COMPUTATIONAL MECHANICS 2022
37th conference with international participation
Srní
November 7 - 9, 2022

Identification of dispersion and attenuation curves of thin non-prismatic heterogeneous viscoelastic rods

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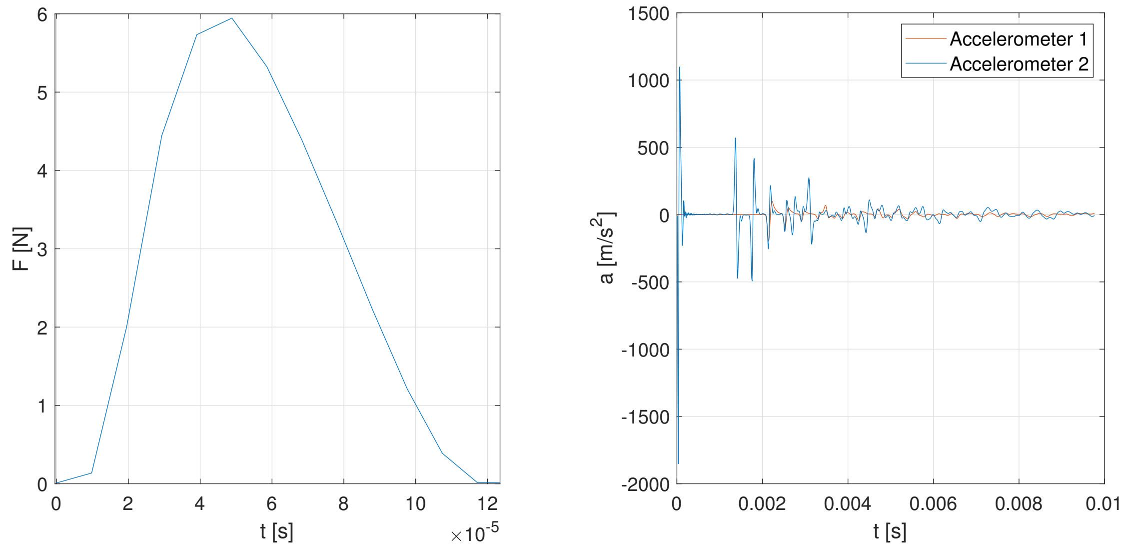
This work deals with the identification of the dispersion and attenuation properties of a thin heterogeneous viscoelastic non-prismatic rod. The first two parts of the text will discuss applications of these properties and the possible identification method. Next, the experiments are described, which are performed on several thin homogeneous prismatic rods but also on heterogeneous and non-prismatic rods. The experimental results are compared with analytically obtained results in the following section. At the end of this work, the results are discussed.

The reason for the identification of dispersion and attenuation properties of viscoelastic materials is mainly to solve the problems of passive vibration damping, and for the SHPB test, which consists of a thin input and output rod, between which a sample of similar impedance is placed (see [3]). Previously, typically elastic thin rods have been used in the SHPB test, which was based primarily on investigating the propagation of longitudinal elastic waves. Over time, during the development of plastic materials, it was also necessary to investigate wave phenomena in rods with significantly lower impedance, for which the theory of elastic wave propagation was no longer sufficient. For the description of wave phenomena in viscoelastic materials, discrete rheological models are advantageously used, which can be drawn schematically as a connection of elastic springs and viscous dashpots.

Each homogeneous part of the layered rod is described by a different dashpot viscosity and spring stiffness. Due to this and the variable cross-section of the rod, different attenuation and dispersion behaviour occurs in these parts. To determine the properties of the layered rod, measuring after the wave has passed through the entire rod under investigation is necessary. Several identification methods can be found that meet these requirements. A method described by Blanc (see [2]) will be used in this work. This method requires a Fourier transform of the acceleration measured at two different locations ($x_1 < x_2$) on the rod caused by the passage of the wave packet. According to Blanc, the relations for calculating the phase velocity $c(\omega)$ and the wave number $\kappa(\omega)$ or the attenuation $\alpha(\omega)$ can be written as [2]

$$\begin{aligned} c(\omega) &= \frac{\omega}{\kappa(\omega)} = -\omega \frac{x_2 - x_1}{\theta(x_2,\omega) - \theta(x_1,\omega)}, \\ \alpha(\omega) &= -\frac{1}{x_2 - x_1} \ln \frac{\vartheta(x_2,\omega)}{\vartheta(x_1,\omega)}, \end{aligned} \tag{1}$$

where ω is the angular frequency, $\theta(x,\omega)$ and $\vartheta(x,\omega)$ are the phase and the modulus of the Fourier transform of the acceleration, respectively.



