COMPUTATIONAL MECHANICS 2021
36th conference with international participation
Srní
November 8 - 10, 2021

Modelling approaches to the stress wave propagation in a cracked specimen

A. Kruisová, J. Kopačka, J. Kober

Institute of Thermomechanics, Czech Academy of Sciences, Dolejškova 5, 182 00 Prague 8, Czech Republic

Abstract

One of the essential tasks of non-destructive testing is to detect a crack in a specimen. It is well known that a component with a crack exposed to a harmonic excitation of a given frequency has a nonlinear response as a function of the excitation amplitude. The focus of this paper is the numerical modelling of this phenomenon using the finite element method with the consideration of the contact constraint at the crack interface. In addition to the nonlinear transient dynamic problem solved by explicit time integration, a more efficient procedure based on the harmonic balance method is developed. The results of numerical simulations are also compared with experimentally obtained data.

1. Introduction

Non-destructive testing is often used to detect defects in solids, but the commonly used linear ultrasonic techniques based on reflection, transmission, and scattering, are not sensitive to closed cracks. However, damaged solids, together with other microheterogeneous media such as rocks and concrete, containing such cracks, exhibit strong nonlinearities [3] when a resonant frequency depends on the amplitude of the ultrasonic signal. The underlying microscopic mechanism of these nonlinearities is still poorly understood [1], thus the numerical simulations are used to decode the effect of clapping or frictional contacts of the crack interfaces or the effect of the crack tip plastic zone. In addition, the comparison of numerical simulations with experimental results can provide internal parameters and thus the complete characterization of the defects.

Since the direct time integration method for computing the resonance of the sample is very time-consuming, the new method based on harmony balance has to be developed. In this work, we present this newly developed method and its comparison with direct time integration. Thus, only the effect of clapping of crack interfaces is studied here.

2. Formulation

The problem of wave propagation in an elastic specimen with a crack can be formulated as a linear elastodynamics problem with contact conditions. In particular, it is an initial boundary value problem where the governing equations are partial differential equations of hyperbolic type expressing the balance of linear momentum. For the spatial discretization, we use the finite element method, which leads to a system of nonlinear ordinary differential equations of the form

$$\mathbf{M}\ddot{\mathbf{u}} + \mathbf{K}\mathbf{u} + \mathbf{f}_{\text{c}}(\mathbf{u}) = \mathbf{f}_{\text{ext}}(t), \tag{1}$$

where $\mathbf{M}$ is the mass matrix, $\mathbf{K}$ is the stiffness matrix, $\mathbf{f}_{\text{ext}}$ is the vector of external forces, $\mathbf{f}_{\text{c}}$ is the vector of contact forces and $\mathbf{u}$ is the vector of nodal displacements. Contact constraints are enforced by the so-called bi-penalty method [2].

The time discretization of the semi-discrete system (1) is preferably performed by the central difference method (CDM), which is particularly suitable for fast phenomena such as the wave propagation problem. In addition to the undeniable advantages of this method, such as simplicity and efficiency of numerical implementation, the main disadvantage of the CDM is the conditional stability, which dictates the maximum allowed time step size. In this work, we use the leapfrog variant of the CDM, whose integration scheme is

$$\mathbf{a}^n = \mathbf{M}^{-1}(\mathbf{f}^n_{\text{ext}} - \mathbf{f}^n_{\text{c}} - \mathbf{K}\mathbf{u}^n), \tag{2}$$

$$\mathbf{v}^{n+1/2} = \mathbf{v}^{n-1/2} + \mathbf{a}^n \Delta t, \tag{3}$$

$$\mathbf{u}^{n+1} = \mathbf{u}^n + \mathbf{v}^{n+1/2}\Delta t, \tag{4}$$

where the acceleration $\mathbf{a}^n \approx \ddot{\mathbf{u}}(n\Delta t)$, velocity $\mathbf{v}^{n\pm 1/2} \approx \dot{\mathbf{u}}(n\Delta t \pm 1/2\Delta t)$, and displacement $\mathbf{u}^n \approx \mathbf{u}(n\Delta t)$ are obtained by integration at discrete time points $\{0, \Delta t, 2\Delta t \ldots, N\Delta t\}$. The system of equations is completed with initial and boundary conditions.

Zero displacement and velocity fields are often considered an initial condition, but this is not compatible with harmonic excitation. Therefore, there is a transient phase in the early stages of time integration before the system response stabilizes at the expected harmonic response. It necessitates considering a sufficiently long simulation time for the solution to transition to steady-state harmonic oscillation. In addition, the finite element mesh must be sufficiently fine to capture waves of the desired frequency correctly. All these limitations impose enormous time demands on the numerical solution.

