A NEW LOOK AT BANKRUPTCY MODELS

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Abstract: *New models for bankruptcy prediction are constantly being formulated and tested against the current ones and current ones are tested to assess their current accuracy and to allow users to determine the reliability of the results when using the model. These models use accounting information as input data. Accounting systems, for example, US GAAP, or IFRS, contain rules that may be applied differently from one company to another without being breached. This leads to input data uncertainty. Likewise, uncertainties may arise due to errors in recording and transcribing input data or in translating the values of assets, equity or liabilities in foreign currencies. This research was focused on the effect of entry data uncertainty on models' ability to accurately predict bankruptcy. The initial assumption was that raising the number of input values would increase the error rate probability in entry data, thus also heightening the uncertainty of the results in the given bankruptcy prediction model. The data set of tested companies contained 1,220 non-bankrupt and 285 bankrupt Czech companies. The tested models – Z' score, Model 1, and – were applied to this sample, and in all cases, the resulting accuracy was lower than the accuracy declared by their authors. A procedure was created for the inclusion of entry data uncertainty in the practical application of a model. This procedure consists of changing the limit value of the model that separates bankrupt and non-bankrupt companies to an interval that "absorbs" such uncertainties. The model cannot classify the companies in this interval. The research shows that the inclusion of uncertainties in entry data further reduces their accuracy. However, the reduction in accuracy between the individual models varies significantly from 2.2% to 39.4% for bankrupt companies, and from 3.5% to 91.8% for non-bankrupt companies, respectively. The analysis of the entry data uncertainty effect shows the need to create models with high precision and minimum of input values because the model error rate grows the higher their number. The findings of this research can be applied in the creation of new models for predicting bankruptcy not only in the Central Europe but globally.*

Keywords: *Accuracy, prediction, bankruptcy, bankruptcy model, data uncertainty, grey zone.*

JEL Classification: *M21, G32, C52.*

Introduction

Managers need to know the situation of companies they manage and what their prospects are in the market. That is why the financial analysis has become a necessary part of the managerial decision-making of any company that intends to succeed in today's competitive environment. It represents an assessment of the past, the present and the future of the company's financial health. One of the tools of financial analysis is the bankruptcy prediction model.

The great advantage of such models is that their primary source of input data is based on internal information from the company, internal accounting statements, included in the final accounts, that is, their balance, profit and loss statement, cash flow. Accounting units are obliged to prepare their final accounts according to legal requirements. For example, in the Czech Republic, under Act No. 563/1991 Coll., on Accounting, there is an exemption for micro and small accounting units that do not need to prepare cash flow statement if their turnover is up to CZK 200 mill. and their assets are not greater than CZK 100 mill. Few bankruptcy models also draw information from outside of company accounting. They rarely utilize external information regarding economic trends in the country within which the company operates, or more specifically, they might include sector development. In general, it can be said that the data necessary for a specific bankruptcy model are easily available, which affects the degree to which such models are used in practice.

We classify bankruptcy models as tools of higher financial analysis. In professional literature, they are also designated as models of early warning, prediction models and summary financial stability indicators, and they are still widely used in practice (Stříteská & Jelínková, 2015). The principles of these models are based on purposefully chosen indicators, and their goal is to assess the financial situation (financial health) of the company, that is, to predict any crisis development or company financial distress. There is also a group of quality models that assess company financial health globally, but more from an investor's perspective. These are mostly focused on and emphasize the company's profitability. According to Kuběnka and Slavíček (2016), the structure of both model types is similar. They are usually based on selected ratio indicators of financial analysis, to which various importance weights are assigned. The models result in a unique complex value, which is compared to the evaluation scale.

The resulting complex value, the so-called evaluation coefficient, classifies the company among bankrupting or non-bankrupting ones. Models often include a grey zone ('zone of ignorance' or 'grey area', as termed by the pioneer of bankruptcy models creation I. Altman in the USA in 1968 – Altman, 1968).

In the case of the grey zone, it is not possible to unambiguously determine whether the company is in good or bad financial health. The analysis is based on a presumption that some anomalies appear in the company several years before it actually goes bankrupt. These contain symptoms of future problems that characterize endangered companies. These limit values, which define the grey zone or directly separate the bankruptcy zone from the non-bankruptcy zone, are sensitive to data uncertainty entered into the evaluation. Bankruptcy model authors are dissatisfied with the available models, and to avoid such uncertainties, they try to achieve better outcomes of company analysis using revised versions. The authors of new models and new versions do not consider the complexity of their models in relation to sensitivity to data uncertainty, which is causing uncertainties in estimates on whether a company is approaching or becoming bankrupt.

The relationship between the sensitivity of models to the uncertainty in input data has so far been dealt with only marginally by researchers, and therefore only a few findings on related topics can be reported. For example, De Bock et al. (2020) focused on the role of cost uncertainty in cost-sensitive business failure prediction. Yuan et al. (2018) found that the level of uncertainties associated with the default risk predictions is correlated with the level of default risks. Zhou and Lai (2017) found alternative for bankruptcy prediction with missing data. Huang et al. (2017) found that the prediction accuracy increases after they discretized the continuous variables of financial ratios.

No further research on this issue has been found in the world's leading citation databases. It follows that so far no one has focused on the influence of input data uncertainties on the accuracy of the model. Likewise, no one directly focused on the sensitivity of the bankruptcy model to the number of variables. The article fills a gap in the analysis of the sensitivity of bankruptcy models to the uncertainty of input data. The aim is to confirm or refute the existence of differences in the sensitivity of models to uncertainties in the input data, and in the opinion of the authors, the chosen region where the models will be tested does not play a role.

The Czech context was chosen because of the agreement between the economy where the models originated and the economy

where the analyzed companies operate. This is the case of the subsequently used model and Model 1. Model Z' score was selected as a complementary, to analyze the effect of uncertainty in degrees data model with a grey zone.

The article aims to find out how much the sensitivity of bankruptcy models to the uncertainty of input data differs. If this is different for the three selected analyzed models, then it can be strongly assumed that it is also different for other models, which also have a construction in the form of a linear function, regardless of the place of origin and place of application of the model.

1. Theoretical Background

Since as early as the first half of the twentieth century, efforts were underway to find the means to predict company bankruptcy based on financial data from accounting books. Then in 1968 Altman added the first multivariate analysis (MDA) to his bankruptcy model, Z score (Altman, 1968), which works with five financial ratios. Ohlson used the logit linear probability to create his bankruptcy model for the first time in 1980 (Ohlson, 1980). In 1985 factor analysis was used to obtain independent variables for the logit model (Zavgren, 1985). Attempts to create more precise models with other methods followed. For example, the hazard model (Shumway, 2001), Ahn and Kim's (2009) hybrid case-based reasoning and genetic algorithm or a combination of the random subspace approach and the binary logit model (Li et al., 2011).