(a) Excited pulse (b) Acceleration response

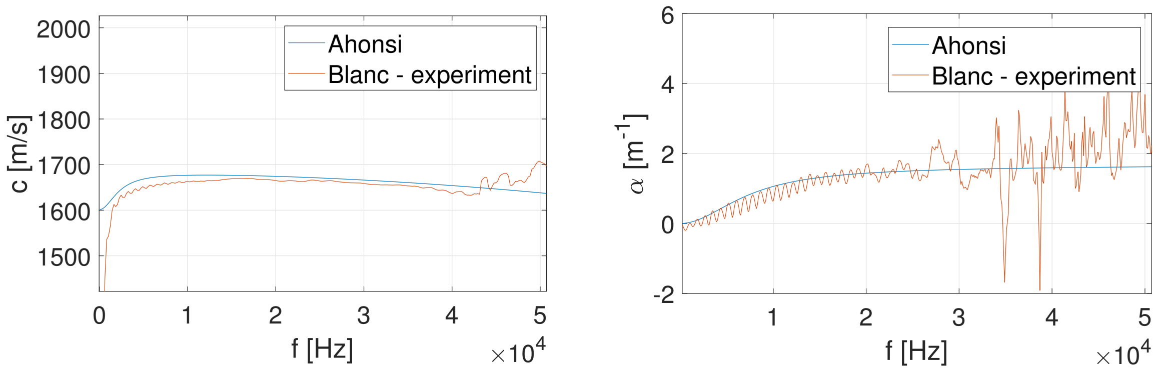
Fig. 1. Experimental data for the heterogeneous non-prismatic rod

Acceleration was measured by two accelerometers of brand *Brüel & Kjær Miniature DeltaTron® Accelerometer – Type 4519* at the ends of the investigated rods. Rods were excited at one end by an impact hammer *Brüel & Kjær Miniature Impact Hammer – Type 8204*. The signals from the accelerometers and the hammer were processed by an eight-channel analyser *OROS OR35* with a sampling rate of $100\,\mathrm{kHz}$. The measurement was performed on four homogeneous prismatic rods of approximate length $1\,\mathrm{m}$ made of polypropylene (PP), polyvinyl chloride (PVC), polyethene terephthalate (PET), polylactic acid (PLA). Also on a homogeneous non-prismatic rod made of PLA of a length $1\,\mathrm{m}$ and on a heterogeneous non-prismatic rod, which was composed of 4 homogeneous prismatic parts of different cross-sections and made of acetal heteropolymer (POM-C), aluminium (AL), polycarbonate (PC1000) and PP in this order with a total length of $4\,\mathrm{m}$. A smooth cosine pulse of length $100 - 200\,\mu\mathrm{s}$ was generated for all rods using the mentioned impedance hammer. This pulse excited the heterogeneous rod at the end of the POM-C segment, and the acceleration response to this pulse can be seen in Fig. 1b. The amplitude of each excited pulse depends proportionally on the weight of the examined rod, so Fig. 1a shows the highest measured amplitude. From Fig. 1b, it is possible to observe a significant decrease in the acceleration amplitude, i.e., the energy dissipation due to the viscosity and various cross-sections of the rod segment.

The measured phases and modulus of the transformed acceleration were used in relation (1), and the frequency dependences of longitudinal wave phase velocity and attenuation were obtained for each rod. These curves were compared with analytically calculated curves for homogeneous prismatic rods whose material is characterised by Zener rheological model. Analytical relations for the evaluation of these curves are provided, for example, by Ahonsi in his work [1]. In his relations

$$\kappa^2(\omega) = \frac{\rho\omega^2}{2E_E}\left(H_1 + H_2\right), \quad \alpha^2(\omega) = \frac{\rho\omega^2}{2E_E}\left(H_1 - H_2\right), \tag{2}$$

the author considers transverse contractions during the propagation of the pulse, i.e., the nonzero Poisson ratio ν and the inertia of the rod elements in the radial direction characterised by the radius of gyration r. The density of the rod is denoted by ρ, and E_E is the modulus of



(a) Dispersion curve of the homogeneous PVC rod

(b) Attenuation curve of the homogeneous PLA rod

Fig. 2. Comparison of analytically obtained and experimentally measured dispersion and attenuation curves for rods made of homogeneous material

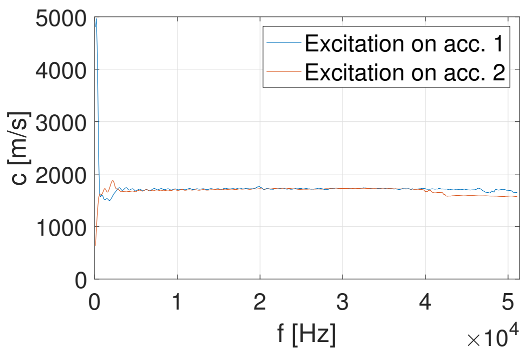
elasticity of the alone-standing spring in the Zener material model. Real functions H_1 and H_2 in (2) are given by

$$
\begin{aligned}
H_1 &= \sqrt{\frac{1+\omega^2 t_R^2}{\left[\left(1+\dfrac{E_1}{E_E}\right)-\dfrac{\rho\nu^2 r^2\omega^2}{E_E}\right]^2 \omega^2 t_R^2+\left(\dfrac{\rho\nu^2 r^2\omega^2}{E_E}-1\right)^2}},\\
H_2 &= \frac{\left[\left(1+\dfrac{E_1}{E_E}\right)-\dfrac{\rho\nu^2 r^2\omega^2}{E_E}\right]\omega^2 t_R^2-\left(\dfrac{\rho\nu^2 r^2\omega^2}{E_E}-1\right)}{\left[\left(1+\dfrac{E_1}{E_E}\right)-\dfrac{\rho\nu^2 r^2\omega^2}{E_E}\right]^2 \omega^2 t_R^2+\left(\dfrac{\rho\nu^2 r^2\omega^2}{E_E}-1\right)^2}.
\end{aligned}
\tag{3}
$$

