

## Two-mass model based vibration suppression feedback control method applied to standard servo control system

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### Abstract:

This paper describes one of the functional methods for two-mass system torsional vibrations suppression that are evoked due to torsion plasticity of kinematical chain links and large moment of inertia. This strategy belongs to the group of feedback methods. To its function, the method uses a mathematical model of mechanism to predict the actual speed of the kinematical chain working link. By means of that signal, the measured motor shaft actual speed and weightings product the required correction signal which is able to significantly damp the torsional vibrations of the kinematical chain end-link. The control structure was tested by simulation and then integrated into the existing cascade structure standard servo control unit Siemens Sinamics S120. The results of the simulations on the two-mass system mathematical model and the experiments on the real device demonstrated the ability of this method to effectively compensate the torsional vibrations of the mechanism end-link.

### INTRODUCTION

The multi-mass systems are widely represented in processing machines. The typical representatives are machine-tools, robots, turntables etc. The first mass is formed by the electric coupling stator-rotor and by rotor inertia. The elasticity of mechanical parts of the kinematical chain, together with their moment of inertia, forms the second and typically the rest of the masses. It is possible to characterize them by their natural frequency and dumping constant. In other words, such a mechanism can be expressed by the second or higher order system with complex poles. During the excitation of the multi-mass system in addition to the required motion there are also evoked torsional vibrations at the end of the mechanism which refers to theirs natural frequencies. This torsional vibration may seriously affect the servomechanism positioning accuracy, especially if the mechanism moves through the long elastic shaft with the masses of large inertia. Depending on the excitation function each mass may produce various amplitude of torsional vibration frequency. This mechanism end-link vibration can be hardly suppressed by the conventional cascade control structure. Usually, another control strategy must be used. There are lots of articles that are devoted to this topic. Various control strategies have been proposed [1], [2], [3], [4]. The typical way to apply those strategies is in programming the control structure into the signal processor. From the whole control structure of the servo controller there is typically used only the power section starting from the current control loop. But the producers are increasingly putting the pressure on substitution of those special solutions by using standard controllers. This is the reason why we investigated the problem to find the functional control

structure that is easily applicable into the standard servo control unit. This paper is focused on the vibration suppression strategy based on the mathematical model of the two-mass system.

### EXPERIMENTAL MODEL OF TWO-MASS SYSTEM

The experimental stand that enables to test the two-mass system has been developed at the Research Institute of Textile Machines in Liberec (VUTS, a.s.). Our department cooperates with this institute on the joint project that is focused on the application of electronic cams in processing machines. The main interest is devoted to the problems of substitution of stepping mechanisms like turntables, indexing gearboxes etc. by means of electronic cams. Such applications belong to the group of the two-mass system.

The principal chart of the experimental mechanism is shown in Fig.1. It consists of a synchronous servomotor with reduced speed by means of a preloaded backlash-free gearbox. For the measuring purposes, an external position sensor is connected to the end-link being formed by a flywheel.

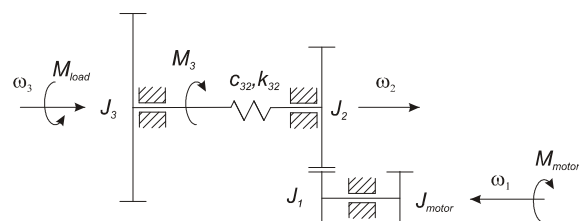


Fig. 1: Chart of mechanical coupling between two masses  
The control strategy doesn't need any signal from the external encoder. It serves only for measuring purposes and for easy verification of the correctness of the two-mass system mathematical model.

For the purposes of testing the electronic cams there were developed three displacement characteristics with the same stroke. They differ only in the definition function (polynomial, harmonic, parabolic). All of them consist of the dynamic and static part. The entire stroke is performed in the starting (dynamic) part. Here could be examined the positioning ability of the servomechanism. For the rest of the cycle there is held the reached position. This is the static part that serves for observing the torsional vibrations inducted by the second mass. Each cam excites the vibration with different amplitude depending on its derivatives. The most problematic cam in this regard seems to be the parabolic cam (see Fig. 2.). That's because of the step change of acceleration that refers to the impulse changes of the jerk. These abrupt changes in acceleration and jerk cause the torsional oscillations on the two-mass system with large amplitude.