Later progress was made in connection with methods of artificial intelligence, especially the use of neural networks (NN) for developing predictive models starting in the 1990s. Tam and Kiang (1991, 1992) are among the pioneers of NN usage. Particular methods (MDA vs. logit vs. NN) of model creation have been compared many times. NN is probably the newest among the current 'publicly/generally' investigated methods. Some experts consider neural networks to be the most appropriate procedure for model creation (Liang, 2005; Rafiei et al., 2011). Other experts (Chih-Fondg & Chihli, 2014; Kim & Park, 2012) claim that although NN are more accurate than previous methods, the difference is only by a few per cent. According to Kuběnka and Honková (2019), the major drawback of this method is that it cannot be published or shared freely for use and analysis due to how the NN method works as a so-called black box. This perspective points to how the inner computer algorithms of the NN method cannot be analysed by the usual methods.

After the Czech Republic and the Slovak Republic transformed into market economies in the 1990s, bankruptcy models also began to appear in these countries in order to predict potential risks of bankruptcy. Such models are supposed to factor in the market specificity of the given countries. The first Czech model (index) to be formulated, IN95 (Neumaierová & Neumaier, 2002), was designed as a creditor's model, as it is mostly used for subjects in the role of creditors (banks and business partners). In 1999 the same authors introduced a so-called ownership model, named IN99. It functions as a prosperity prediction based on positive economic value added (EVA). In 2001 the duo created IN01, a model that combines the properties of both its predecessors, that is, it predicts bankruptcy as well as prosperity. An updated version called IN05 was released in 2005 (Neumaierová, 2005).

After the economic crisis in 2008, more bankruptcy models appeared within the territory of the Czech Republic and the Slovak Republic. Namely, these were models based on MDA and logit regression.
1. Models based on multidimensional discrimination analysis:
   a) CZ model and FLKp model (Kalouda & Vaníček, 2013);
   b) index of Karas and Reznakova (2014);
   c) prediction models of financial health for construction companies (Slavíček, 2015);
   d) V4 model, Model CZ (Klieštik et al., 2018).
2. Models based on logit regression:
   a) JI Index (Jakubík & Teplý, 2011);
   b) microeconomic scoring model of Czech companies' bankruptcy (Valecký & Slivková, 2012);
   c) bankruptcy model by Slavíček and Kuběnka (2016);
   d) model to Predict Survival of Transportation and Shipping Companies (Vochozka et al., 2015)

Čámská (2013) emphasizes that the application of these types of models is user friendly as they do not require any specific mathematical or statistical knowledge of the user.

Some models are specialized in companies from a specific industry, of a certain size or a specific business activity. For example, the models focused on transport companies Vochozka et al. (2015), plastic producers and metal manufacturing companies (Homolka et al., 2014), construction companies (Slavíček, 2015), the Czech model BAMF of Kraftová and Kašparová (2017) for assessing the financial health of regional emergency medical services, the hospitality industry (hotels/accommodation) (Youn & Gu, 2010; Kim, 2011), internet companies (Chandra et al., 2009), spa enterprises (Čabinová & Onuferová, 2019), etc (for more examples, see Tab. 1). Financial diagnostic models can be divided into the categories of bankruptcy models and prosperity models. The accuracy of bankruptcy models is known from their creation thanks to the use of statistical methods and test samples of companies.

On the contrary, the prosperity models were created on the basis of logical assumptions without empirical research, and these models do not have determined accuracy. Examples of the latter sort include Grünwald's Bonita Index from 1995 (Grünwald & Holečková, 2007), Doucha's Balance Analysis I, II, III from 1996 (Doucha, 1996), Tamari's risk index (Tamari, 1966) and Index of Creditworthiness (for more, see Zalai, 2010). The Czech index IN99 (Neumaierová, 2002), which posits that financially healthy companies are those that have positive economic value added, is an exception. Its accuracy was at 85% at the moment of its creation. The adapted Index of Creditworthiness (IC) with an added evaluation scale is the second exception. According to Kuběnka (2015), if focused only on the IC prediction ability of EVA, the accuracy of the model is 76.39% pursuant to the chosen methodology of research.

For the meaningful use of a bankruptcy model, it is important to know its predicative capability. This means to know its accuracy of bankruptcy prediction. The model accuracy is given by its authors at the moment of creation and in some cases by other scientists who apply the model and verify the current accuracy in a specific economic environment or find and evaluate factors that affect company bankruptcies.

Company performance, bankruptcy development and company risks (not only financial performance) are discussed by Gorzeń-Mitka (2016), Fuka et al. (2014), Korableva and Kalimullina (2016), Gorzeń-Mitka (2019), Honková (2019).

The effect of various methods on bankruptcy prediction accuracy using an identical test sample has been studied by many researchers (Klepac & Hampel, 2018; Tseng & Hu, 2010; Muller et al., 2009). For example, Kuběnka and Myšková (2019) deal with the effect of the structure of the test sample on the resulting accuracy of the model. Diversities of fiscal systems (US GAAP vs. IFRS vs. CAS) that are data sources for bankruptcy models were discussed by Honková and Výbora (2015), among others. Many authors conduct comparisons of original and current accuracy, whereas they primarily focus on how much the accuracy has decreased due to the model's obsolescence or its implementation in a different region than the site of its origin (USA vs. Europe, the United Kingdom vs. the Czech Republic etc). This topic continues to be of major importance as model accuracy unambiguously determines its current usability and credibility for its user. For example, Gissel et al. (2007) present a summary of the accuracy of bankruptcy models as stated by their authors.

Bankruptcy models were derived from the contemporary data of companies that went bankrupt in the past or, on the contrary, have prospered. They can help company management to correctly interpret the indicators of possible future problems and to identify them in a timely manner and adapt before serious problems occur or bankruptcy takes place.

Thus bankruptcy models represent systems of early warning as based on the behaviour of chosen indicators, which show possible dangers to the company's financial health.

Nevertheless, the forecasts of company financial health are not 100% reliable, and thus new bankruptcy models continue to be created or revised, and many authors intend to design their own models or to improve on established models.

Corporate bankruptcy prediction represents one of the problems referred to as strongly affected by uncertainty (Andres et al., 2011; Chen et al., 2011; Čapek & Kraftová, 2003). However, no attention has been paid to the uncertainty of the data in connection with Altman's model and others similar to it. To fill this gap uncertainty was taken into account in this study.