A promising alternative that could reduce the described drawbacks of the direct time integration solution is the harmonic balance method (HBM) [4]. This method offers an alternative to time-domain methods for analyzing problems where a steady-state, periodic solution to the equation of motion is sought. The idea of this method is to represent the time history of the displacement function, $\mathbf{u}(t)$ by its frequency content, $\mathbf{U}(\omega)$, to obtain a set of equations for the corresponding frequencies (harmonics) and iteratively balance the related terms. Specifically, the displacements and forces are represented as truncated Fourier series with N harmonics

$$\mathbf{u}(t) \approx \sum_{n=1}^{N} \mathbf{U}_n e^{j(\omega_n t)}, \qquad \mathbf{f}_{\text{c}}(t) \approx \sum_{n=1}^{N} \mathbf{F}^n_{\text{c}} e^{j(\omega_n t)}, \qquad \mathbf{f}_{\text{ext}}(t) \approx \sum_{n=1}^{N} \mathbf{F}^n_{\text{ext}} e^{j(\omega_n t)}, \tag{5}$$

where $\mathbf{U}_n$, $\mathbf{F}^n_{\text{c}}$, $\mathbf{F}^n_{\text{ext}}$ are the vectors of Fourier coefficients. Substituting these expressions into the equation of motion, (1), and balancing the harmonic terms yields, for a harmonic n

$$\left(\mathbf{K} - (n\Omega)^2\mathbf{M}\right)\mathbf{U}_n = \mathbf{F}^n_{\text{c}} + \mathbf{F}^n_{\text{ext}}. \tag{6}$$

As the Fourier coefficients, $\mathbf{F}^n_{\text{c}}$, of the non-linear contact forces are functions of the displacements, equation (6) is non-linear and must be solved iteratively. This iteration process can be sketched as

$$\mathbf{U}(\omega_n)^{(k)} \xrightarrow{\text{FFT}^{-1}} \mathbf{u}(t)^{(k)} \rightarrow \mathbf{f}_{\text{c}}(t)^{(k+1)} \xrightarrow{\text{FFT}} \mathbf{F}_{\text{c}}(\omega_n)^{(k+1)} \rightarrow \mathbf{U}(\omega_n)^{(k+1)}. \tag{7}$$

While the CDM approach achieves steady-state solutions by direct time integration, the HBM solves systems of nonlinear equations. For large problems, HBM proves to be a more efficient approach.

3. Numerical example

As a numerical example, we studied a resonant frequency of a plate with the dimension of $100 \times 20 \times 8\,\mathrm{mm}^3$ with a V-shaped crack in the middle of the longest side of the sample. The crack length is half of the plate, and the aperture of the crack on the side of the plate is $1\,\mu\mathrm{m}$, see Fig. 1. Plate was made from 2024 aluminium alloy with Young's modulus of elasticity $E = 73.1$ GPa, Poisson's ratio $\nu = 0.33$ and density $\rho = 2.78$ g.cm^{-3}. The sample was modelled by 20×100 plane strain 4-node elements. The direct integration was provided by the central difference method with time step 7×10^{-8} s. Contact pairs were prescribed on the clapping crack interfaces, and the bi-penalty method was used to simulate the contact nonlinearity.

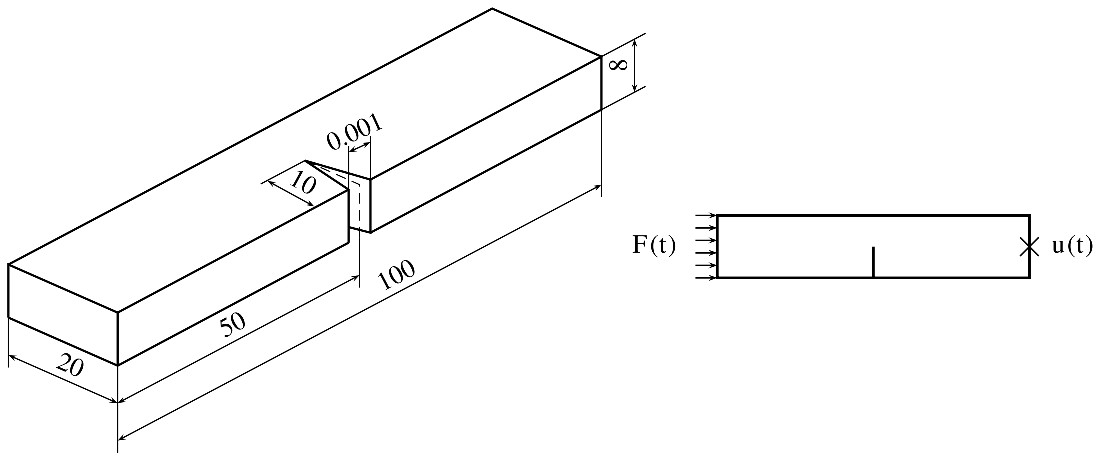

Fig. 1. Sample geometry (*left*) and loading (*right*)

During the first 10 ms of computation, on one side of the sample was applied force signal shown in Fig. 2 causing the longitudinal waves. It is the linear chirp signal with frequencies from 20 kHz to 23 kHz modified by the cosine-tapered window on the first and last sixths of the signal. Next 50 ms, the sample was already unloaded. The displacement in the longitudinal direction was captured in the middle of the opposite face to loading, see Fig. 3. After the fast Fourier transform (Fig. 4), the resonance frequency was stated (here, it is 21.978 kHz).

