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Flow in poro-piezoelectric media induced by persitaltic deformation waves – homogenization approach

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We consider a fluid-saturated porous medium subjected to deformation waves which generate peristaltic driven flow. The deformation is actuated by piezoelectric elements periodically distributed in the structure and controlled locally by electrodes inducing the electrostatic field. The presented research is aimed to explore functionality of such metamaterial structures by in silico experiments. For this we employ a two-scale modelling approach based on the homogenization method [1] combined with the sensitivity analysis. We extend the homogenized model of the fluid-saturated piezo-poroelastic medium equipped with the controlling conductor networks [5] to describe the fluid-structure interaction respecting influence of the deformation of the micro-configuration.

The computational model arises from the homogenization of the fluid-saturated porous medium. To treat the large deformation phenomenon, we follow the Eulerian approach leading to the updated-Lagrangian incremental formulation in the two-scale setting [2, 3]. In the context of locally periodic structures, local cell problems are obtained which provide characteristic responses of the microstructures with respect to macroscopic strains, fluid pressure and electric potentials. Within the homogenization scheme introduced for the incremental fluid-structure interaction problem, the macroscopic nonlinearity of the device is captured using the first order expansions of the homogenized coefficients with respect to macroscopic variables [6], cf. [4]. For this, the sensitivity analysis approach is employed. We present examples of microstructures and results of the simulations as the proof of concept aimed at designing smeared peristaltic pumps in a bulk medium.

The heterogeneous periodic structure of the two-phase medium is constituted by a piezoelectric skeleton interacting with a viscous fluid saturating the pores in the skeleton. The structure characterized by the pore size $\ell^\varepsilon \approx \varepsilon$, where the parameter $\varepsilon \to 0$ is related to the asymptotic analysis leading to a model of the homogenized fluid saturated piezo-poroelastic medium.

Micromodel In the piezoelectric (PZ) solid, the Cauchy stress tensor $\boldsymbol{\sigma}^\varepsilon$ and the electric displacement $\vec{D}^\varepsilon = (D_i^\varepsilon)$ depend on the strain tensor $\boldsymbol{e}(\boldsymbol{u}^\varepsilon) = (\nabla \boldsymbol{u}^\varepsilon + (\nabla \boldsymbol{u}^\varepsilon)^T)/2$ defined in terms of the displacement field $\boldsymbol{u}^\varepsilon = (u_i^\varepsilon)$, and on the electric field $\vec{E}^\varepsilon = \nabla\varphi^\varepsilon$ defined in terms of the electric potential, $E_i^\varepsilon = \partial_i^x \varphi^\varepsilon$, such that

$$
\begin{aligned}
\boldsymbol{\sigma}^\varepsilon(\boldsymbol{u}^\varepsilon, \varphi^\varepsilon) &= \mathbb{A}^\varepsilon \boldsymbol{e}(\boldsymbol{u}^\varepsilon) - \underline{\boldsymbol{g}}^T \vec{E}^\varepsilon(\varphi^\varepsilon) \;, \\
\vec{D}^\varepsilon(\boldsymbol{u}^\varepsilon, \varphi^\varepsilon) &= \underline{\boldsymbol{g}}^\varepsilon \boldsymbol{e}(\boldsymbol{u}^\varepsilon) + \boldsymbol{d}^\varepsilon \vec{E}^\varepsilon(\varphi^\varepsilon) \;,
\end{aligned}
\tag{1}
$$

where $\mathbb{A}^\varepsilon = (A_{ijkl}^\varepsilon)$ is the elasticity fourth-order symmetric positive definite tensor of the solid, where $A_{ijkl} = A_{klij} = A_{jilk}$, the deformation is coupled with the electric field through the 3rd