Tab. 1: **Models for assessing bankruptcy**

Author of the study	Sample characteristics	Method/ Components	Model accuracy
Altman, 1968	Manufacturing firms	MDA/5	Model accuracy for hold-out sample – 79%
Altman, 1973	Railroads	LDA	Model accuracy for hold-out sample – 83%
Wilcox, 1973	General	Binominal/2	Year before failure 1 2 3 4 5 Model accuracy 94% 90% 88% 90% 76%
Taffler, 1974	UK manufacturing	MDA/5	Model accuracy (failed firms) – 60%
Beerman, 1976	German firms	LDA/10	Year before failure 1 2 3 4 Model accuracy 90.5% 81.0% 71.4% 61.9%
Tisshaw, 1976	Privately owned UK manufact. firms	MDA/5	Model accuracy: Failing firms – 97%, Non-failing firms – 97%
Altman et al., 1977	General	LDA/7	Year before failure 1 2 3 4 5 Bankrupt firms 92.5% 84.9% 76% 61.7% 62.8% Non-bankrupt 91.4% 91.4% 91.4% 93.0% 84.0%
Taffler, 1980	UK distribution firms	MDA/4	Failed firms – 96%, Non-failed firms – 100%
Taffler, 1982	UK private Mig & Construction firms	MDA/4	Failed firms – 95%, Non-failed firms – 96%
Takahashi et al., 1984	Japanese firms	MDA/8	Bankrupt firms – 100%, Non-bankrupt firms – 53% to 75%
Zmijewski, 1984	General	Probit/6	Bankrupt firms – 20.0% Non-bankrupt firms – 99.5%
Kiviluoto, 1998	Small & medium-size Fljwish firms	LVQ/4 NN/4 LDA/4 QDA/4	Bankrupt firms – 34.8%, Non-bankrupt firms – 97.3% Bankrupt firms – 24.8%, Non-bankrupt firms – 98.5% Bankrupt firms – 52.9%. Non-bankrupt firms – 93.4% Bankrupt firms – 44.1%, Non-bankrupt firms – 93.5%
Zordan, 1998	Retail/Wholesale & manuf. firms	MDA/30	Failed firms 78.7% to 85.2% Non-failed firms – 66.7% to 68.5%
Patterson, 2001	Casinos	MDA/12	Failed firms – 100%, Non-failed firms – 89%
Gaereminck & Willekens, 2003	Belgian private	Logit/8	Model accuracy – 72.4%
Grover, 2003	Manufacturing firm	MDA/6	Model accuracy for hold out sample – 78.17%
Anandarajan et al., 2004	General	NN Genetic algorithm/5 Backpropagation NN MDA	Model accuracy for hold-out sample: Bankrupt firms – 95.5%, Non-bankrupt firms – 93.8% Bankrupt firms – 93.8%, Non-bankrupt firms – 70.0% Bankrupt firms – 82.8%, Non-bankrupt firms – 21.7%
Wang, 2004	Internet firms	Logit/8	Bankrupt firms – 26.7%, Non-bankrupt firms – 90.8%

Source: Bellovary et al., 2007

Note: MDA – Multivariate Discriminant Analysis; LDA – Linear Discriminant Analysis; QDA – Quadratic Discriminant Analysis; NN – Neural Network.

The authors of such models draw on the presumption that the data used for their models are entirely accurate, free of errors brought into the data by the complexities of real life. It is our understanding that failing to consider these errors which leads into data uncertainties can result in a lower reliability of the forecasts made from the given model. Besides the methods stated in Tab. 1 for model creation, the following tools were or are used as well: UDA – unidirectional discrimination analysis, LSR – method of least-squares

regression, ID3 – induction dichotomiser 3, SOFM, etc.

Following the Introduction and Section 1, which provides the theoretical background, Section 2 describes the data set and the method applied to discover data uncertainty, which is the backbone of this paper. Methodology is introduced for the modification of boundaries for Model 1 and the model and the boundaries of the grey zone for Altman's Z' score. Section 3 presents and discusses the results. Finally, the paper's findings are summarized in the conclusion.

2. Data and Research Methodology
2.1 Data Structure

The examined data set contained accounting statements of Czech companies in time t (t is the year 2012) from groups of companies that were a) financially healthy (non-failed) and b) bankrupted (failed).
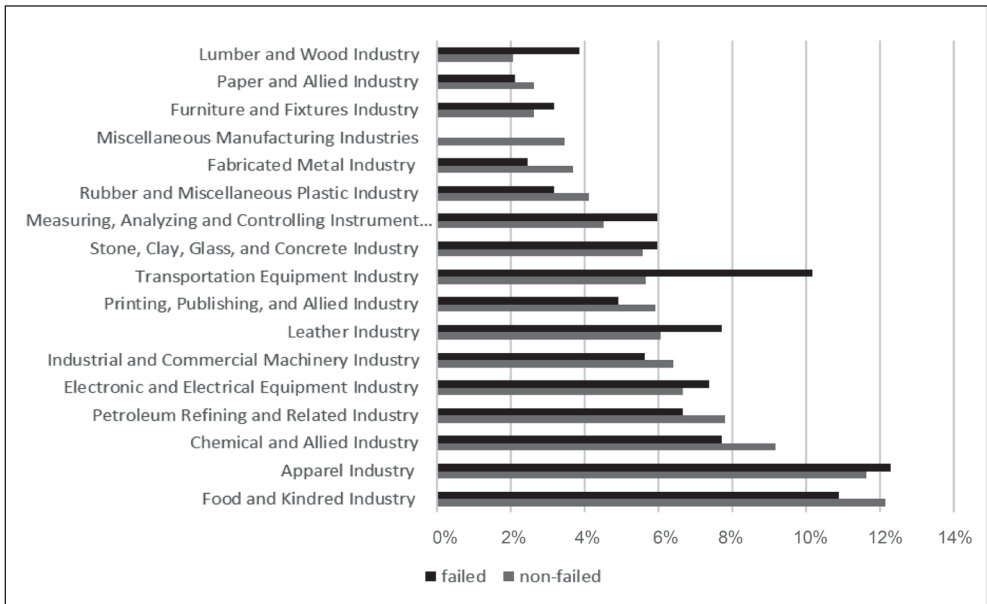
Description of the groups:
a) financial 'non-failed' statements are statements according to IFRS accounting standards from the year t, whereas it was confirmed that in time $t+1$, the company showed no financial issues. This non-bankrupt group showed no negative signs (insolvency, failure, extinction, execution, negative shareholders' capital) in the year 2012;

b) financial 'failed' statements are statements according to IFRS accounting standards from the year t, whereas it was confirmed that in time $t+1$, the company went bankrupt (according to Act No. 182/2006 Coll., on Insolvency, as amended in Czech law).

The sample contained 1,220 companies of type a) and 285 companies of type b). Data were gathered from the Bisnode's MagnusWeb database (www.magnusweb.cz). All the companies operated in the manufacturing industry sector. The manufacturing industry has 24 categories. The data sample covers most of them. Specifically, the following sectors were included, see Fig. 1. In general, the manufacturing industry is based on the processing or fabrication of products from raw materials or commodities.

