Non-linear dynamics of kinematically excited bistable pendulum

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Introduction

In many devices, such as switches, clasps and closures, mechanisms having stable equilibria in two distinct positions are desired. As an example, to achieve a bistable configuration of a mechanical system, buckling of a beam has been mentioned by Kovacic (2011). Here, we deal with dynamical properties of a bistable pendulum that is kinematically excited. Dynamic properties are investigated both computationally and experimentally using high-speed camera Smolik (2020) to capture the large scale motion of the pendulum in different operational regimes.

System description and dynamical analysis

Presented system consists of physical pendulum and two identical permanent magnets. The first one is placed at the free end of the pendulum and the second one is fixed with the moving frame to which is the pendulum connected by a revolute joint, see Fig. 1 right. The orientation of the poles of the magnets causes the magnets repel each other. Therefore, two stable and one unstable equilibria exist. The dynamical behaviour is studied both experimentally and computationally. The equation of motion can be witten in following normalized form, as was derived in Steinbach (2021)

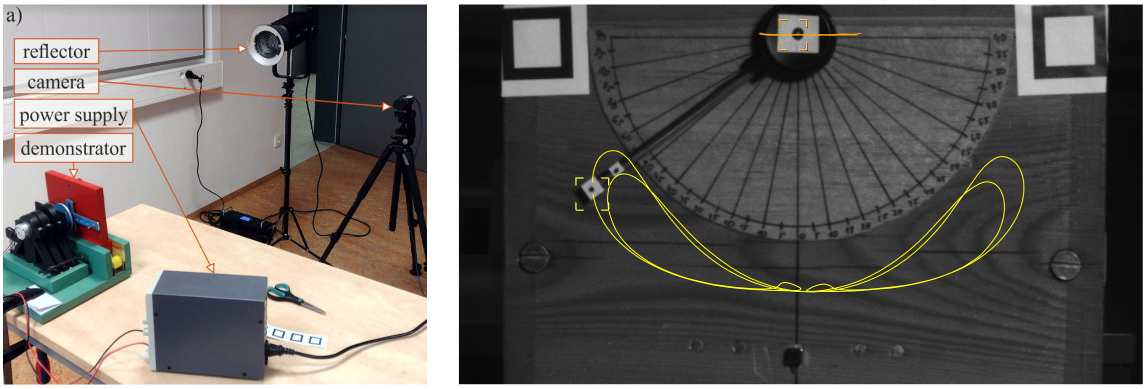

Figure 1: Setup of the experiment (left) and trajectory of the end point at $\omega = 21.6$ rad/s (right)

$$\ddot{\varphi} + 2D\Omega + \Omega^2 \sin\varphi = f_m l \cos\psi \sin\varphi + u(t)\frac{\Omega^2}{g}\omega^2 \sin\varphi, \tag{1}$$

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where the kinematic excitation is assumed in a form $u(t) = U \cos \omega t$, l represents the length of the pendulum and the mentioned angular quantities are defined in Fig. 1 right. Normalized magnetic force f_m, based on Gilbert model presented in Vokoun (2009), utilizes following quatities: distance between poles of magnets p, area of magnet cross-section S, length of magnet d, permeability of vacuum μ_0, magnetic induction B_0, which can be expressed as $B_0 = \frac{\mu_0}{2} M$, where M is magnetization of magnets.

Dynamic properties of this system were investigated using numerical integration in time domain for different excitation frequencies. Based on that bifurcation diagram was created, see Fig. 2 left. It helps to visualize qualitative changes in the system response. Some chaotic responses were found and for illustration, chaotic attractor is visualized using Poincaré's section in Fig. 2 right.

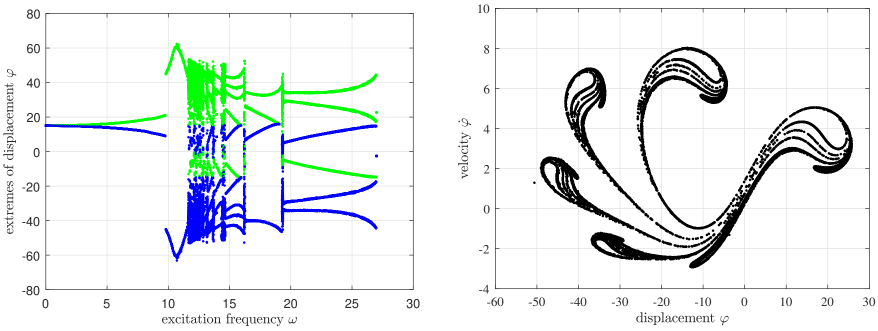

Figure 2: Bifurcation diagram of pendulum angular displacement (left), chaotic attractor represented by Poincaré's section corresponding to excitation frequency $\omega = 13.7$ (right)

The demonstrator design is intertwined with modelling. Therefore, many parameters of the model were chosen with respect to physical limitations of the demonstrator. Kinematic excitation is realized by means of a DC motor. Since the motor is capable of rotational motion only, a Scotch Yoke mechanism was employed to convert rotational motion to a linear reversible one, see Fig. 1 left. The motion of the demonstrator was captured using an IRIS M high-speed camera and Genaray AK-230 monolight LED reflector. The recordings were captured with a framerate of 256 fps in HD resolution (1280 × 720 px) and the trajectories of chosen reference points were tracked and compared with those gained using computational model based on equation (1).

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