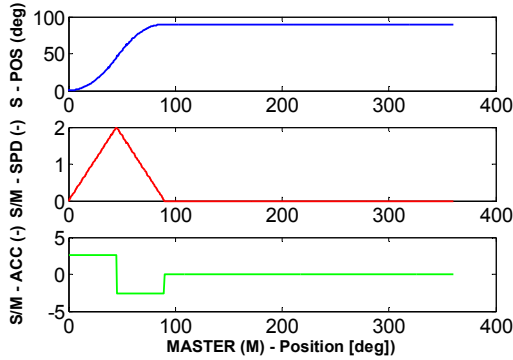


Fig. 2: Parabolic cam position profile

## EXPERIMENTAL MODEL OF TWO-MASS SYSTEM

As mentioned above, for the purposes of this vibration suppression method there is necessary to create an accurate model of the two-mass system.

The first mass of the system forms the electromagnetic compliance of the stator and the rotor. It can be represented by the D-Q model of the synchronous servomotor described in [5], [6]. This model is based on the separate controlling each component of current vector (torque-generating and field-generating). This kind of the model shows a great match with the real servomotor responses. But the simple model, which considers the current loop as ideal, also suits the purposes of our experiment. Assuming the higher-order speed of the current loop in comparison with the superior speed loop, this kind of simplification is possible [7]. The motor model is formed only by the rotor moment of inertia and the torque constant, see Fig. 5.

In order to set the load model correctly, there is necessary to know how to calculate and to measure the mechanical parameters of each member of the mechanism kinematical chain.

In our case the parameters were calculated by means of the finite element method. Their rightness could be

verified by measuring the natural frequency of the two-mass system. The parameters for setting up the mathematical model of the synchronous servomotor were measured by means of the servo-control unit functions. The required parameters for setting up the model of the two-mass system are summarized in Table 1.

Tab. 1: Parameters of two-mass system

<b>Mechanism parameters</b>	
Torsion stiffness	1000 [Nm/rad]
Flywheel moment of inertia	0,105525 [kgm <sup>2</sup> ]
Gearbox input moment of inertia	0,003935[kgm <sup>2</sup> ]
Gearbox output moment of inertia	0,000798[kgm <sup>2</sup> ]
Gearbox ratio	33
<b>Servomotor parameters</b>	
Lq inductance	9,9767 [mH]
Ld inductance	10,6695 [mH]
Stator and cables resistance	1,081 [Ω]
Voltage constant	144 [V]
Moment of inertia	0,0048 [kgm <sup>2</sup> ]
Pole pair number	4

In comparison with other links of the kinematical chain the gearbox elasticity is negligible. Therefore, the parameters such as stiffness and damping constants are taken as ideal. Under this assumption there is possible to involve the gearbox moment of inertia into the motor moment of inertia and to express the motor with the gearbox as the one-mass system. This kind of simplification cannot be done with the elastic shaft that connects the gearbox with the flywheel. That is because the flexible connection together with the flywheel creates the second mass of the system. The model of the two-mass system can be described with the following equation:

$$\begin{aligned}
 J_{Tot} &= J_{motor} + J_1 + \frac{J_2}{p^2} \\
 M_{Motor} &= J_{Tot} \frac{d\omega_1}{dt} + \frac{M_3}{p} \\
 M_3 &= c_{32} \left( \frac{\varphi_1}{p} - \varphi_3 \right) + k_{32} \left( \frac{\omega_1}{p} - \omega_3 \right) \quad (1) \\
 &= \left( \frac{\varphi_1}{p} - \varphi_3 \right) \left( c_{32} + \frac{d}{dt} k_{32} \right) = \\
 &= J_3 \frac{d\omega_3}{dt} + M_{Load}
 \end{aligned}$$

Where  $c_{32}$  is constant of elasticity and  $k_{32}$  is constant of viscous damping.  $\varphi_1$  and  $\varphi_3$  are torsion angles.  $J_{Tot}$  is the sum of all moments of inertia described above and  $J_3$  the flywheel moment of inertia.  $\omega_1$  represents the angular velocity of the motor and  $\omega_3$  the velocity of the flywheel. Using the Laplace transformation there is possible to transform into the s-domain and to illustrate as the block diagram shown in Fig. 3.