Fig. 1: **Branch structure of the data set**

Source: own

2.2 Tested Bankruptcy Models

The Z' score model is a modification of the original Z score model from 1968, which was specifically created for listed firms that operated in the USA. In this newer model (1) from 1983, the indicator X_4 is changed, where the market value of equity is replaced by the book value of equity. The model was created based on a study of 53 companies under bankruptcy and 58 non-bankrupt companies. It used a multiple discriminant analysis in its creation. The author of the model declared a bankruptcy prediction success level of 90% with one year in advance. The limit value to classify a company as prospering is 2.90, companies in risk of bankruptcy display a Z' score under 1.23. There is a grey zone between these values in which a clear conclusion cannot be determined. The Z' score takes the following form (Altman & Hotchkiss, 2005):

$$Z' = 0.717X_1 + 0.847X_2 + 3.107X_3 + \\ + 0.420X_4 + 0.998X_5 \quad (1)$$

where X_1 = working capital/total assets; X_2 = retained earnings/total assets; X_3 = profit before interest and tax/total assets; X_4 = book value of equity/book value of liabilities; X_5 = sales/total assets.

Model 1 was created in 2016 by Slavíček and Kuběnka (2016). The data set for its model creation comprised 22 non-bankrupt companies and 11 companies under bankruptcy operating in the Czech Republic. Logistic regression was used for its creation. The model works with four variables. The authors declared a bankruptcy prediction success level of 91% and a non-bankruptcy prediction success level of 95% for financially healthy companies. The limit value is 0.5 (bankruptcy above 0.5, non-bankruptcy under 0.5). The model is somewhat unusual in that the probability of bankruptcy increases the higher the resulting value is. Model 1 is defined thus (Slavíček & Kuběnka, 2016):

$$\text{Model } 1 = 0.0173V_1 - 4.7107V_2 + \\ + 0.0412V_3 + 0.0918V_4 - 7.5378 \quad (2)$$

where V_1 = inventories/(sales/360); V_2 = financial property/current liabilities; V_3 = operating profit/total assets × 100; V_4 = (liabilities/total assets) × 100.

The youngest tested bankruptcy model Y_{CZ} (the creators named it the CZ model) also works with Czech companies and thus it will be tested on the sample of Czech companies as well. Klieštik et al. analysed 50,058 non-bankrupt companies and 12,736 bankrupt companies. They created the model using multiple discriminant analysis. The model works with 10 ratio indicators and one constant. The overall model accuracy is 84.8% according to the authors. The separating limit between bankrupt and non-bankrupt companies is at the 0 value (bankruptcy-endangered companies take a value <0, non-bankrupt ones are above 0). The model is formulated thus (Klieštik et al., 2018):

$$Y_{cz} = -1.016 + 0.007\,Y_2 - 0.884\,Y_4 + \\ + 2.168\,Y_7 - 0.343\,Y_8 + 2.526\,Y_{10} + \\ + 0.416Y_{12} - 0.592Y_{21} - 2.561Y_{27} + \\ + 0.352Y_{28} - 1.075\,Y_{35} \quad (3)$$

where Y_2 = current assets/current liabilities; Y_4 = net income/equity; Y_7 = earnings after taxes/total assets; Y_8 = working capital/total assets; Y_{10} = liabilities/total assets; Y_{12} = cash & cash equivalents/total assets; Y_{21} = non-current liabilities/total assets; Y_{27} = ROA (return on assets); Y_{28} = ROE (return on equity); Y_{35} = profit margin (ROS, return on sales).

2.3 Methodology of the Analysis of the Entry Data Uncertainties Effect

The essence of financial analysis tools are multidimensional models, which are to be determined by the target evaluation of the company and what it shows in the model, and the weights assigned to them. As a rule, there is a multiple discriminant analysis for the selection of indicators and the determination of their applicability. The general form of these indicators can be written as follows (4):

$$I = \sum_{i=1}^{n} w_i X_i \quad (4)$$

where $w_i = i^{th}$ weight; $X_i = i^{th}$ indicator.

The authors creating the multidimensional models based on synthetic indicators according to relation (4). They assume in their reasoning that the individual variables enter the relation without uncertainty of the data accuracy. The authors of such models draw on the presumption that the data used for their models are entirely accurate, free of errors brought into the data by the complexities of real life.

We are convinced that disregarding these errors can result in lower reliability of predictions made by the model. However, due to the nature of the individual variables, such an idea is out of place. Many authors make their model more complex by increasing the number of indicators X_i in their effort to consider more factors and thus refine the resulting index. The following steps show, how to incorporate the uncertainty of the data entering the models.

First step:
The synthetic indicator (4) represents an unknown quantity I, which is determine by indirect measurement (i.e., one cannot determine it directly, because no company reports it directly and must be calculated from auxiliary quantities). In fact, the synthetic indicator (index) I is a function of the variables $x_1, x_2, ..., x_n$, that is, it can be expressed as the relation (5):

$$I = f(x_1, x_2, ..., x_n) \tag{5}$$

Let it be supposed that each variable can be determined in the relation (5) with a total uncertainty of $\pm \Delta x$. The total uncertainty covers systematic error as well as random error. The relation (5) then changes to the form:

$$I + \Delta I = f(x_1 + \Delta x_1, x_2 \pm \Delta x_2, ... \\ ... x_n \pm \Delta x_n) \tag{6}$$

Developing (6) in its Taylor series gives:

$$I \pm \Delta I = f(x_1, x_2, ..., x_n) + \frac{\partial f}{\partial x_1} \Delta x_1 + \\ + \frac{\partial f}{\partial x_2} \Delta x_2 + ... + \frac{\partial f}{\partial x_n} \Delta x_n + \\ + \frac{1}{2} \left[\frac{\partial^2 f}{\partial x_1^2} (\Delta x_1)^2 + ... \right] \tag{7}$$

Second step:
If higher derivations are omitted, a comparison of the relations (4) and (6) results in the following relation for the absolute total uncertainty of the indicator/denoted as ΔI.

$$\Delta I = \left| \frac{\partial f}{\partial x_1} \Delta x_1 \right| + \left| \frac{\partial f}{\partial x_2} \Delta x_2 \right| + ... \\ ... + \left| \frac{\partial f}{\partial x_n} \Delta x_n \right| \tag{8}$$

The issue will now be illustrated on Altman's Z' score, which will now be applied on the relation (4):

$$Z' = 0.717X_1 + 0.847X_2 + 3.107X_3 + \\ + 0.420X_4 + 0.998X_5 = 0.717 \, WC/TA + \\ + 0.847 \, RE/TA + 3.107 \, EBIT/TA + \\ + 0.420 \, BVE/TL + 0.998 \, S/TA$$

where X_1 = working capital (WC)/total assets (TA); X_2 = retained earnings (RE)/total assets (TA); X_3 = earnings before interest and tax (EBIT)/total assets (TA); X_4 = book value of equity (BVE)/book value of liabilities (TL); X_5 = sales (S)/total assets (TA).

