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Application of Kalman Filter to oversampled data from Global Position System for Flight Path Reconstruction

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Abstract:
Many applications exist that exploit Global Positioning System (GPS) as a source of position and velocity data. A main problem of the GPS data is its inconsistency on higher sampling frequencies. The usage of enhanced Kalman Filter usage to fill the gaps between the GPS data samples is presented. The presented Kalman filter is enhanced to contain the velocity data on the output. The output velocity data meet the condition of integration and differentiation between the values of the position and the velocity.

INTRODUCTION

Many technologies based on Global Position System (GPS) exist. They use the GPS as a source of position and velocity with respect to the Earth. However, GPS has relatively fairly low sampling frequency. Therefore, it is needed to use sophisticated filtering algorithms to fill the gaps between the GPS data samples.

The measured data from a flight data acquisition system usually contains the GPS position and velocity with respect to the Earth. It is important to filter this raw measured data to enhance its information consistency.

This filtration step is, for example, used in the Flight Path Reconstruction (FPR). FPR is a complex process intended for enhancement of information value of the measured flight data. This process uses kinematic equation of motion for description of the relations between variables measured during the flight. One of the best–known algorithms for the FPR is the Kalman Filter.

It is not possible to use rolling median or mean filters because these filters significantly increase the delay between input and output. This delay is in matter of several samples.

Three main algorithms to filter nonlinear differential–equation–based systems are available. Gross et al. described in his summarizing article [1] Extended Kalman Filter, Unscented Kalman Filter, and Particle Filter. By his experiments, it is clear that Extended Kalman Filter has the biggest ratio between precision and computational cost.

Particle Filter, unlike the Extended Kalman Filter, uses a random number generator to simulate the noise in the measurement. This approach simplifies the assumptions but the computational cost is rising rapidly and the precision rising slowly [1].

Unscented Kalman Filter (UKF) is a modification of the Kalman Filter [4]. UKF avoids the use of the Jacobian matrix of uncertainty, which is a major problem for EKF. UKF realizes modeling of uncertainty by manipulation with sigma points [2]. This approach slightly increases the computational cost in the comparison with EKF.

This paper presents the enhancement of the Kalman Filter. The enhanced Kalman Filter enables a velocity data output based on the numerical differentiation. This enhancement of the Kalman Filter could be helpful to reduce artifacts in real–world applications.

This filtering step is a prerequisite for the visualization of the position in the modern flight deck instruments onboard of the general aviation aircraft. This filtered data could be the source for a later processing or could be used in the identification of flight parameters.

This paper is organized as follows. The following section shows the structure of general system and its discretization. Construction of the Kalman Filter is described in next section. The model of our system is described in following section. Next section presents the enhanced Kalman Filter. The tests results are presented in section Results of tests.

MODEL OF GENERAL SYSTEM

For further thoughts, we have to describe general system. The definition of continuous system is taken from [3]:

$$\dot{\mathbf{x}} = \mathbf{F}\mathbf{x} + \mathbf{D}\mathbf{u} + \mathbf{w} \quad (1)$$

$$\mathbf{z} = \mathbf{H}\mathbf{x} + \mathbf{v} \quad (2)$$

where $\dot{\mathbf{x}}$ is a vector of a time derivative of the system state, $\mathbf{z}$ is a vector of the measured variables, $\mathbf{F}$, $\mathbf{D}$, and $\mathbf{H}$ are matrices of coefficients, $\mathbf{u}$ is a vector of system inputs, $\mathbf{x}$ is a vector of the system state, $\mathbf{w}$ is a vector of a process noise and $\mathbf{v}$ is a vector of a measurement noise. These equations could be described by schema in Fig. 1:

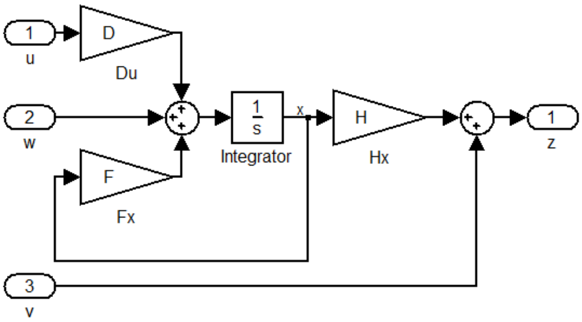

Fig. 1: Block diagram describing continuous system.

After discretization, it is possible to use discrete linear equations to describe the system [5]:

$$\mathbf{x}(k)=\mathbf{A}\mathbf{x}(k-1)+\mathbf{B}\mathbf{u}(k-1)+\mathbf{w}(k-1) \quad (3)$$

$$\mathbf{z}(k)=\mathbf{H}\mathbf{x}(k)+\mathbf{v}(k) \quad (4)$$

where $\mathbf{A}$, $\mathbf{B}$, and $\mathbf{H}$ are matrices of coefficients.
These equations could be described by a block diagram shown in Fig. 2:

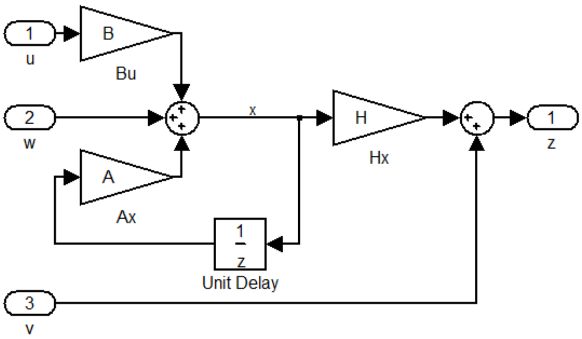

Fig. 2: Block diagram describing discrete system.