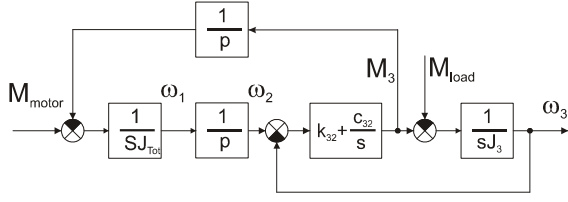


Fig. 3: Block diagram of two-mass system  
If we express the transfer function from  $\omega_1$  to  $\omega_3$ , we can get the formula for calculating the oscillation of the mechanism, so called anti-resonance frequency.

$$\frac{\omega_3}{\omega_1} = \frac{1}{p} \cdot \frac{k_{32}s + 1}{\frac{s^2}{\Omega_L^2} + \frac{2\zeta_L}{\Omega_L}s + 1} \quad (2)$$

Where  $\Omega_L$  is the locked motor frequency (natural frequency of the mechanism) and  $\zeta_L$  is the damping ratio of the mechanism.

$$\Omega_L = \sqrt{\frac{c_{32}}{J_3}} \quad \zeta_L = \frac{k_{32}}{2} \cdot \sqrt{\frac{1}{J_3 \cdot c_{32}}} \quad (3)$$

The transfer function from  $\varphi_1$  to  $M_1$  gives the relation for the natural frequency of the entire system, so called resonance frequency.

$$\begin{aligned} \frac{\varphi_1}{M_1} &= \frac{p^2}{s^2} \cdot \frac{J_3 s^2 + k_{32}s + c_{32}}{(p^2 J_1 + J_3) \cdot \left( \frac{p^2 J_1 J_3}{p^2 J_1 + J_3} s^2 + k_{32}s + c_{32} \right)} = \\ &= \frac{p^2}{s^2} \cdot \frac{\frac{s^2}{\Omega_L^2} + \frac{2\zeta_L}{\Omega_L}s + 1}{(p^2 J_1 + J_3) \left( \frac{s^2}{\Omega_{LM}^2} + \frac{2\zeta_{LM}}{\Omega_{LM}}s + 1 \right)} \quad (4) \end{aligned}$$

Where  $\Omega_{LM}$  is the resonance frequency and  $\zeta_L$  is the damping ratio of the two-mass system.

$$\begin{aligned} \Omega_{LM} &= \sqrt{\frac{(p^2 J_1 + J_3)c_{32}}{p^2 J_1 J_3}} \\ \zeta_L &= \frac{k_{32}}{2} \cdot \sqrt{\frac{(p^2 J_1 + J_3)}{p^2 J_1 J_3 c_{32}}} \quad (5) \end{aligned}$$

Fig. 4 shows the frequency characteristics of two transfer functions. The blue line shows the transfer function from  $M_1$  to  $\omega_1$  and the red line the transfer from  $M_1$  to  $\omega_3$ . The arrows highlight two important points, resonant and anti-resonant frequencies, which refer to the natural frequencies (3) and (5).

Another simplification could be done. The damping constant of such systems is usually very low. That's why it is possible to neglect the damping completely. As it is shown on the simulation and experiment results, this kind of simplification has not any significant effects to the system stability.

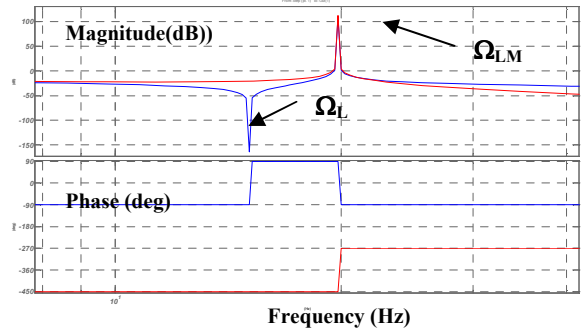


Fig. 4: Frequency characteristics of transfer functions from  $M_1$  to  $\omega_1$  and from  $M_1$  to  $\omega_3$

## SUPPRESSION METHOD BASED ON LOAD MODEL

The principals of the suppression method based on the load model are derived from the method based on the external end-link position sensor which has been proposed earlier [8]. The principle of this method is based on comparing weighted velocity signals from the external encoder located at the end-link of the mechanism and the internal (motor) encoder. The advantage of this method is that there is a high resistance to the changes of the system parameters. However, not every machine allows the application of the direct measuring of the end-link position.

That is why the effort is directed to the external encoder signal substitution. The idea is to predicate that signal by means of the mathematical model of the mechanism.

At the top of the block diagram in Fig. 5 there is shown the mathematical model of the system (interaction between speed controlled motor and load) and underneath the vibration suppression structure (green illustrated).

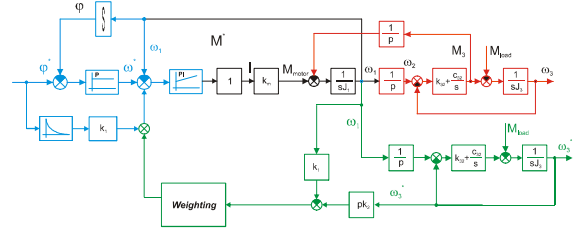


Fig. 5: Block diagram of vibration suppression control structure based on load model.