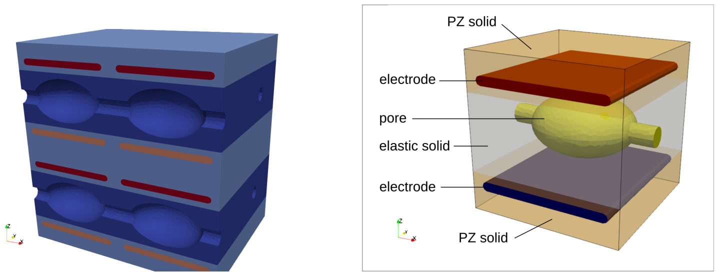

Fig. 1. The periodic microstructure with two electrodes (*left*); the representative unit cell Y (*right*)

order tensor $\underline{\boldsymbol{g}}^\varepsilon = (g^\varepsilon_{kij})$, $g^\varepsilon_{kij} = g^\varepsilon_{kji}$ and $\boldsymbol{d}^\varepsilon = (d^\varepsilon_{kl})$ is the permittivity tensor. The skeleton includes also conducting elastic parts (electrodes) constituting equipotential surfaces. Obviously tensors $\underline{\boldsymbol{g}}^\varepsilon$ and $\boldsymbol{d}^\varepsilon$ vanish in these electrodes which are used to control deformation of the PZ solid, thus, modifying the shape of the pores saturated by an incompressible viscous fluid characterized by the viscosity $\mu^\varepsilon = \varepsilon^2\bar{\mu}$. The fluid flow in the pores is governed by the linearized Navier-Stokes equations. On the fluid-solid interface, standard interaction conditions are prescribed guaranteeing continuity of the velocities and traction stresses of the two-phase medium.

In order to account for the localized electric field control inducing steep electric potential gradients $\approx 1/\varepsilon$, a proper scaling of the PZ material parameters must be considered when passing to the limit $\varepsilon \to 0$, as suggested in [5], such that

$$\underline{\boldsymbol{g}}^\varepsilon(x) = \varepsilon\bar{\underline{\boldsymbol{g}}}\;, \quad \boldsymbol{d}^\varepsilon(x) = \varepsilon^2\bar{\boldsymbol{d}}\;. \tag{2}$$

Homogenized model The limit macroscopic equations obtained for $\varepsilon \to 0$ involve the homogenized coefficients expressed in terms of the characteristic responses which are obtained upon solving the so-called local problems imposed in the representative cell Y, comprised of the solid and fluid parts, see Fig. 1.

The homogenization procedure leads to a model describing the fluid flow in the deforming PZ-poroelastic medium situated in the macroscopic domain $\Omega \subset \mathbb{R}^3$. Due to the non-stationary characteristic flow response , the hydraulic permeability $\boldsymbol{K}^H$ depends on time and constitutes the dynamic Darcy law governing the fluid seepage. The scaling (2) leads to a macroscopic model of a porous medium with a modified constitutive law involving the macroscopic potentials $\bar{\varphi}^k$, $k = 1, \dots, \bar{k}$ which provide a control handle. The model is represented by following system of equations imposed in a time-space domain, $x \in \Omega$, $t > 0$, involving the macroscopic fields, the displacements $\boldsymbol{u}$, the fluid seepage velocity $\boldsymbol{w}$, and the pore fluid pressure p satisfying

$$\begin{aligned}
&\bar{\rho}\ddot{\boldsymbol{u}} + \rho_f\dot{\boldsymbol{w}} - \nabla\cdot\boldsymbol{\sigma}^H(\boldsymbol{u},p) = \boldsymbol{0}\;,\\
&\qquad\text{where}\quad \boldsymbol{\sigma}^H(\boldsymbol{u},p) = \mathbb{A}^H\boldsymbol{e}(\boldsymbol{u}) - p\boldsymbol{B}^H + \sum_k \boldsymbol{H}^k\bar{\varphi}^k\;,\\
&\boldsymbol{B}^H : \boldsymbol{e}(\dot{\boldsymbol{u}}) + M^H\dot{p} + \nabla\cdot\boldsymbol{w} = \sum_k Z^k\dot{\bar{\varphi}}^k\;,\\
&\qquad\qquad\text{where}\quad \boldsymbol{w} = -\int_0^t \boldsymbol{K}^H(t-\tau)[\nabla p(\tau,\cdot) + \rho_f\ddot{\boldsymbol{u}}(\tau,\cdot)]\mathrm{d}\tau\;,
\end{aligned} \tag{3}$$

whereby the voltage potentials $\bar{\varphi}^k(t,x)$, $k = 1,\dots,k^*$ are assumed to be known functions of time t and macroscopic position x. This model involves homogenized coefficients $\mathbb{A}^H, \boldsymbol{B}^H$,

