COMPUTATIONAL MECHANICS 2023
38th conference with international participation
Srní
October 23 - 25, 2023

Time periodic solution of linear vibrating systems with time dependent stiffness using periodic collocation

J. Dupal[a]

[a]Department of Mechanics, Faculty of Applied Sciences, University of West Bohemia, Univerzitní 8, 301 00 Plzeň, Czech Republic

This paper deals with approach to solution of periodic response of systems having periodic time dependent stiffness. The methodology is prepared for systems with $n<\infty$ degrees of freedom (DOF) but we restrict to 1 DOF systems in this presentation. Let suppose that presented system can be described by equation

$$m\ddot{q}(t)+b\dot{q}(t)+\left[k_0-\mu\tilde{k}(t)\right]q(t)=f(t),\quad q(t)=q(t+T),\ \tilde{k}(t)=\tilde{k}(t+T),\ f(t)=f(t+T),\tag{1}$$

where all symbols are well known and used and it is not necessary to explain them. As the first stage of investigation let us build periodic Green function (PGF), which is a response of the system without time dependent stiffness $\tilde{k}(t)$ to Dirac train of impulses with period *T*. This PGF has form

$$H_T(t)=\frac{1}{T}\sum_{k=-N}^{N}L_k e_k(t),\quad T=\frac{2\pi}{\omega},\ e_k(t)=e^{ik\omega t},\quad L_k=\left(-k^2\omega^2 m+ik\omega b+k_0\right)^{-1}.\tag{2}$$

Solution can be written in form

$$q(t)=\mu\int_0^T H_T(t-s)\tilde{k}(s)q(s)ds+\int_0^T H_T(t-s)f(s)ds,\tag{3}$$

where

$$H_T(t)=\frac{1}{T}\sum_{k=-N}^{N}L_k e_k(t-s)=\frac{1}{T}\sum_{k=-N}^{N}L_k e^{ik\omega t}e^{-ik\omega s}=\frac{1}{T}\sum_{k=-N}^{N}L_k e_k(t)e_{-k}(s).\tag{4}$$

The time periodic dependent stiffness can be expressed by relations over whole time scale or over only one period respectively can write [1]

$$\tilde{k}(t)=\sum_{l=1}^{m}\sum_{j=-N}^{N}k_l\delta(t-t_l-jT),\quad \sum_{l=1}^{m}k_l\delta(t-t_l),\tag{5}$$

where *m* is number of time steps over one period and *N* is number of upper respected harmonic contribution. After substitution relations (4) and (5) into (3) and after some rearrangements and respecting relation

$$\int_{-\infty}^{\infty}g(t)\delta(t-t_0)dt=g(t_0),$$

we can rewrite the relation (3) in compact form

$$\mathbf{q}(t)=\frac{\mu}{T}\mathbf{E}^T(t)\mathbf{L}\hat{\mathbf{I}}\mathbf{E}(t)\mathbf{K}\mathbf{q}(t)+\mathbf{E}(t)\mathbf{L}\mathbf{f},\tag{6}$$

The second term was in detail subscribed in [2]. The remaining symbols present a form

$$\mathbf{e}(t)=\left[e_{-N}(t),e_{-N+1}(t),...,e_N(t)\right]^T,\quad \mathbf{f}=\left[f_{-N},f_{-N+1},...,f_N\right]^T, \tag{7}$$

$$\mathbf{E}^T(t)=\begin{bmatrix} e_{-N}(t_1), & e_{-N+1}(t_1), & \dots, & e_N(t_1) \\ e_{-N}(t_2), & e_{-N+1}(t_2), & \dots, & e_N(t_2) \\ \vdots & \vdots & \ddots & \vdots \\ e_{-N}(t_m), & e_{-N+1}(t_m), & \dots, & e_N(t_m) \end{bmatrix}=\begin{bmatrix} \mathbf{e}^T(t_1) \\ \mathbf{e}^T(t_2) \\ \vdots \\ \mathbf{e}^T(t_m) \end{bmatrix}\in\mathbf{C}^{m,2N+1}, \tag{8}$$

$$\mathbf{L}=\begin{bmatrix} L_{-N} & & & \\ & L_{-N+1} & & \\ & & \ddots & \\ & & & L_N \end{bmatrix}\in\mathbf{C}^{2N+1,2N+1},\ \hat{\mathbf{I}}=\begin{bmatrix} & & & 1 \\ & & 1 & \\ & \cdot^{\cdot^{\cdot}} & & \\ 1 & & & \end{bmatrix}\in\mathbf{R}^{2N+1,2N+1}, \tag{9}$$

$$\mathbf{q}(t)=\left[q(t_1),\quad q(t_2),\quad \dots,\quad q(t_m)\right]^T\in\mathbf{R}^{m,1}, \tag{10}$$

$$\mathbf{K}=\Delta t\begin{bmatrix} k_1 & & & \\ & k_2 & & \\ & & \ddots & \\ & & & k_m \end{bmatrix}=\Delta t\begin{bmatrix} \tilde{k}(t_1) & & & \\ & \tilde{k}(t_2) & & \\ & & \ddots & \\ & & & \tilde{k}(t_m) \end{bmatrix}=\mathbf{R}^{m,m}. \tag{11}$$

The individual terms of the vector correspond to Fourier coefficients whose can be expressed in form

$$f_k=\frac{1}{T}\int_0^T f(s)e_{-k}(s)ds,\qquad k=-N,\,-N+1,....,\,N\,.$$

The final expression of the response follows from (6) and has form

$$\mathbf{q}(t)=\left[\mathbf{I}-\frac{\mu}{T}\mathbf{E}^T(t)\mathbf{L}\hat{\mathbf{I}}\mathbf{E}(t)\mathbf{K}\right]\mathbf{E}^T(t)\mathbf{L}\mathbf{f}\,, \tag{12}$$

where $\mathbf{I}\in\mathbf{R}^{2N+1,2N+1}$ is identity matrix. The properties of the matrix in brackets are significant for stability assessment [3] and existence of total solution of (1). The details of stability assessment and existence of solution will be explained during the presentation.

Acknowledgement

This publication was supported by the project TAČR No. FW06010052.

Refrences

[1] Babitsky, V. I., Krupenin, V. L., Vibration of strongly nonlinear discontinuous systems, Springer, Berlin, 2001.

[2] Dupal, J., Zajíček, M., Analytical periodic solution and stability assessment of 1 DOF parametric systems with time varying stiffness. Applied Mathematics and Computation 243 (2014) 138-151.

[3] Dupal, J., Zajíček, M., Existence of analytical solution, stability assessment and periodic response of vibrating systems with time varying parameters, Applied and Computational Mechanics 14 (2) (2020) 123–144.