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Solving the moving mass problem on large finite element models with modal analysis – estimation of the discrete movement error

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1. Introduction

Solving transient dynamic problems on large finite element (FE) models using a direct integration requires a high sampling rate and therefore also considerable computing times because of the large system matrices. Movement of a mass on a FE model has to follow the discrete pattern of the FEs, and is therefore of a discrete character. This introduces an unknown error into the analytical results. Applying modal analysis (MA) reduces the number of equations used in the subsequent numerical integration by orders of magnitude. Resampling of the mode shapes to the required sampling rate makes it possible to solve the moving mass problem much more quickly and quasi-continuously. The performed comparative analytical study using ANSYS and MATLAB showed that using a discrete movement on the FE mesh when solving a moving mass problem can cause a considerable error.

The problem of moving vehicles on bridges is practically as old as the first railway bridges. Theoretical studies were in focus until around the late 1950s. Since that time, numerical methods have prevailed. The problem of a moving vehicle on a structure can be formulated as a coupled or uncoupled system, or by using Lagrange multipliers.

The difficulty of the problem lies in the changing inertia relations in the system during the passage of the vehicle across the bridge. This means that the natural frequencies and the mode shapes of the system in fact change during the movement of the mass. If only a force travels, a closed form solution exists [5], and it can be solved using Duhamel's Integral. A closed-form solution for a moving mass on a beam has been around since 1985 [4], using operational calculus to obtain eigenfunctions, finally leading to an integral equation.

The closed-form solution is however not applicable for more complicated practical problems, and the FE method must then be applied. Discussions about discretisation problems with moving loads started in the 90-ties [3]. An efficient way of attaining quasi-continuous solutions using FE analysis and post-processing in MATLAB is applied in the article. This approach makes it possible to estimate the discrete movement error, which is difficult to estimate when only using FE programs. It is demonstrated that even a rather fine FE mesh can cause a considerably biased transient solution of the moving mass problem. This is considered to be the main contribution of the presentation next to the description of the model transfer from ANSYS into MATLAB.

Discrete movement is not only the problem with using software like ANSYS and similar, but also of all stepper motors frequently used nowadays. The solution of moving mass requires modification of the system mass matrix at each integration step. In addition, the system matrices are large (unless model reduction is applied), and therefore long solution times are required.

2. Theoretical background

According to the well-known principles of modal analysis [1], the dynamic behaviour of common building structures can be described with FE models using the following equations (1, 4-7).

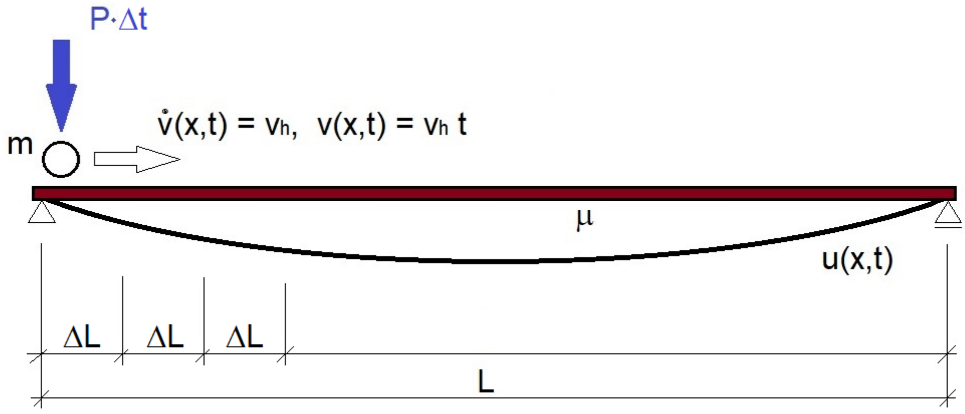

Fig. 1. Schema of the coupled system "Structure – Mass"

$$M \cdot \ddot{u} + C \cdot \dot{u} + K \cdot u = F \,, \qquad (1)$$

where F and u are time-dependent. Formulation for the coupled system shown in Fig. 1 changes the equation (1) to

$$\mathrm{M} \cdot \ddot{u} + \mathrm{C} \cdot \dot{u} + \mathrm{K} \cdot u = -\delta_j \cdot \left(m \cdot g - \mathrm{m} \cdot \ddot{u}_j - \mathrm{m} \cdot v_h^2 \cdot u''_j - \mathrm{m} \cdot 2 \cdot v_h \cdot \dot{u}'_j - F_{st}\right). \qquad (2)$$

