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Assessment of various computational approaches for airfoil stability analysis with two degrees of freedom

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The paper deals with the solution of fluid-structure interaction (FSI) problems, specifically the external aerodynamics of airfoils and turbine blades. This problem is of high importance since structural instabilities may occur if the flow field's input parameters and the airfoil's properties are not properly configured. A typical example of such instability is flutter, a self-excited oscillation of the structure induced by the flowing fluid. A long-term problem in solving FSI tasks with classical numerical approaches (FVM, FEM) is their high computational complexity, which does not allow solving these tasks in real-time. This deficit is usually compensated by using semi-empirical methods, which, however, do not offer comparable accuracy compared to numerical techniques. This paper aims to compare the two mentioned approaches and their advantages and disadvantages in the context of the two degrees of freedom (DoF) airfoil wrapping problems.

For the analysis, a 2D perfectly rigid airfoil model with 2 DoF, namely in heave h and pitch θ, was chosen, a sketch of which can be seen in Fig. 1. Based on the Lagrange equations, the system's equations of motion were then derived, which have the following form

$$\left.\begin{aligned} &m\ddot{h} - m\ddot{\theta}e_{MS}\cos(\theta+\tilde{\varphi}_0) + m\dot{\theta}^2 e_{MS}\sin(\theta+\tilde{\varphi}_0) + k_h h = mg - L_{\mathrm{dyn}},\\ &\left(me_{MS}^2 + I_{M_C}\right)\ddot{\theta} - m\ddot{h}e_{MS}\cos(\theta+\tilde{\varphi}_0) + k_\theta\theta = -mge_{MS}\cos(\theta+\tilde{\varphi}_0) + M_{\mathrm{dyn}}, \end{aligned}\right\} \qquad (1)$$

where m is the mass of the body, k_h is the stiffness of the vertical spring, k_θ is the stiffness of the torsion spring, g is the gravitational acceleration and I corresponds to the moment of inertia, the form of the aerodynamic force effects L_{dyn} and M_{dyn} depends on the choice of the particular solver.

The first approach to solving the problem was a complex CFD analysis using the ANSYS Fluent software, which is based on solving a nonlinear Reynolds-averaged Navier-Stokes equa-

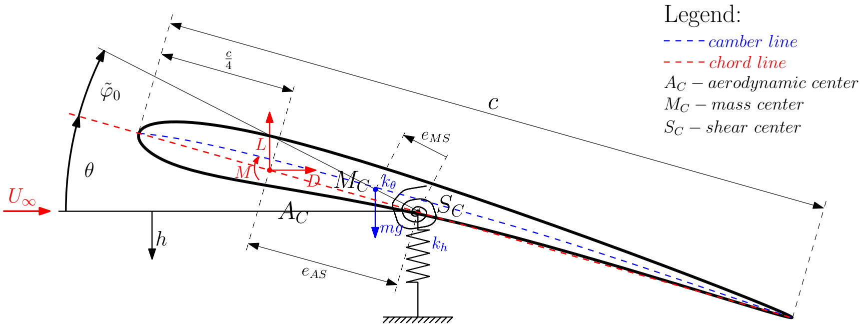

Fig. 1. Sketch of the analyzed airfoil profile with 2 DoF, including force effects on the system

tions (RANS) using the finite volume method (FVM). In this problem, a computational domain of type C was chosen, and the airfoil was the symmetric profile NACA0012. In addition, a pressure-based solver and a single-equation Spalart-Allmaras (SA) turbulence model were used to solve the external aerodynamics problems. To minimize the resulting numerical calculation errors, a mesh resolution study was calculated, especially in the boundary layer region of the airfoil where $y^{+}_{\text{max}} \sim 1$. The influence of the choice of the turbulent model (specifically SA, SST k–ω, and LES) was also investigated. Further specifications of the chosen domain, the quality of the discretization, and the settings of other parameters are given in the paper [2].