$$Z = 0.717X_1 + 0.847X_2 + 3.107X_3 + \\ + 0.420X_4 + 0.998X_5 \tag{9}$$

Final step:
For further considerations, it is preferable to use the relation (9) in the form of relative total uncertainty.

$$\delta = \frac{\Delta I}{I} 100 \, [\%] \tag{10}$$

As the variables X_i are expressed with a proportion, the partial relative uncertainties of individual variables entering the resulting relation are determined first.

$$\Delta X_1 = dX_1 = \left| \frac{\partial X_1}{\partial WC} dWC \right| + \left| \frac{\partial X_1}{\partial TA} dTA \right| = \left| \frac{\partial X_1}{\partial WC} \Delta WC \right| + \left| \frac{\partial X_1}{\partial TA} \Delta TA \right| = \left| \frac{1}{TA} \Delta WC \right| + \left| \frac{-WC}{TA^2} \Delta TA \right|$$

$$\delta_{X_1} = 100 \frac{\Delta X_1}{X_1} = 100 \left[\frac{\left| \frac{1}{TA} \Delta WC \right| + \left| \frac{-WC}{TA^2} \Delta TA \right|}{\frac{WC}{TA}} \right] = 100 \left[\frac{\Delta WC}{WC} + \frac{\Delta TA}{TA} \right] = \delta_{WC} + \delta_{TA}$$

Similarly, the partial relative uncertainties are expressed:

$$\delta_{X_2} = \delta_R + \delta_{TA}; \delta_{X_3} = \delta_{EBIT} + \delta_{TA}; \delta_{X_c} = \delta_{MVE} + \delta_{TL};$$
$$\delta_{X_5} = \delta_S + \delta_{TA}$$

$$\delta Z = 100Z/Z = \delta WC + \delta TA + \delta RE + \\ + \delta TA + \delta EBIT + \delta TA + \delta BVE + \delta TL + \\ + \delta S + \delta TA = \delta WCA + \delta RE + \delta EBIT + \\ + \delta BVE + \delta TL + \delta S + 4\delta TA \quad (11)$$

It can be seen from the relation (11) that a total of 10 quantities participate in Altman's Z' score accuracy, whereas the total assets, which are enteredinto the calculation 4 times in this specific case, greatly affect the resulting *I* indicator accuracy. The uncertainty of data accuracy will be taken into account in further considerations.

3. Research Results and Discussion

For the purpose of comparison, we chose Altman's model, Model 1 and the Y_{cz} model. The Altman model was chosen as it is worldwide recognized model of which the methodology and form are a pattern for many other model authors. Model 1 and Y_{cz} were chosen as they are regionally focused, specifically targeting the market environment of the Czech Republic, while also being of relatively recent origin. Thus their accuracy should not differ from the original accuracy declared by their authors.

3.1 Altman's Model

There is no literature that would provide the individual values of absolute or relative uncertainties entering into the calculation of any version of the Z' score. If it is assumed that

individual quantities enter the calculation with the same relative uncertainty, which can be supposed for the purpose of simplification to be at 1%, then the specific indicator evinces the total relative uncertainty of 10% because 10 quantities of relation (11) participate in the calculation of the Z' score value. Note that a relative uncertainty of 1% is hardly achievable in practice.

Altman sorts resulting values into three groups. For Z > 2.90, the prosperity zone, for Z < 1.23, the bankruptcy zone and for Z within the interval of <1.23; 2.9>, the so-called grey zone.

Result	Assessment
Z' ≥ 2.9	non-bankruptcy company
1.23 ≤ Z' < 2.9	grey zone
Z' < 1.23	bankruptcy company

Tab. 2 focuses solely on the limit points of the 'grey zone'. Let it be supposed that the indicator determination uncertainty increases or decreases by 1 to 5%, always in one partial variable, whereas the total number of variables shall be considered. The results are shown in Tab. 2.

Descriptive statistics of predictors are given in Tab. A2 in Appendix. It follows from Tab. 2 and Fig. 2 that the inclusion of uncertainties in the source data causes a considerable enlargement of the grey zone. In the case of 5% uncertainty, the grey zone grows to the interval of <0.62; 4.35>, which consequently means that a higher number of companies in the sample cannot be classified by the model as either bankrupt or non-bankrupt ones.
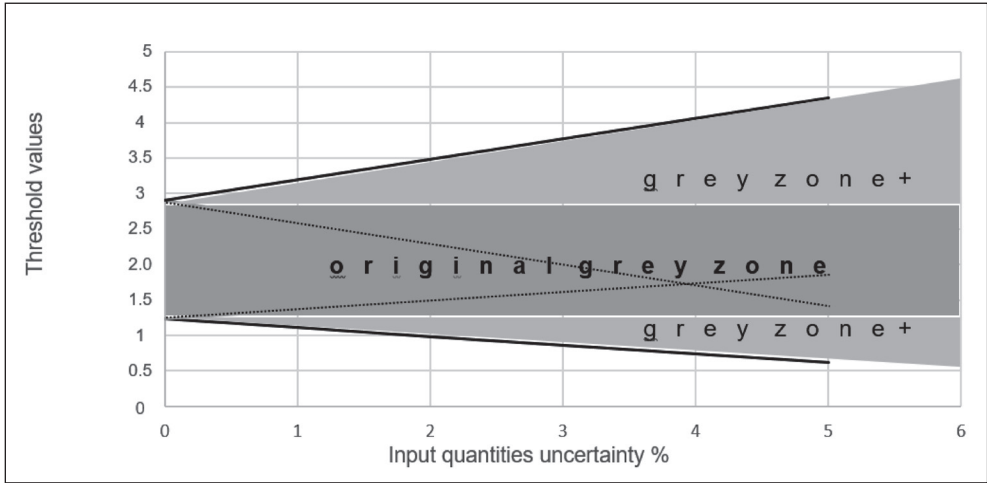
Tab. 2: Values of limit points of the Z' score grey zone, uncertainties included

δ_i	Limit points of Z' score			
0	1.23		2.9	
1	1.11	1.35	2.61	3.19
2	0.98	1.48	2.32	3.48
3	0.86	1.60	2.03	3.77
4	0.74	1.72	1.74	4.06
5	0.62	1.85	1.45	4.35

Source: own

Fig. 2: Development of limit point values of the Z' score grey zone with regard to uncertainties

Source: own

The largest move in the grey zone limit occurred with the addition of 5% uncertainty in 10 entry quantities, where the upper limit increased by 1.45 and the lower limit decreased by 0.61.