The structure is based on the mathematical model of the mechanism that serves for the external encoder signal substitution. The signal from the internal encoder excites the load model structure that predicts the actual speed at the end-link of the mechanism. As shown in Fig. 5, the correction signal is obtained from the difference of the weighted actual motor speed and the predicted end-link speed. If the weighting factors of both signals are set identically, we get the actual torsional vibrations from the difference. But in practice, it has proved useful to set them differently. The result from the subtraction has been consequently adjusted by the output weighting

factor to the required scale and set up into the correct phase by means of time delay elements. After the final adjustment the signals can be connected to the control process. A usual connection point there is the speed or the current feed-forward input.

## SIMULATION AND EXPERIMENT RESULTS

In order to get the comparable results from the simulation and experimental measurements on the real system it was necessary to set up the parameters of the control structure correctly to get the similar reactions to the same excitation function from both systems. The parameters of the conventional cascade control structure controllers were firstly set up by means of the auto-tuning functions supported by the servo control unit and consequently refined by means of the pole placement method. After the two-mass models described above had been successfully verified with the real system, the first suppression vibration structure experiments could start.

### Simulations in MATLAB

The first simulation was made on the system with disabled compensation structure. As mentioned above, the parabolic cam has the worst possible option for the torsional vibration excitation. That's why all the presented resulting signals are excited just with this type of cam. Fig. 6 shows the response of the two-mass system to the parabolic cam excitation (red line). For the better recognition of the torsional vibration there is shown in the same figure also the position setpoint signal (blue line) as the end-link position signal.

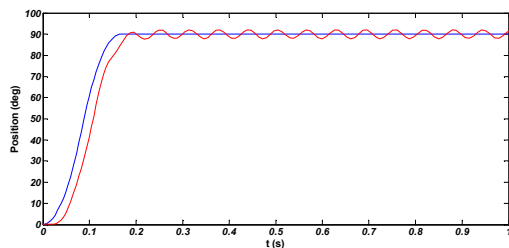


Fig. 6: Response to parabolic cam excitation of two-mass system with disconnected suppression.

The commissioning of the vibration suppression structure was firstly made experimentally according to the tracking of the end-link position and searching the weight constants of both subtracted signals. Their results were consequently refined with the results from the pole placement method.

The fine tuning of the suppression structure helped us to reach the satisfactory results of the end-link actual position. As in the previous figure, in Fig.7 there are shown two characteristics in the same diagram, the setpoint and the actual position of the end-link. In Fig. 8 there is shown the shape of the correction signal in time. At the first glance it can be seen that the weightings have different values. In the transition

part the correction signal has the shape like inverse shape of the speed (see Fig.2). This shape has an influence on the accuracy of the end-link positioning already in the transition phase. Its influence in the regulatory process slightly adjusts the shape of the desired displacement characteristic and possibly avoids the abrupt changes in acceleration and jerk. The estimated torsional vibration signal operates in phase opposite with the vibrations coming from the end-link. This helps a great to compensate the torsional oscillations of the kinematics chain end-link.

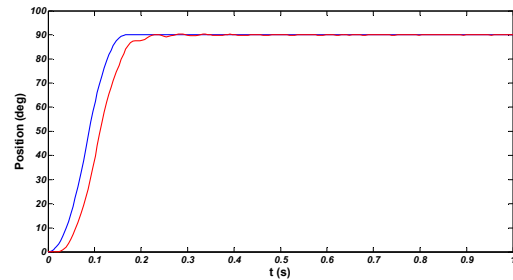


Fig. 7: Response to parabolic cam excitation of two-mass system with connected vibration suppression circuit.

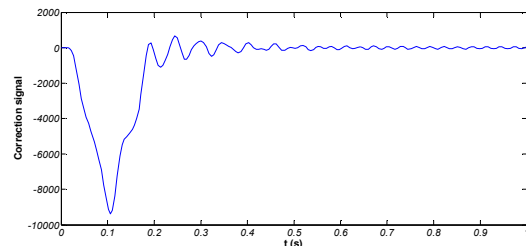


Fig. 8: Correction signal after subtraction of weighted signals of measured and calculated velocity.