The main disadvantage of the numerical model based on the FVM is its high computational cost, where several tens of hours of machine time of a standard desktop computer are needed to obtain a solution of one-time response to initial conditions with a total length of 6 s. On the other hand, the advantage of this approach is the high complexity of the obtained outputs and their accuracy. The graphical outputs demonstrate this in Fig. 2, which offer a comparison with the experimental data available in the literature [5]. The plots show comparisons in the time and frequency domain for four cases of initial-boundary conditions for the flow field and airfoil. The time-domain output quantity is the vertical displacement z_w located at the top of the airfoil at a distance of $0.7c$ from the wing's leading edge.

Comparisons with the experiment are made for both the one and two DoF cases, the first three cases with zero flow field velocity, and the last case (bottom right) shows a comparison of the time domain response, which the author of the experiment refers to as the limiting cyclic oscillation, i.e., the limit of instability (flutter) directly. Except for the last case, the time domain responses achieved a good agreement, and the higher amplitude of the experimental data is seen in the last case. More importantly, however, the stability limit has been reached in the numerical calculation for the same flow field velocity. Thus, despite the mentioned differences, the outputs of ANSYS Fluent software represent a suitable reference to which the outputs from the application of other methods can be related.

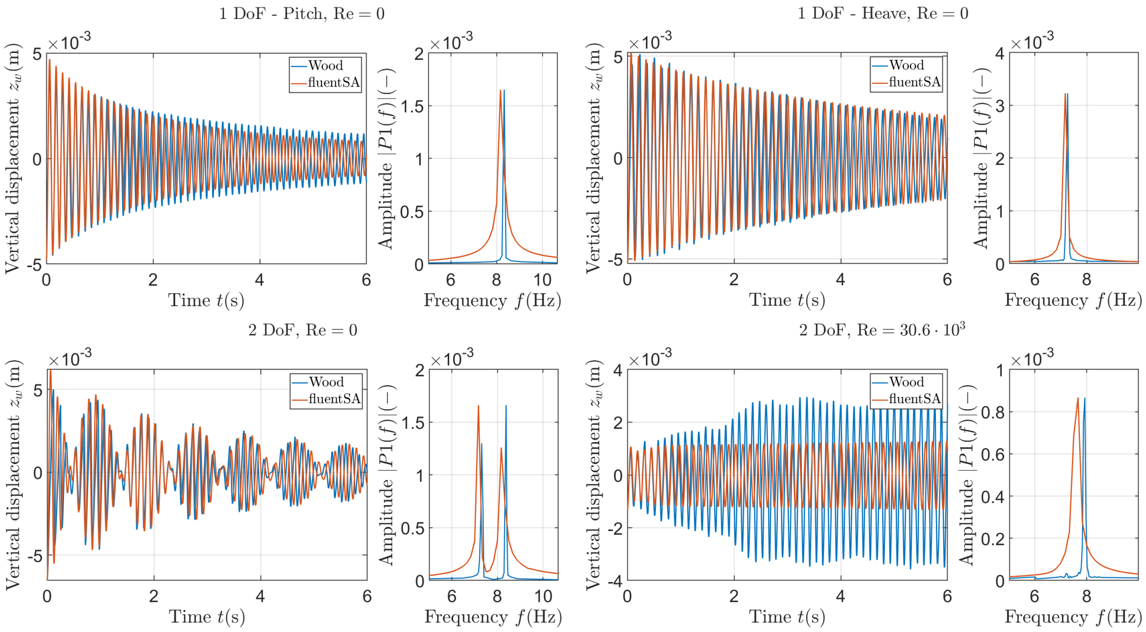

Fig. 2. Comparison of ANSYS Fluent generated outputs with experimental data presented in literature [5]. Top left case with 1 DoF in pitch and for $\text{Re} = 0$, the top right case with 1 DoF in heave and $\text{Re} = 0$, the bottom left case with 2 DoF and for $\text{Re} = 0$ and bottom right case with 2 DoF and $\text{Re} = 30.6 \times 10^3$ corresponding to the flutter limit