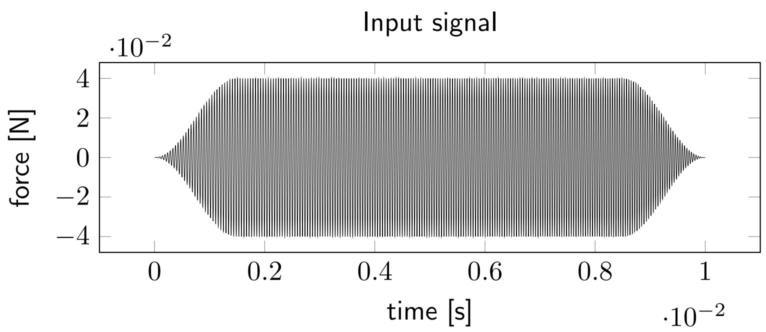

Fig. 2. Input loading signal is a chirp with frequency from 20 kHz to 23 kHz

4. Conclusions

The direct time integration method for the determination of the resonance frequency is time-consuming. On the other hand, the harmonic balance method proves to be more efficient, espe-

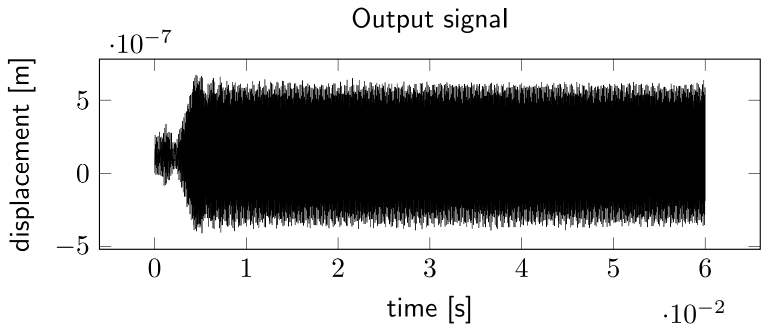

Fig. 3. Computed displacement

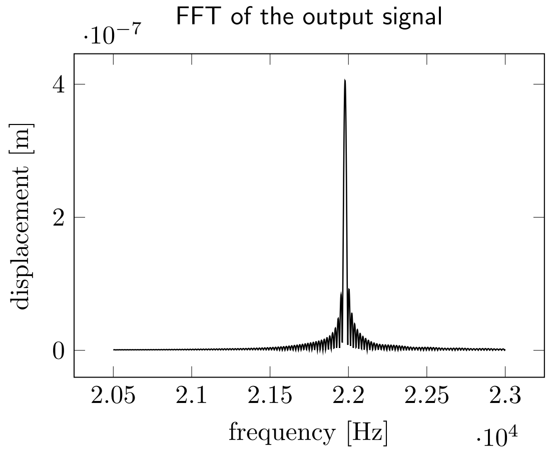

Fig. 4. Fast Fourier transform of the displacement signal

cially for large 3D problems with a fine finite element mesh. It is because the demanding time integration with a small time step is replaced by a sequence of iterative solutions of systems of nonlinear equations. Moreover, due to the orthogonality of the basis functions of the Fourier expansion, the process can be parallelized efficiently. The proposed methodology will be further employed to study the behavior of a crack subjected to harmonic excitation.

Acknowledgements

This research was founded by the Czech Science Foundation (grant number GA 19-142375) and institutional support RVO: 61388998.

References

[1] Gao, K., Rougier, E., Guyer, R. A., Lei, Z., Johnson P. A., Simulation of crack induced nonlinear elasticity using the combined finite-discrete element method, Ultrasonics 98 (2019) 51-61.

[2] González, J. A., Kopačka, J., Kolman, R., Park, K. C., Partitioned formulation of contact-impact problems with stabilized contact constraints and reciprocal mass matrices, International Journal for Numerical Methods in Engineering 122 (17) (2021) 4609-4636.

[3] Nazarov, V. E., Sutin, A. M., Nonlinear elastic constants of solids with cracks, The Journal of the Acoustical Society of America 102 (1997) No. 3349.

[4] von Groll, G., Ewins, D., The harmonic balance method with arc-length continuation in rotor / stator contact problems, Journal of Sound and Vibration 241 (2) (2001) 223-233.