$\mathbf{A}$, $\mathbf{B}$ after the expansion in Taylor series are equal to [3]:

$$\mathbf{A}=e^{\mathbf{F}\Delta t}\approx\mathbf{I}+\mathbf{F}\Delta t+\mathbf{F}^2\frac{\Delta t^2}{2!}+\dots \quad (5)$$

$$\mathbf{B}=\mathbf{D}\int_0^{\Delta t}e^{F\tau}d\tau\approx\mathbf{D}\Delta t+\mathbf{D}\mathbf{F}\frac{\Delta t^2}{2!}+\dots \quad (6)$$

where $\mathbf{I}$ is an identity matrix and τ is the integration step.

KALMAN FILTER

The Kalman Filter gives us opportunity to increase precision of state of the system by using input data and redundant measurements. Kalman Filter uses a discrete system. The Kalman Filter consists of two parts: a prediction part and a correction part. The prediction part is defined by the following equations:

$$\tilde{\mathbf{x}}(k+1)=\mathbf{A}\hat{\mathbf{x}}(k)+\mathbf{B}\mathbf{u}(k) \quad (7)$$

$$\tilde{\mathbf{P}}(k+1)=\mathbf{A}\hat{\mathbf{P}}(k)\mathbf{A}^T+\mathbf{Q} \quad (8)$$

where $\tilde{\mathbf{x}}$ is an estimated value of $\mathbf{x}$, $\hat{\mathbf{x}}$ is the corrected estimation of value $\mathbf{x}$, $\tilde{\mathbf{P}}$ is an estimated error covariance matrix, $\hat{\mathbf{P}}$ is the corrected estimation of the error covariance matrix, $\mathbf{Q}$ is a covariance matrix of the process errors and $\mathbf{A}^T$ is transposition of the matrix $\mathbf{A}$.
The initial conditions are:

$$\hat{\mathbf{x}}(0)=\mathbf{x}_0 \quad (9)$$

$$\hat{\mathbf{P}}(0)=\mathbf{P}_0 \quad (10)$$

The Extended Kalman Filter uses the equations (7) and (8) with just first elements of Taylor series. This step simplifies the computing in the nonlinear Kalman Filtering.
The correction part is defined by following equations:

$$\mathbf{K}(k)=\tilde{\mathbf{P}}(k)\mathbf{H}^T\left[\mathbf{H}\tilde{\mathbf{P}}(k)\mathbf{H}^T+\mathbf{R}\right]^{-1} \quad (11)$$

$$\hat{\mathbf{x}}(k)=\tilde{\mathbf{x}}(k)+\mathbf{K}(k)\left\{\mathbf{z}(k)-\mathbf{H}\tilde{\mathbf{x}}(k)\right\} \quad (12)$$

$$\hat{\mathbf{P}}(k)=\left\{\mathbf{I}-\mathbf{K}(k)\mathbf{H}\right\}\tilde{\mathbf{P}}(k) \quad (13)$$

where $\mathbf{K}$ is a Kalman gain matrix, $\mathbf{H}^T$ is a transposition of the matrix $\mathbf{H}$ and $\mathbf{R}$ is a covariance matrix of measurement errors.
Kalman gain matrix computation is presented in Fig. 3.

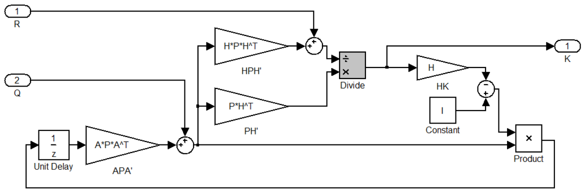

Fig. 3: Block diagram describing computing of Kalman gain matrix

The “divide” component, shown in grey, is one of the most time consuming operation in Kalman Filter.
The UKF uses slightly different equations. See [4] for further details.

THE MODEL OF THE PROPOSED SYSTEM

The proposed system could be simplified to a point near the Earth surface with the position and the velocity derived with respect to the Earth’s surface. The input data describing the system are based on the GPS measurements:

$$\mathbf{z}=\begin{bmatrix}\ell & \lambda & h\end{bmatrix}^T \quad (14)$$

$$\mathbf{u}=\begin{bmatrix}v_\ell & v_\lambda & v_h\end{bmatrix}^T \quad (15)$$

where $\mathbf{z}$ is a position vector, ℓ is the terrestrial latitude, λ is the terrestrial longitude, h is the geodetic height over the geoid, $\mathbf{u}$ is a velocity vector, v_ℓ is the terrestrial latitude speed, v_λ is the terrestrial longitude speed and v_h is the vertical geodetic speed over the geoid.