The terms on the right hand side represent the mass weight, the inertia force, the centripetal force, the Coriolis force and the F_{st} is inertia force due to the static deformations (from the curved path of the mass). For the vertical movement of the mass it holds that

$$w = \delta_j \cdot \mathrm{u}_j \,, \qquad (3)$$

where δ_j is the Kronecker delta. The following expressions are valid for the natural modes

$$\emptyset^T \cdot M \cdot \emptyset = I; \quad \emptyset^T \cdot C \cdot \emptyset = D; \quad \emptyset^T \cdot K \cdot \emptyset = \Omega \,. \qquad (4a\text{-}c)$$

$$u = \emptyset \cdot \mathrm{Q}\,; \quad \dot{u} = \emptyset \cdot \dot{\mathrm{Q}}\,; \quad \ddot{u} = \emptyset \cdot \ddot{\mathrm{Q}}\,; \qquad (5a\text{-}c)$$

$$u'' = \frac{d^2\emptyset}{dx^2} \cdot \mathrm{Q}\,; \quad \dot{u}' = \frac{d\emptyset}{dx} \cdot \dot{\mathrm{Q}}\,; \qquad (5d\text{-}e)$$

$$\mathrm{Q} = [q_1; \dots; q_n]\,, \qquad (6)$$

$$\emptyset = [\varphi_{1,1}; \dots; \varphi_{p,n}]\,, \qquad (7)$$

where n is the number of modes used, and p is a number of finite element nodes.

From rewriting (2) using (4) – (7) it follows that

$$\mathrm{I} \cdot \ddot{\mathrm{Q}} + \mathrm{D} \cdot \dot{\mathrm{Q}} + \Omega \cdot \mathrm{Q} = \emptyset^T \cdot \left(\delta_j \mathrm{m} \cdot \left(-\mathrm{g} - \emptyset_{..,j}\ddot{\mathrm{Q}} - 2v_h \frac{d\emptyset_j}{dx}\dot{\mathrm{Q}} - v_h^2 \frac{d^2\emptyset_j}{dx^2}\mathrm{Q}\right) - F_{sw}\right), \qquad (8)$$

$$\ddot{u}_j(\mathrm{x}, \mathrm{t}) = \sum_{i=1}^{n} \varphi_{i,j} \cdot \ddot{q}_i = \emptyset_{..,j} \cdot \ddot{\mathrm{Q}}\,, \qquad (9)$$

$$\mathrm{I} \cdot \ddot{\mathrm{Q}} + \mathrm{D} \cdot \dot{\mathrm{Q}} + \Omega \cdot \mathrm{Q} = -\delta_j \cdot \mathrm{m} \cdot \emptyset^T \cdot \left[\left(\emptyset_j \cdot \ddot{\mathrm{Q}} - \mathrm{g} - a_{sw}\right) - \left(\frac{d\emptyset}{dx} \cdot \dot{\mathrm{Q}}\right)_j - \left(\frac{d^2\emptyset}{dx^2} \cdot \mathrm{Q}\right)_j\right]. \qquad (10)$$

Because $a_{sw} = F_{sw}/\mathrm{m}$ can usually be neglected, eq. (10) can be written as

$$\left(\mathrm{I} + \mathrm{m} \cdot \emptyset_{\mathrm{j}}^{\mathrm{T}} \cdot \emptyset_{\mathrm{j}}\right) \cdot \ddot{\mathrm{Q}} + \left(\mathrm{D} + \mathrm{m} \cdot \emptyset_{\mathrm{j}}^{\mathrm{T}} \cdot \frac{\mathrm{d}\emptyset}{\mathrm{dx}}\right) \cdot \dot{\mathrm{Q}} + \left(\Omega + \mathrm{m} \cdot \emptyset_{\mathrm{j}}^{\mathrm{T}} \cdot \frac{\mathrm{d}^2\emptyset}{\mathrm{dx}^2}\right) \cdot \mathrm{Q} = -\delta_{\mathrm{j}} \cdot \mathrm{m} \cdot \mathrm{g} \cdot \emptyset_{\mathrm{j}}^{\mathrm{T}} . \quad (11)$$

The vertical displacement under the moving mass is

$$w = \delta_j \cdot \emptyset_j \cdot \mathrm{Q} = \mathrm{diag}(\emptyset \cdot \mathrm{Q}^T) . \quad (12)$$

Only s selected degrees of freedom (DOFs, $s < p$) can be imported from the FE model. But the imported set $\emptyset_a$ must contain all the DOFs on the driving path and can also contain other nodes of interest $\emptyset_b$ like e.g. measured nodes

$$\emptyset = \begin{bmatrix} \emptyset_a \\ \emptyset_b \end{bmatrix} . \quad (13)$$

The damping matrix D can be assumed to be proportional, and therefore also of the diagonal form

$$\mathrm{D} = \alpha \cdot \mathrm{I} + \beta \cdot \Omega . \quad (14)$$

The quasi continuous solution is achieved through resampling of the $\emptyset_a$ using a spline interpolation from the model resolution into the resolution resulting from the horizontal moving velocity and the applied sampling frequency.

