author. As an illustrative example, rings with the geometry parameters h_{min} = 0.04 mm, d = 125 mm, D = 180 mm, C = 450 mm, $\varphi = \Phi = 89°$ (see Fig. 3) are considered below. Their contact is lubricated by oil of ISO VG 46 at temperature 40ºC, the value $\alpha = 0.0215 \text{xMPa}^{-1}$ is given from [3]. Angular frequencies are $\omega = 10791$ rpm, $\Omega = 1490$ rpm. Fig. 4 shows the oil pressure field calculated under boundary conditions with atmospheric pressure. In accordance with the inequality $p \geq 0$, the field is zero in the right hand part of the contact as opposite points of cone surfaces are receding. The most important result is the dependence of the total contact axial force on the minimal thickness, $h_{min} - F$, (see Fig. 5). That the force increases steeply for h_{min} below the value 0.05 mm is apparent. It was also worked out that friction losses increases simultaneously with the contact force.

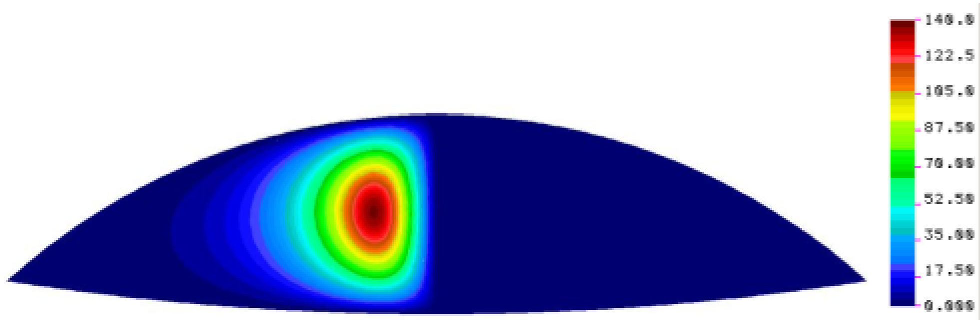

Fig. 4. Oil pressure field, p, in MPa for geometry of rings from Fig. 3, for angular frequencies $\omega = 10791$ rpm, $\Omega = 1490$ rpm, and for the mineral oil ISO VG 46

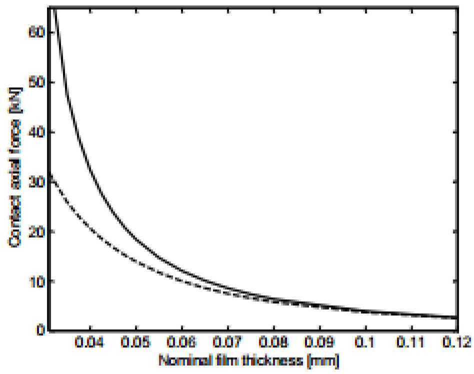

Fig. 5. Dependence of the contact axial force on the minimal thickness, $h_{min} - F$, for the parameters from Fig. 3 and 4. Solid line: $\alpha = 0.0215 \text{xMPa}^{-1}$; dashed line: $\alpha = 0$

Acknowledgements

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