3.2 Model 1

Model 1 is represented by the relation (12). We define the relative uncertainty of entry quantities of Model 1 similarly as in case of Altman's model.

$$Model\ 1 = 0.0173\ V_1 - 4.7107\ V_2 + \\ + 0.0412\ V_3 + 0.0918\ V_4 - 7.5378 \quad (12)$$

where V_1 = inventories/(sales/360); V_2 = financial property/current liabilities; V_3 = (operating profit/total assets) × 100; V_4 = (liabilities/total assets) × 100.

The relative uncertainty for Model 1 is derived as follows:

$$\delta_{V_1} = \delta_Z + \delta_R;\ \delta_{V_2} = \delta_{FM} + \delta_{FV}; \\ \delta_{V_3} = \delta_{HV} + \delta_{TA};\ \delta_{V_4} = \delta_{FS} + \delta_{TA}$$

$$\delta Mod1 = \frac{100M1}{M1} = \delta V1 + \delta V2 + \delta V3 + \\ + \delta V4 = \delta Z + \delta R + \delta FM + \delta FV + \quad (13) \\ + \delta HV + \delta FS + 2\delta TA$$

It can be seen from the relation (13) that a total of 8 quantities participate in the accuracy of Model 1, whereas the total assets, which are entered into the calculation 2 times in this specific case, greatly affect the resulting Model 1 indicator accuracy. Limit values are shown in Tab. 3.

Descriptive statistics of predictors are given in Tab. A3 in Appendix. Possible percentage errors in the entry data create a grey zone, even though this model has no grey zone of its own and classifies all companies as either bankrupt or non-bankrupt ones. The greater the error percentage, the larger the grey zone, thus reducing the effective application range of the model. In this case the grey zone forms and grows uniformly on both sides. The move of limits due to possible errors in 8 variables is the lowest of the three tested models, compared to Z' score and Y_{cz}, at a mere +/− 0.2.

3.3 Y_{cz} Model

The Y_{cz} model is represented by the relation (14); when compared to the relation (6), the entry quantities are detailed for the purpose of the analysis of possible error occurrence in entry quantities.
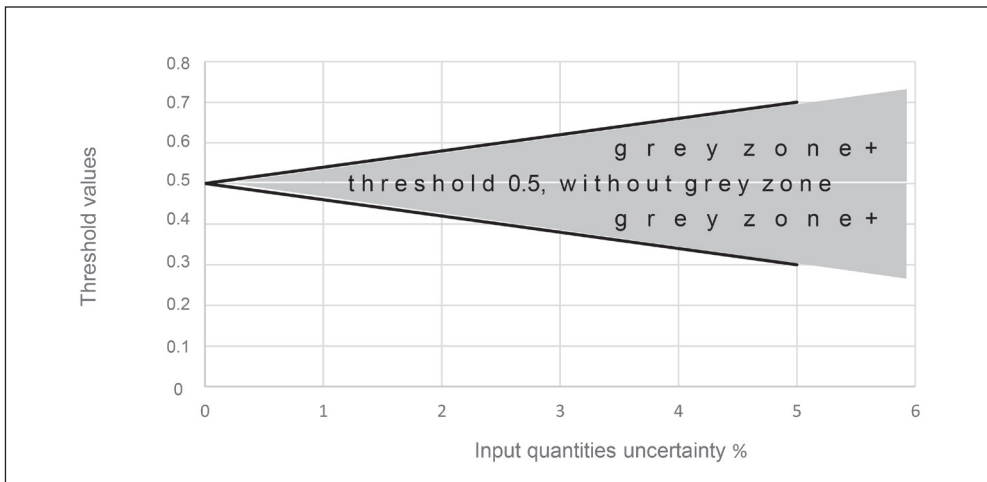
$$Y_{cz} = -1.016 + 0.007Y_2 - 0.884Y_4 + \\ + 2.168Y_7 - 0.343Y_8 + 2.526Y_{10} + \\ + 0.416Y_{12} + 0.592Y_{21} - 2.561Y_{27} + \quad (14) \\ + 0.352Y_{28} - 1.075Y_{35}$$

Tab. 3: Values of limit points of the Model 1 grey zone, uncertainties included

δ_i	Limit point of Model 1	
0	0.5	
1	0.46	0.54
2	0.42	0.58
3	0.38	0.62
4	0.34	0.66
5	0.30	0.70

Source: own

Fig. 3: Limit values of Model 1

Source: own

where Y_2 = current assets/current liabilities; Y_4 = net income/equity; Y_7 = earnings after taxes/total assets; Y_8 = working capital/total assets; Y_{10} = liabilities/total assets; Y_{12} = cash & cash equivalents/total assets; Y_{21} = non-current liabilities/total assets; Y_{27} = ROA (return on assets); Y_{28} = ROE (return on equity); Y_{35} = profit margin (ROS, return on sales).

The separating limit between bankrupt and non-bankrupt companies is at the 0 value (bankrupt companies achieve a value higher than 0, non-bankrupt ones lower than 0). Modified limit values will be derived as follows:

$$\delta_{Y_2} = \delta_{OA} + \delta_{KZ} \; ; \; \delta_{Y_4} = \delta_{EAT} + \delta_{VK};$$
$$\delta_{Y_7} = \delta_{EAT} + \delta_{TL} \; ; \; \delta_{Y_8} = \delta_{PK} + \delta_{TA} \; ;$$
$$\delta_{Y_{10}} = \delta_{DZ} + \delta_{TA} \; ; \; \delta_{Y_{27}} = \delta_{TA} + \delta_{ROE} \; ; \quad (15)$$
$$\delta_{Y_{28}} = \delta_{CZ} + \delta_{VK} \; ; \; \delta_{Y_{35}} = \delta_{CZ} + \delta_T \delta_{Y_{27}} = \delta_{TA} +$$
$$+ \delta_{ROE} ; \; \delta_{Y_{28}} = \delta_{CZ} + \delta_{VK} \; ; \; \delta_{Y_{35}} = \delta_{CZ} + \delta_T$$

$$\delta Y_{CZ} = \frac{100 Y_{cz}}{Y_{cz}} = \delta OA + \delta KZ + 2\delta EAT +$$
$$+ 2\delta VK + \delta EAT + \delta TL + \delta PK + 2\delta TA + \quad (16)$$
$$+ \delta DZ + 2\delta TA + \delta ROE + \delta ROS$$

Descriptive statistics of predictors are given in Tab. A4 in Appendix. It results from relations (15) and (16) that the relative uncertainty is

Tab. 4: Values of limit points of the Y_{cz} grey zone, uncertainties included

δ_i		Y_{cz}
0	0	0
1	−0.16256	0.16256
2	−0.32512	0.32512
3	−0.48768	0.48768
4	−0.65024	0.65024
5	−0.81280	0.81280

Source: own

Fig. 4: Values of limit points of the Y_{cz} grey zone, uncertainties included

Source: own

a compound of 16 quantities. This leads to a grey zone being formed and growing to the entry data interval of <−0.8128; 0.8128> at the 5% uncertainty level. When compared to the other two models, the limit movement is the second largest. Calculated limit values are shown in Tab. 4 and Fig. 4.