As far as semi-empirical methods are concerned, they offer an advantage over the complex CFD approach in terms of much lower computational cost (for the identical problem described above, the solution is computed in higher units of seconds). However, the disadvantage is the lower accuracy of the results obtained. The popular Quasi steady theory approach and the Theodorsen theory approach were selected for comparison from several possible variants. The definition of the aerodynamic force effects L_{dyn} and M_{dyn} from Eq. (1) including their detailed derivation is given in the literature [3]. Both methods were implemented in MATLAB, and the more complex approach of Theodorsen theory was verified on the outputs reported in the literature [1]. As with the ANSYS Fluent software, a time-step resolution study was performed.

The analyses show that for the case of both semi-empirical approaches, there is higher damping of the model oscillations across different variations of the structural parameter settings and initial conditions. This is well illustrated in Fig. 3 showing the values of the dynamic force effects L_{dyn} and M_{dyn} over time for the approaches represented by Fluent, Quasti steady theory and Theodorsen theory. In addition to the noticeably higher dissipation of the two semi-empirical models, on closer examination, the plots show a good indication of the evolution of the quantities of interest. This shows that although greater damping is typical of both semi-empirical methods, only the curve corresponding to the Theodorsen theory calculation follows the Fluent outputs in a trend direction. We note here that a similar result can be observed for the outputs of both displacements over time and also for other settings of the model input.

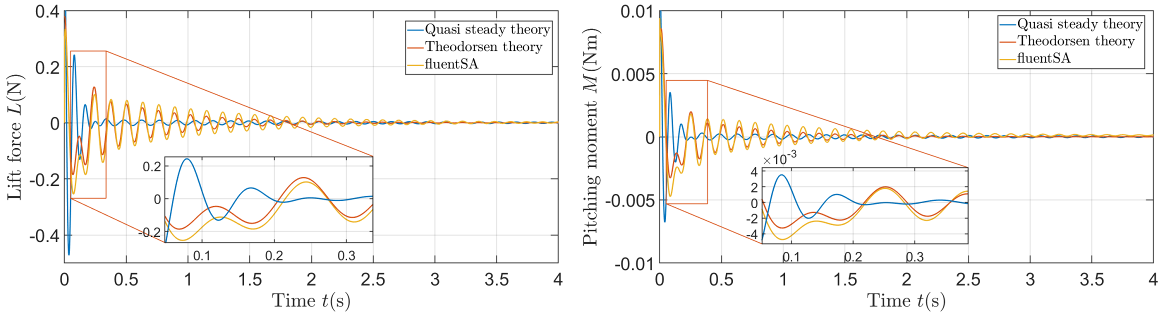

Fig. 3. Comparison of the dynamic force effects of the lift force L_{dyn} (*left*) and pitching moment M_{dyn} (*right*) for three different numerical approaches. In the zoom-in, the trend of the individual force effects can be seen, in which there is an agreement between Thedorsen theory and the results of the Fluent

The phenomenon of the higher damping of Theodorsen theory compared to the Fluent software outputs is evident even when directly comparing the value of the flutter threshold for a particular choice of input parameters. This fact is well illustrated in Fig. 4, which shows the time responses to the flutter limit for Theodorsen theory (top left) and Fluent (top right). It is also possible to find the stability limit directly using methods based on modal analysis. A variety of approaches based on modal analysis principles have been developed for airfoil problems, of which the p method (see, e.g., [4] for a derivation) and the U–g method (e.g., [1]) have been implemented here. These methods do not need to iterate solutions for different levels of input flow field velocity U_∞ and thus have the potential to streamline the flutter limit detection process further. Outputs of the modal analysis in Fig. 4 (bottom) operate with the definition of force effects based on Theodorsen theory, and the resulting values of the limiting velocity of the flow field U_∞ should be comparable to the outputs of Theodorsen theory.