### Experiments on Sinamics S120 servo control unit

In the experiment there was used the servo control unit Sinamics S120 for controlling the motion of the synchronous servomotor. As a master unit for controlling the synchronous operation, e.g. cams, there is the motion controller Simotion C240. In contrast to the servo systems of other producers, Siemens offers the programmable memory and the control structure variability. These options give the end-user possibility to use any signal from the list of readable parameters, to create the control structure in the programming environment (DCC Chart) similar to Matlab-Simulink and to lead back into the control structure by means of the write access parameters. In this way it is possible to influence the standard control structure and to implement our vibration suppression methods into the standard control unit. The same experiment as in simulations was made with the real system on the experimental stand. Because the function blocs are similar, the conversion to the DCC chart does not present any problems. The only disadvantage of the DCC chart there is minimal execution cycle time of 1ms. That's why it was better to use the slower speed feed-forward input rather than the faster torque feed-forward input.

After the required signals had been connected into the structure and the program set up into the control unit execution system, the first experiments could start. The first measurement was performed with disabled compensation structure using the external encoder. In Fig. 9 is visible the torsional vibration of the end-link during the excitation by the parabolic cam. By means of the oscillation frequency measurement and its comparison with the estimated value were verified the two-mass model parameters.

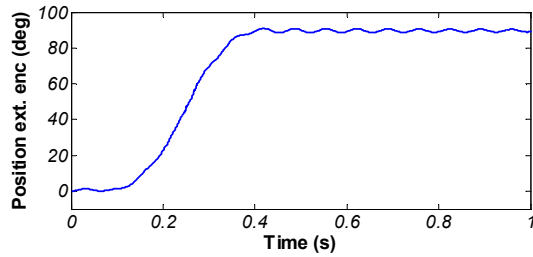


Fig. 9: Response to parabolic cam excitation of real two-mass system with disconnected suppression.

For the proper function of the proposed control structure there is necessary to set up suitable parameters of individual controllers from the classics cascade regulation structure as well as the compensation structure weighting factors. For searching the optimal parameters was used pole placement method. After that was also necessary to tune the phase of the correction signal in order to get inverse signal to the torsional vibration. All the parameters are summarized in the following table.

Tab. 2: Parameters of the control structure

<b>Cascade structure</b>	
I - controller gain	28 [V/A]
I - controller integration time	2 [ms]
S - controller gain	4 [Nm*s/rad]
S - controller integration time	17 [ms]
P - controller gain	150 [1/s]
<b>Compensation structure in DCC</b>	
Internal encoder weighting	0,0033 [-]
Estimated speed signal weighting	1 [-]
Output weighting	$3e^{-4}$ [-]
Output time delay	10 [ms]

The efficiency of proposed method is shown on the external position encoder signal in Fig.10. In Fig. 11 there is shown the shape of the correction signal.

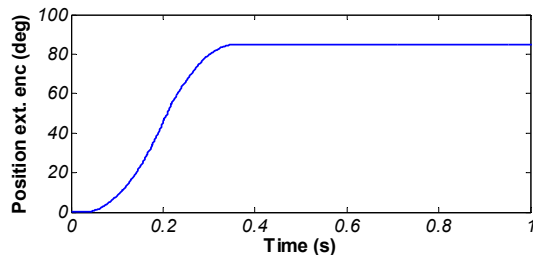


Fig. 10: Response to parabolic cam excitation of real two-mass system with connected vibration suppression circuit

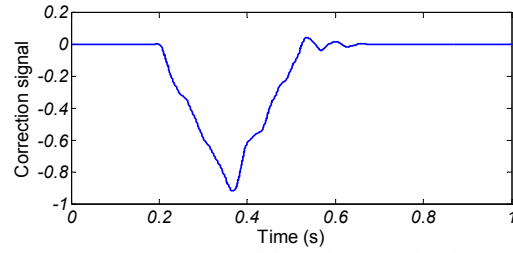


Fig. 11: Correction signal of the real system after the signal correction

## CONCLUSION

A method based on the load model for suppression of the two-mass system end-link vibration is described in this article. The results from the simulation as well as of the experiments on the real system demonstrate its functionality. The advantage of this method is that it doesn't need any additional position sensors. For full functionality the cascade structure uses only the input encoder speed signal.

On the other hand, in comparison with other methods based on the external encoder sensing, this method has a great disadvantage. It is not robust enough against changing the load parameters. For the proper function of this method, the mathematical model of the mechanism must be adjusted precisely. This model estimates the end-link speed from the actual motor speed signal. If the parameters of the model differ from the actual mechanism parameters, the model products another frequency than the real system and the suppression of vibrations is not efficient enough or doesn't work at all.

That's why we are currently concentrated on the self commissioning methods which would be able to correct the model parameters in real time and in this way to react to the system parameter changes.

## ACKNOWLEDGMENT

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