Originally the model had no grey zone, but it now has a grey zone from −0.8128 to 0.8128, giving an overall range of approximately 1.6. This can considerably affect the usability of the model. Companies that fall within the interval of the grey zone cannot be marked as either

bankrupt or non-bankrupt. The larger the grey zone, the lesser the usability of the model.

3.4 Total Model Accuracy before and after the Inclusion of Uncertainty

The chosen bankruptcy models were applied to the test sample of 1,120 non-bankrupt and 285 bankrupt companies. At the time of their creation, the respective authors declared accuracy levels that differ from the currently tested ones. This is due to gradual model obsolescence caused by changing market conditions, or by the model's application on an

entirely different market environment or due to data taken from another accounting system. This can be seen in the case of Altman's globally respected Z' score, in which its author declares the total average accuracy for non-bankruptcy and bankruptcy classification as equal to 90%. The results in Tab. 5 show that there is no sense in using this model today under the conditions of the Czech economy, as 63% of non-bankrupt and 22% of bankrupt companies are eliminated from classification by the model. Of the remaining companies, it is able to correctly determine 72% cases of non-bankruptcy and 82% of bankruptcy. The authors of Model 1 claim an accuracy of 93% at the moment of its creation and did not consider any grey zone. When applied to the companies' data set in this investigation, it showed an accuracy of 89% for non-bankrupt companies and 87% for bankrupt companies, with a total accuracy of 88% based on the arithmetic average. The authors of the Y_{cz} model declared its accuracy to be 85%, the current measured total accuracy is 80%. Specifically, the non-bankruptcy prediction accuracy for non-bankruptcy companies is about 70%, and the bankruptcy prediction accuracy for bankruptcy companies is 91%. The current accuracy measured in the tested sample was reduced for all models. See more in Tab. 5.

Tab. 6 summarizes the shifts in model accuracy with regard to possible uncertainties in entry values. The inclusion of uncertainties causes the creation of a grey zone (Model 1, Y_{cz}), or its enlargement if already existing (Altman's Z' score) – see row 4 in Tab. 6. The smallest grey zone is found in Model 1, the largest one in Z' score. This meant that Z' score was only able to classify a small part of the companies. For bankrupt companies, it was 66.6% of the tested sample, for non-bankrupt companies, it was a mere 8.2% of the sample with a total accuracy (row 5) of 29.9%. The highest accuracy with regard to uncertainties was achieved by Model 1 with an accuracy of 86.4% (row 5) with a grey zone of 2.2%, or 3.5%. Provided that the tested sample is cleaned of companies that the models are not able to classify (that is, companies in the grey zone), the base for the calculation of resulting accuracy is decreased and the model accuracy is increased. Accuracy results without the grey zone are shown in row 6 of Tab. 6. This accuracy was calculated as the arithmetic average of the models' success rate in correctly determining bankrupt companies as bankrupt ones and in correctly determining non-bankrupt companies as non-bankrupt ones. Row 6 shows that Z' score increased its total accuracy from 29.9% (row 5) to 76.3% in relation to the number of classified companies. In the same way, Model 1 increased its accuracy from 86.4% to 88.9%, thus increasing its accuracy when compared to the tested current accuracy of the model without the inclusion of uncertainties (Tab. 5), which has an empirically determined total model accuracy of 88.0%. Similarly, the Y_{cz} model evinces the lowest level of accuracy when taking into account uncertainty – 21.6% (row 5 in Tab. 6). If we compare the successfully classified companies with only the number of companies in the data set, ignoring the number of grey zone companies, then the accuracy increases again. Row 6 shows a total accuracy of 86.9%. Such a major change in accuracy is due to the fact that the Y_{cz} model included 18.6% of bankrupt companies and 66.0% of non-bankrupt companies in its grey zone. The

Tab. 5:	Current accuracy of chosen models determined on test sample

MODEL	Non-bankruptcy accuracy	Bankruptcy accuracy	Classified correctly (total)
Z' score contains 10 quantities	72.22% (*26.64%)	81.90% (*63.51%)	77.06% (*45.06%)
Model 1 (2016) contains 8 quantities	89.45%	86.62%	88.03%
Y_{cz} model (2018) contains 16 quantities	68.93%	91.08%	80.01%

Source: own

Note: *Tested accuracy when including companies classified in the grey zone (the Z' score did not classify 770 non-bankrupt companies, that is, 63.11% of the sample, and 64 bankrupt companies, that is, 22.46% of the sample).

Tab. 6: Accuracy of chosen models with included uncertainty

1.		Z' score			Model 1			Y_{cz} model		
	Real data set structure		Predicted		Real data set structure		Predicted		Real data set structure		Predicted
			Group 1	Group 2			Group 1	Group 2			Group 1	Group 2
2.	Group 1	285	155 (54.4%)	24 (8.4%)	Group 1	285	243 (85.1%)	36 (12.6%)	Group 1	285	216 (75.8%)	16 (5.6%)
3.	Group 2	1,220	35 (2.8%)	66 (5.4%)	Group 2	1,220	108 (8.9%)	1,070 (87.7%)	Group 2	1,220	81 (6.6%)	333 (27.3%)
4.	Grey zone:		39.4% group 1 91.8% group 2				2.2% in group 1 3.5% in group 2				18.6% in group 1 66.0% in group 2	
5.	Classified correctly: (basement 1,220)		(54.4 + 5.4)/2 = **29.9%**				(85.1 + 87.7)/2 = **86.40%**				(75.8 + 27.3)/2 = **51.55%**	
6.	Classified correctly without GZ: (basement 1,220 – GZ)		(86.6 + 66.0)/2 = **76.30%**				(87.1 + 90.8)/2 = **88.94%**				(93.2 + 80.5)/2 = **86.84%**	

Source: own

Note: Group 1 – bankrupt firms; Group 2 – non-bankrupt firms; GZ – grey zone.

inclusion of uncertainties in the entry data of Model 1 and Y_{cz} increases their accuracy while at the same time reducing their usability due to the creation of a grey zone. For Altman's Z' score, the accuracy did not increase.

Altman's model has always worked with a grey zone, without which its accuracy would be rather small in the Czech environment. Thus the inclusion of uncertainties enlarged the grey zone to quite extreme values that only few companies achieve. This explains why the model accuracy of Z' score did not increase even after the deduction of the original grey zone enlarged by entry data uncertainty.