As the presented paper shows, complex CFD numerical methods can be used to solve real problems of complex FSI external aerodynamics tasks. However, despite the currently available computational performance, their high computational cost remains a disadvantage. On the other

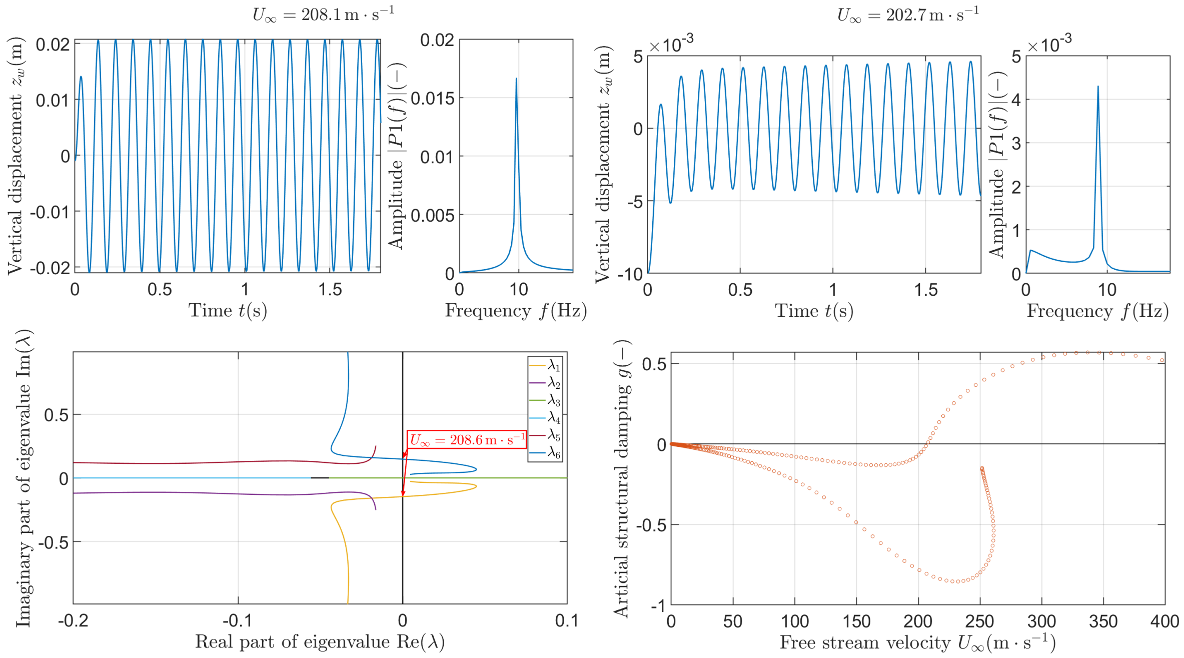

Fig. 4. Comparison of different approaches for flutter limit detection for a specific set of the 2 DoF rigid airfoil problem. Top left output for Theodorsen theory ($U_\infty = 208.1\,\mathrm{m\cdot s^{-1}}$), top right for ANSYS Fluent software ($U_\infty = 202.7\,\mathrm{m\cdot s^{-1}}$), bottom left for p method ($U_\infty = 208.6\,\mathrm{m\cdot s^{-1}}$) and bottom right for U–g method ($U_\infty = 207.1\,\mathrm{m\cdot s^{-1}}$)

hand, semi-empirical approaches to solving this problem offer lower computational costs but at the loss of a certain accuracy of the solution. Of the semi-empirical approaches investigated, the one based on Theodorsen theory appears to be the most appropriate approach. Despite still significant differences in the outputs shows trend-like characteristics with the Fluent outputs. Eliminating the differences between the Theodorsen approach and the Fluent outputs would provide an accurate and efficient tool for flutter limit detection. This elimination is the subject of further investigation.

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