Error analysis in finite element modelling of induction heating processes

Jesus Oswaldo Garcia
Cemef / Mines ParisTech
Sophia Antipolis, France
jesus-oswaldo.garcia-carrero @mines-paristech.fr

Jose Alves
Transvalor
Sophia Antipolis, France
jose.alves@transvalor.com

François Bay
Cemef/Mines ParisTech and Transvalor
Sophia Antipolis, France
francois.bay@mines-paristech.fr

Julien Barlier
Transvalor
Sophia Antipolis, France
julien.barlier@transvalor.com

***Abstract*—This paper focuses on error analysis for modelling induction heating processes**

Keywords— finite element modelling, error analysis, mesh refinement

I. Introduction

The Induction heating process is widely used today in manufacturing processes - for instance, to preheat the material before forming process (forging, stamping, rolling, brazing), or during heat treatment (quenching), surface treatment [1]. This process relies on the use of eddy currents generated inside a workpiece by an AC current running through a coil (Fig.1). The computational modelling tools for designing these processes require dealing with at least electromagnetic/heat transfer couplings.

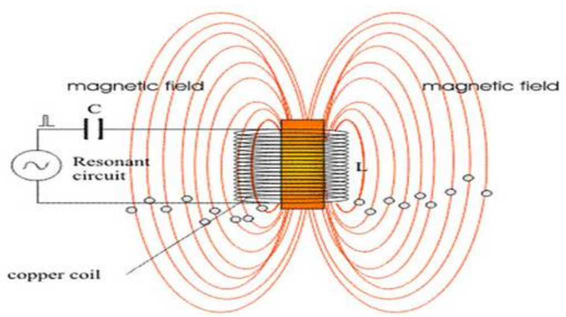

Fig. 1. Induction heating processes

However, one of the problems with the finite element tools used for modelling these processes is to make sure that these tools can be used in a reliable way.

We shall introduce here cases for which we will carry out an error analysis by studying convergence with the mesh size.

These cases will be modelled with the FORGE Induction software [2], which we will describe in the next section.

II. The Computational Tool

A. The mathematical model

The model couples the electromagnetic model with the heat transfer one. The electromagnetic model is classically based on the quasi-static Maxwell equations (1) completed by the electromagnetic constitutive laws (2).

$$\begin{aligned} \vec{\nabla} \times \vec{E} &= -\partial_t \vec{B} \\ \vec{\nabla} \times \vec{H} &= \vec{J} \\ \vec{\nabla} \cdot \vec{B} &= 0 \end{aligned} \quad (1)$$

$$\begin{aligned} \vec{J} &= \sigma \vec{E} \\ \vec{B} &= \mu \vec{H} \end{aligned} \quad (2)$$

The heat transfer model is given by (3)

$$\rho C \frac{\partial T}{\partial t} - \vec{\nabla}.(k \vec{\nabla} T) = \vec{J}.\vec{E} \quad (3)$$

B. The numerical approximation

The electromagnetic equations are then integrated in global A-V potential formulation. Finite element discretisation is then carried out using tetrahedral finite elements; edge finite elements [3] are used for determining the magnetic vector potential field ‘A’ while classical nodal elements are used for the electric potential ‘V’. In order to be more efficient in terms of parallel computations, it has been decided to use a global finite element approach to solve the problem – rather than a mixed finite element/boundary element approach. A global domain is thus defined - embedding the workpiece, the inductors, as well as an air domain wide enough in order to model accurately electromagnetic wave propagation. A weak coupling strategy is used for coupling the electromagnetic and thermal problems; EM computations are carried out only when variations of electromagnetic parameters with temperature exceed a given threshold specified by the user – typically 5%.

III. Error analysis for specific cases

A. The a posteriori error estimator

The error estimator implemented in this work is based on a smooth recovery method, which does not require solving problems on patches and involves global problems that can be solved in parallel. The methodology consists on building a smooth magnetic field [4], starting from one that is already known. In order to build the smooth field for determining the numerical solution error, we have implemented the Galerkin or residual minimization method to obtain a stable approach based on a conservation problem. Let $\vec{X}_{P_0}$ be the field obtained from the finite element analysis and $\vec{X}_{P_1}$ the recovered field. The method consists in solving the global minimization problem

$$min\|\vec{X}_{P_1} - \vec{X}_{P_0}\|^2 \quad (4)$$

Now, let $\vec{\psi}_d$ be the interpolation function on the edges. The field can then be expressed in a discrete form by

$$\vec{X}_{P_1} = \sum_j x_{P_1 j}\vec{\psi}_j \quad ; \quad \vec{X}_{P_0} = \sum_i x_{P_0 i}\vec{\psi}_i \quad (5)$$