In general, 'uncertainty' means information that is intentionally misleading or a state of limited knowledge where it is impossible or impracticable to describe exactly an existing state. In business practice, we do not know how much this uncertainty in the accounting of a particular company. But it is clear that it does exist, although there is no research in this area. The results of this survey confirmed the different sensitivity of the models to the uncertainty of the input data. The authors failed to find a study directly focused on this issue in order to compare these findings with alternative research. The issue of this survey can be considered unique.

Thematically the closest are the investigations of De Bock et al. (2020) focused on the role of cost uncertainty in cost-sensitive business failure prediction. However, this research did not address the sensitivity of the model in context to other variables. The research did not even deal with the influence of the number of variables on the sensitivity of the bankruptcy model. Yuan et al. (2018) found that the level of uncertainties associated with default risk predictions is correlated with the level of default risks. This confirms the conclusions of this article, which are the recommendation to create models with low sensitivity to data uncertainties. Huang et al. (2017) found that a better prediction performance can be achieved by including fewer, but relatively more significant variables. This finding is consistent with the findings of the investigation carried out because the number of variables has a proven impact on the emergence of uncertainty in the information. It follows from the above findings that the creation of models that will minimize the sensitivity to inaccuracies in the input data can clearly be beneficial.

Conclusions

There are many factors that can affect the accuracy of models designed for the prediction of financial distress. The quality of entry data is undoubtedly the key factor in model

creation, both for model accuracy verification and for the model's usability in practice. This investigation focused solely on the practical use of the chosen models with regard to the quality – that is, uncertainty – of entry data. This uncertainty was considered within the scope of 1–5%. Uncertainty can be caused by erroneous records or by fully legally different methodology used within the given accounting system, as for example US GAAP in the USA, IFRS in the EU or CAS in the Czech Republic. So far the authors of this research have failed to find any effective solution to this issue in the literature.

Research focused on the effect of entry data uncertainty on model accuracy. If this uncertainty is to be ignored in model application in practice, the limit model value needs to be changed to an interval that 'absorbs' such uncertainties. This interval can be termed the grey zone, similarly to Altman's Z' score. Companies that fall within the model's grey zone cannot be determined as either bankrupt or non-bankrupt ones. The model is unable to assess them. In fact, such companies are eliminated from the classification.

The data set of tested companies contained 1,220 non-bankrupt and 285 bankrupt companies. The tested models – Z' score, Model 1 and Y_{cz} – were applied to this sample. In all cases the resulting accuracy (see Tab. 5) was lower than the accuracy given by their authors at the moment of the models creation. The inclusion of uncertainties in the entry data caused a further reduction of their accuracy in relation to the size of whole tested sample, as shown in Tab. 6, row 5. If the accuracy of correctly classified bankruptcy and non-bankruptcy is expressed as a relation of correctly classified companies to the sample size without the grey zone (the number of companies in the sample minus the number of companies in the grey zone) then the model accuracy is considerably increased, see Tab. 6, row 6. This accuracy increase occurs despite the fact that a part of the companies cannot be classified by the given model. In the case of Z' score, an extreme result was achieved, and although the grey zone has been part of the model since its creation, its enlargement due to the inclusion of uncertainties within 10 input quantities meant that the model was unable to assess 39.4% of bankrupt companies and 91.8% of non-bankrupt companies. This severely limits the practical use of the model. In contrast, Model 1 proved to be up-to-date and highly accurate, using 8 input quantities. This is the lowest number of the three tested models. Model 1 showed the highest current accuracy both before the inclusion of uncertainties (Tab. 5) and after (Tab. 6), while its grey zone creation effect was the smallest. It should be emphasized that the survey results apply only to models that take the form of a linear function. The question is what impact taking into account uncertainty in other models would have. For example, the implementation of uncertainties in models created on the basis of neural networks.

The research on the effect of entry data uncertainty on model accuracy shows that it is necessary to find a model with a high basic accuracy that also uses the least amount of input quantities. Model 1 conforms to these requirements the best out of the tested models – Z' score, Model 1 and Y_{cz}. These findings clearly confirm that the number of input quantities plays a significant role in model accuracy. The more input quantities used by a model, the greater the growth of the model's error rate.

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Appendix

Tab. A2:	Descriptive statistics

Predictor		Obs.	Mean	Median	Min	Max	Q1	Q3	Std. dev.	Var. coef.
Z' score	X_1	1,505	−0.209	0.143	−236.896	0.951	−0.071	0.318	6.207	−2969.370
	X_2	1,505	0.145	0.099	−2.069	1.385	0.000	0.288	0.267	184.190
	X_3	1,505	−0.149	0.045	−159.719	1.095	−0.007	0.110	4.163	−2798.900
	X_4	1,505	0.039	0.427	−244.561	0.979	0.192	0.656	6.438	16,337.970
	X_5	1,505	1.976	1.503	−0.100	45.218	1.079	2.196	2.414	122.130

Source: own

Tab. A3:	Descriptive statistics

Predictor		Obs.	Mean	Median	Min	Max	Q1	Q3	Std. dev.	Var. coef.
Model 1	V_1	1,505	67.507	40.890	0.000	4,385.410	19.243	68.870	228.284	338.160
	V_2	1,505	0.337	0.067	−1.000	14.310	0.013	0.243	0.973	288.610
	V_3	1,505	−20.391	3.002	−16,608.800	117.160	−2.787	8.622	468.274	−2,296.480
	V_4	1,505	99.791	57.411	1.900	24,556.060	33.856	82.311	694.642	696.100

Source: own

Tab. A4:	Descriptive statistics

Predictor		Obs.	Mean	Median	Min	Max	Q1	Q3	Std. dev.	Var. coef.
Y_{cz}	Y_2	1,505	2.151	1.315	−0.785	243.264	0.864	2.166	7.175	333.580
	Y_4	1,505	0.178	0.109	−44.537	65.545	0.021	0.265	3.252	1,826.910
	Y_7	1,505	−0.205	0.029	−166.087	0.872	−0.027	0.086	4.682	−2,283.440
	Y_8	1,505	0.581	0.573	−2.658	1.000	0.432	0.737	0.243	41.750
	Y_{10}	1,505	0.963	0.557	0.010	245.561	0.321	0.792	6.941	720.980
	Y_{12}	1,505	0.062	0.030	−7.944	0.775	0.007	0.085	0.257	413.250
	Y_{21}	1,505	0.135	0.031	−0.060	10.012	0.001	0.139	0.455	337.110
	Y_{27}	1,505	0.013	0.093	−13.416	23.311	0.019	0.161	0.919	6,858.680
	Y_{28}	1,505	0.367	0.250	−43.920	101.370	0.132	0.446	3.389	924.410
	Y_{35}	1,505	2.450	0.066	−171.667	3,343.000	0.010	0.121	93.801	3,828.090

Source: own