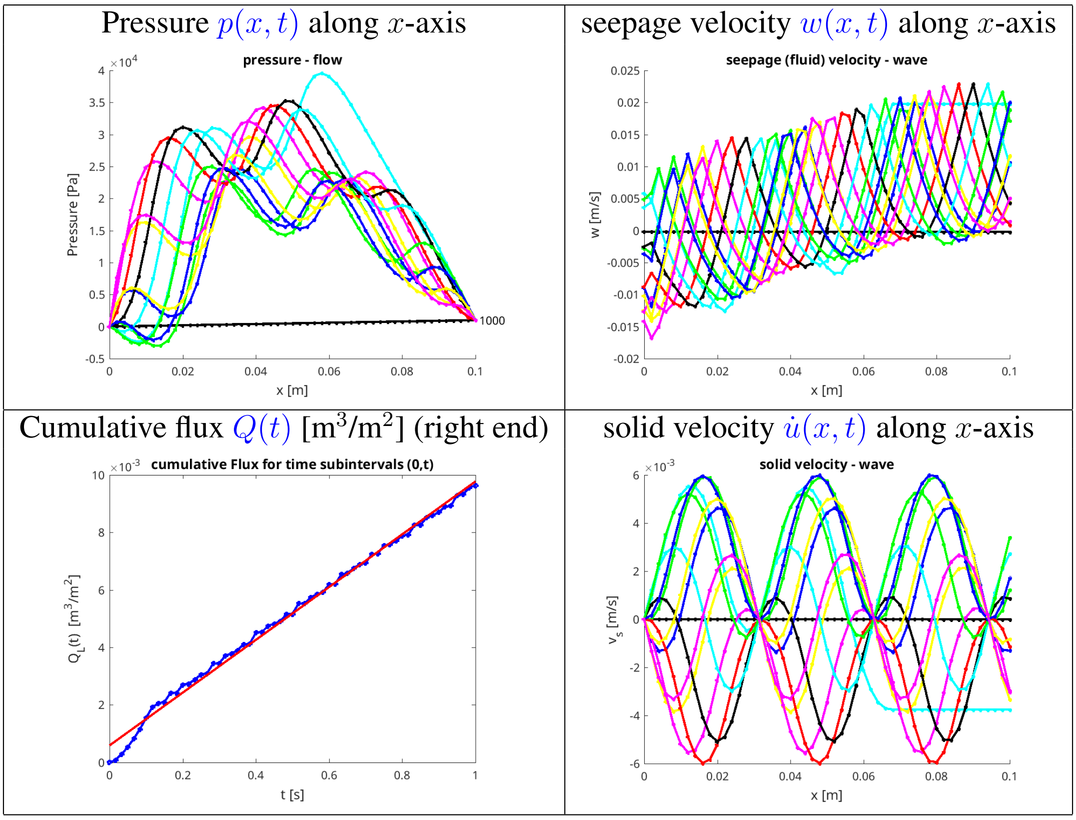

Fig. 2. Characterization of the peristaltic flow in the 1D continuum due the PZ actuation controlled by electric field

$\boldsymbol{K}^H$, M^H, $\underline{Z}$, $\boldsymbol{H}$, further denoted by $\mathbb{H}$ in a generic sense. The inertia effects are pronounced by the terms involving effective densities $\bar{\rho}$ and ρ_f, and also by the dynamic permeability.