Eq. (11) is a nonlinear system of n differential equations of the second order with a time dependent mass matrix. Solving it is generally not an easy task. However, under the assumption that the mass matrix changes only marginally between two successive time steps, numerical integration using the Hilber–Hughes–Taylor-α method (HHT-α) solver [2] with fixed integration stepping can be applied.

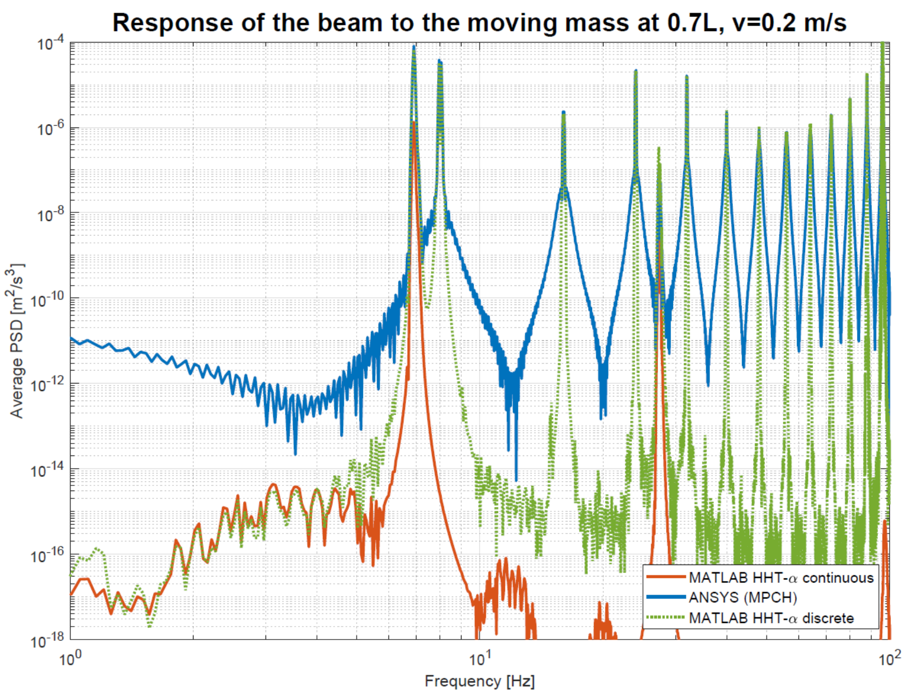

Fig. 2. Comparison of the solutions obtained from ANSYS and the above described procedure

3. Numerical Simulations

For demonstration purposes, an FE model was assembled in the ANSYS program. The applied model corresponded to Fig. 1, with a span of 3.975 m, made from U-Jäckel steel 210x50x4 mm, and weighting 33.3 kg. The moving mass was 0.5 kg, thus the mass ratio between the structure and the moving mass was only ca. 1.5 %. The first natural frequency of the beam without the moving mass was 6.98 Hz. Two passing velocities were considered: v_h = 0.2 m/s, and v_h = 0.16 m/s. The relatively low driving speed was chosen due to practical requirements. Proportional damping was used with the mass multiplier $\alpha = 0.1$ and a stiffness multiplier of $\beta = 2*10^{-5}$.

The FE model was assembled in ANSYS 17. A resolution of 160 nodes per driving path was chosen in order to minimise the discrete steps in movement of the mass, forming a model of ca 2600 nodes and about 2400 SHELL181 elements.

The first seven natural modes with distinct amplitudes on the driving path (bending modes) in the frequency band 0-200 Hz were exported from ANSYS into MATLAB using the APDL commands. The solution then followed the schema described above.

Fig. 2 compares the average acceleration power spectral density (PSD) obtained from ANSYS and MATLAB. The discrete solution obtained in MATLAB complies well with the ANSYS solution, showing that the transfer of the modal model into MATLAB was successful. The continuous solution however has a considerably lower response at the peaks corresponding to the natural frequencies, and the harmonic peaks caused by the discrete motion are missing entirely. The difference in amplitudes at the first natural frequency is approximately 10 times lower in the case of the continuous solution than with the discrete movement.

4. Conclusions

The described approach offers an efficient tool for solving interaction problems between structures and moving bodies, reducing the required computing time by orders of magnitude. If a continuous movement is solved as if it were a discrete movement, a considerable error can be expected in the transient analysis. This has to be considered also in application of stepper motors drives.

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