Since $\vec{X}_{P_1}$ is the unknown, the minimisation problem is solved by projecting $\vec{X}_{P_1} - \vec{X}_{P_0}$ on the base functions of the edge mesh as follows:

$$\langle\sum_j x_{P_1 j}\vec{\psi}_j - \sum_i x_{P_0 i}\vec{\psi}_i, \vec{\boldsymbol{\psi}}_{\boldsymbol{k}}\rangle = 0 \quad (6)$$

$$\sum_j \langle\vec{\psi}_j, \vec{\boldsymbol{\psi}}_{\boldsymbol{k}}\rangle x_{P_1 j} = \sum_i \langle x_{P_0 i}\vec{\psi}_i, \vec{\boldsymbol{\psi}}_{\boldsymbol{k}}\rangle \quad (7)$$

Knowing that the term $x_{P_0 i}\vec{\psi}_i$ is provided by the finite element solution, we have:

$$\sum_j \langle\vec{\psi}_j, \vec{\boldsymbol{\psi}}_{\boldsymbol{k}}\rangle x_{P_1 j} = \sum_i \langle\vec{X}_{P_0}, \vec{\boldsymbol{\psi}}_{\boldsymbol{k}}\rangle \quad (8)$$

The field calculated by system (8) is represented on the edges elements ($\vec{X}_{P_1}^{edges}$). Thus, a transformation must be performed to compare it with the initial field. This technique allows to transform the field from the edges to the integration points ($\vec{X}_{P_1}^{int.p}$), where the initial field is also stored. The transformation is performed by the following expression:

$$\vec{X}_{P_1}^{int.p} = \sum_{i=1}^{edges} \sum_{k=1}^{int.p} \left(\vec{X}_{P_1}^{edges}\vec{\psi}_i\right)_k \quad (9)$$

The linear field specified above has a one order higher accuracy compared to the initial field. By comparing the smooth P_1 field to the P_0 solution, we can calculate the error as:

$$e_H = \|\vec{H}_{P_1} - \vec{H}_{P_0}\| \quad (10)$$

B. Case study

We present the well know benchmark problem "TEAM7" (Testing Electromagnetic Analysis Methods) of the International Compumag Society. This problem consists of a thick aluminium plate with a hole, which is placed eccentrically, is set unsymmetrically in a non-uniform magnetic field. The field is produced by the exciting current which varies sinusoidally with time.

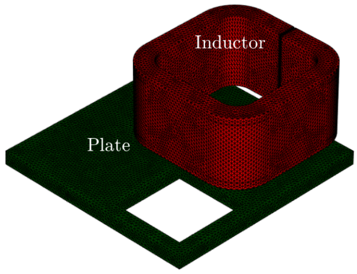

Fig 2: Benchmark problem TEAM 7

The error estimator is verified by performing a convergence analysis on decreasing uniform mesh sizes. The mesh size and the estimator results are show in the Fig 3.

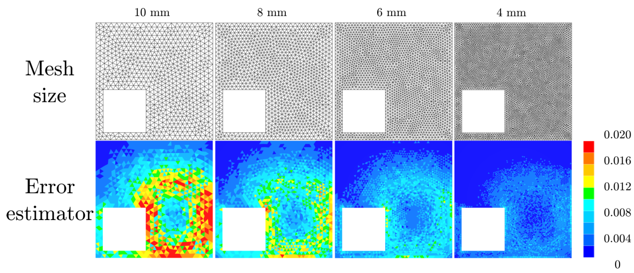

Fig. 3: Magnetic field error.

C. Conclusion

We presented recovery-based *a posteriori* error estimator for the Induction heating process modelling, applied to a fully immersed finite element method approach in conjunction with a full-time integration of the Maxwell equations. This estimator uses the Galerkin method, and has been implemented to build a magnetic field with a higher interpolation order. Finally, results on a Benchmark case test were presented, showing the convergence of the estimator.

REFERENCES

[1] "Conduction and induction heating (Livre, 1990) [WorldCat.org]." [Online]. Available: https://www.worldcat.org/title/conduction-and-induction-heating/oclc/20016205. [Accessed: 19-Oct-2020].

[2] J. R. Alves Zapata and F. Bay, "Modeling and Analysis of Electromagnetism in Magnetic Forming Processes," *IEEE Trans. Magn.*, vol. 52, no. 5, May 2016, doi: 10.1109/TMAG.2015.2501758.

[3] J. C. Nédélec, "A new family of mixed finite elements in ℝ3," *Numer. Math.*, vol. 50, no. 1, pp. 57–81, Jan. 1986, doi: 10.1007/BF01389668.

[4] M. Ainsworth and A. Craig, "A posteriori error estimators in the finite element method," *Numer. Math.*, vol. 60, no. 1, pp. 429–463, Dec. 1991, doi: 10.1007/BF01385730.