Geometrical nonlinearity Since the peristaltic flow is driven by the pore deformation, it is crucial to capture the influence of the deformation on the permeability and other effective model parameters, though it is derived using the linear kinematics framework. As a compromise between the linear modelling leading to model (3) and a fully nonlinear treatment, cf. [2], we suggest to apply the approach proposed in [6] which is based on the domain method of the shape sensitivity analysis of the characteristic responses defining homogenized coefficients $\mathbb{H}$. This enables to introduce perturbed coefficients $\tilde{\mathbb{H}}(\boldsymbol{e}(\boldsymbol{u}), p, \{\bar{\varphi}^k\})$ using the first order expansion formulae which have the generic form applicable to each of the homogenized coefficients,

$$\tilde{\mathbb{H}}(\boldsymbol{e}(\boldsymbol{u}), p) = \mathbb{H}^0 + \delta_{\boldsymbol{e}}\mathbb{H}^0 : \boldsymbol{e}(\boldsymbol{u}) + \delta_p \mathbb{H}^0 p + \sum_k \delta_{\varphi,k}\mathbb{H}^0 \bar{\varphi}^k \,. \tag{4}$$

Although the two-scale problem becomes nonlinear, for a periodic initial configuration all the characteristic responses and the sensitivities are computed for the unperturbed cell Y, thus, independently of the macroscopic solutions.

Numerical illustration To illustrate capability of the material to transport the fluid against the pressure slope, a 1D macroscopic model was considered, whereby the effective material parameters are computed for a 3D microstructure, see Fig. 1, where the skeleton is made of the piezoceramic PZT-5 material. In this example we neglect the inertia effects. A pressure slope $\Delta p = \bar{p}_2 - \bar{p}_1 > 0$ is prescribed, which defines the boundary conditions $p(0,t) = \bar{p}_1$ and

$p(L,t) = \bar{p}_2$ of the 1D homogenized continuum. The reduced 1D model of a prism obtained from (3) due to the symmetry assumptions involves displacement $u = u_1$, seepage $w = w_1$, strain $e(u) = e_{11}(u) = u'$ and voltage φ being functions of (x,t), whereby the prime $()'$ denotes d/dx. Noting that the total stress $\sigma = -\bar{p}_2$ is constant, the following equations hold

$$\begin{aligned} \sigma := Ae(u) - pB + H\varphi = -\bar{p}_2 \ , \\ M\dot{p} + w' + Be(\dot{u}) = Z\dot{\varphi} \ , \\ w = -\tilde{K}p' \ , \quad \text{where } \tilde{K}(e,p,\varphi) = K_0 + \partial_e K_0 e + \partial_p K_0 p + \partial_\varphi K_0 \varphi \ . \end{aligned}$$

Upon eliminating $e(u)$, we obtain the following equation for $p(x,t)$ to be solved for a given the electric potential wave $\varphi(x,t)$,

$$C\dot{p} - (K_0 + \partial_e K_0 A^{-1}\sigma + K_p p + K_\varphi \varphi)p'' + (K_p p' + K_\varphi \varphi')p' = F\dot{\varphi} \ ,$$

where

$$\begin{aligned} C = M + BA^{-1}B \ , \quad F = Z + BA^{-1}H \ , \\ K_p = \partial_e K_0 A^{-1} B + \partial_p K_0 \ , \quad K_\varphi = \partial_e K_0 A^{-1} H + \partial_\varphi K_0 \ . \end{aligned}$$

In Fig. 2, the response is depicted in terms of pressure $p(x,t)$, the fluid seepage velocity $w(x,t)$, the bulk material velocity $\dot{u}(x,t)$ and the cumulative flux $Q(t) = \int_0^t w(0,\tau)\mathrm{d}\tau$ which reveals the pumping effect. The pressure slope was $\bar{p}_2 - \bar{p}_1 = 10^3$ Pa.

Conclusions and perspectives Respecting the geometrical nonlinearity by virtue of the solution-dependent homogenized coefficients of the model is crucial to capture the performance of the peristaltic pump, i.e., a "smart structure" transporting the fluid against the flux due to the pressure gradient. Several issues to be explored in the further research include the control through the potential $\varphi(x,t)$, fully collapsible pores, and the influence of the acoustic wave propagation.

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