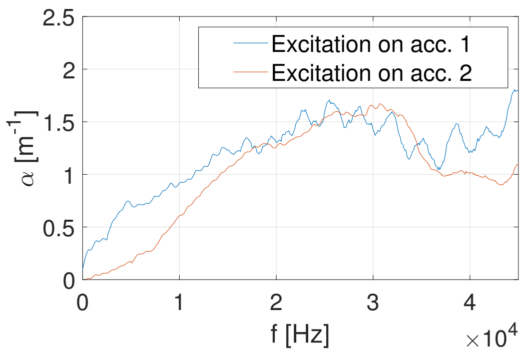
Symbols E_1 and t_R denote the modulus of elasticity of the spring and the relaxation time characterising the Maxwell branch in the Zener model, respectively.

When comparing the analytically obtained dispersion curves for all homogeneous materials, it was possible to achieve agreement within 20 to $50\,\mathrm{kHz}$. For example, the comparison for a rod made of PVC material is shown in Fig. 2a. For the attenuation curves, it was possible to achieve a much smaller agreement with the analytics (curves marked as Ahonsi), i.e., from 8 up to $25\,\mathrm{kHz}$, see Fig. 2b showing the attenuation of the PLA rod. The accuracy of the results, especially the attenuation, was influenced by the relatively low sampling rate of the signal. The transverse contraction of the rod in the relation (2) positively influenced the agreement of the results. In the case of a non-prismatic and heterogeneous rod, the comparison of the curves identified from the excitation at both ends of the rod was used to verify the results. In this way, it was possible to verify the dispersion curve up to $40\,\mathrm{kHz}$ for the non-prismatic rod (see Fig. 3a), and the attenuation curve only up to $35\,\mathrm{kHz}$ (see Fig. 3b). For the heterogeneous rod, dispersion and attenuation have been verified up to $18\,\mathrm{kHz}$ as shown in Fig. 3c and 3d.

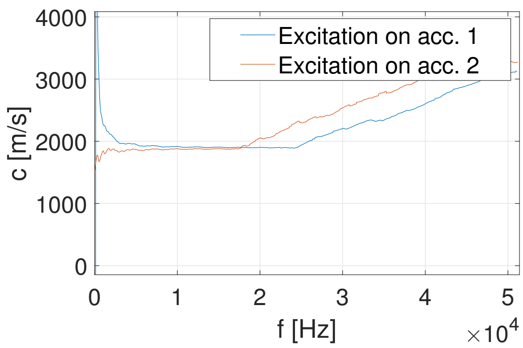
The identified dispersion and attenuation curves in Fig. 2 are similar to the curves presented in [1] and [2]. The phase velocity and the attenuation for a homogeneous viscoelastic rod increase with higher frequencies. For thicker rods, where the measured acceleration is more influenced by transverse contractions, it was shown that with increasing frequency, the wave speed decreases but the attenuation increases. The dispersion curve of homogeneous non-prismatic and heterogeneous non-prismatic rods is constant on the identified part, and the attenuation



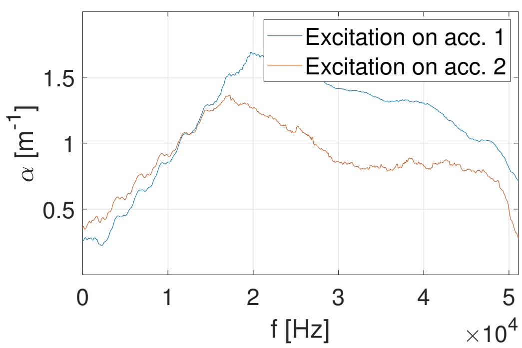
(a) Dispersion curve for non-prismatic rod



(b) Attenuation curve for non-prismatic rod



(c) Dispersion curve for heterogeneous rod



(d) Attenuation curve for heterogeneous rod

Fig. 3. Experimentally measured dispersion and attenuation curves for homogeneous non-prismatic and heterogeneous non-prismatic rod

curve is increasing (see Fig. 3). The maximum frequency to which these properties can be determined is mainly influenced by the weight of the accelerometers, the low sampling frequency and the shape of the excited cosine pulse.

In this work, a method of identifying the dispersion and attenuation properties of homogeneous prismatic as well as heterogeneous non-prismatic thin viscoelastic rods was described. Using this method, it was possible to determine these properties from the measured acceleration and verify them up to tens of kHz.

Acknowledgements

The publication was supported by the project SGS-2022-008 and by the grant GA 22-00863K.

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