The GPS data representation is based on a EGM96 geoid [6], which is de facto a WGS84 ellipsoid with a correction grid. The WGS84 ellipsoid has been used for simplification of our estimations. The difference between the WGS84 and the EGM96 is less than 107 m [6], which is significantly less than semi–major axis of the Earth (a). The WGS84 is used for position transformation from the polar (terrestrial latitude and longitude) coordinates to the Cartesian coordinates and for the definition of the radius of curvature in the prime vertical N is represented by the equation [6]:

$$N = \frac{a}{\sqrt{1-e^2\sin(\ell)^2}} \tag{16}$$

where a is the semi–major axis of the Earth (by the definition of WGS84, it is equal to 6,378,137.0 m) and e is the first eccentricity of the Earth which is defined by following equation [6]:

$$e = \sqrt{\frac{2}{1/f} - \frac{1}{(1/f)^2}} \tag{17}$$

where $1/f$ is a flattening (it is equal to 298.257223563 by the definition of WGS84).

The continuous model of the system can be described by a following set of differential equations:

$$\dot{\ell} = v_\ell(a+h) \tag{18}$$

$$\dot{\lambda} = v_\lambda(N+h) \tag{19}$$

$$\dot{h} = v_h \tag{20}$$

where $\dot{\ell}$ is a time derivative of the terrestrial latitude, $\dot{\lambda}$ is a time derivative of the terrestrial longitude, $\dot{h}$ is a time derivative of the geodetic height over the geoid.

Using continuous system equations (1) and (2), we can rewrite equations (18), (19) and (20) to form, where $\mathbf{F}$ matrix is zero matrix, $\mathbf{H}$ matrix is identity matrix and $\mathbf{D}$ are equal to:

$$\mathbf{D} = \begin{bmatrix} a+h & N+h & 1 \end{bmatrix}^T \tag{21}$$

Using discretization by equations (5) and (6), it is possible to express EKF matrices $\mathbf{A}$ and $\mathbf{B}$ much simpler and allows us to express them as:

$$\mathbf{A} = \mathbf{I} \tag{22}$$

$$\mathbf{B} = \mathbf{D}\Delta t \tag{23}$$

This simplification changes the Kalman Filter to a form, which is equal to the Extended Kalman Filter.

ENHANCED KALMAN FILTER

The Extended Kalman Filter uses the velocity as input variable. The output of the Extended Kalman Filter is the position vector. For our purposes, it is necessary to filter velocity as well. The numerical differentiation can produce the velocity data on the output. This solution could be used in the real–time applications. For these purposes, the usage of a causality system is needed. One of the numerical differentiation methods compliant with the causal system is the finite backward differentiation. It is described by the following equation [7]:

$$\dot{\mathbf{z}}'(k) = \frac{\mathbf{z}'(k) - \mathbf{z}'(k-1)}{\Delta t} \tag{24}$$

The equation (24), in combination with the equations (18), (19), and (20) could be used for the enhancement of the Extended Kalman Filter to reveal the velocity data from the filtered position data.

This enhancement of the Extended Kalman Filter can be described by the following set of equations:

$$v_\ell = \frac{\ell(k) - \ell(k-1)}{\Delta t(a+h)} \tag{25}$$

$$v_\lambda = \frac{\lambda(k) - \lambda(k-1)}{\Delta t(N+h)} \tag{26}$$

$$v_h = \frac{h(k) - h(k-1)}{\Delta t} \tag{27}$$

This enhancement of the Extended Kalman Filter is shown in grey color in Fig. 4:

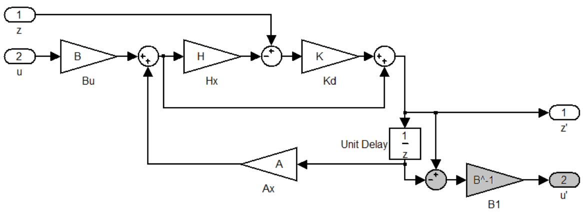

Fig. 4: Block diagram representation of the eKF. The grayed part is the added enhancement to the EKF.

After application of the enhanced Kalman Filter (eKF), the measured data fulfills the condition of the differentiation and integration, e.g. $\mathbf{v} = \dot{\mathbf{p}}$ and $\mathbf{p} \approx \int \mathbf{v}\,dt$. The using of position data and internal delay in Extended Kalman Filter to realize numerical differentiation ensures this condition. This differentiation increases computational cost minimally (by less than 20%).

RESULTS OF TESTS

Firstly, the enhanced Kalman Filter has been tested on a generated signal with amplitude equal to 1.0 and frequency equal to 0.1 rad/s. The derivative of this signal is used as a velocity data and the generated signal as a position data. This signals are sampled on frequency 1 Hz and later oversampled to 5 Hz. Results of such sampling are showed in Fig. 5:

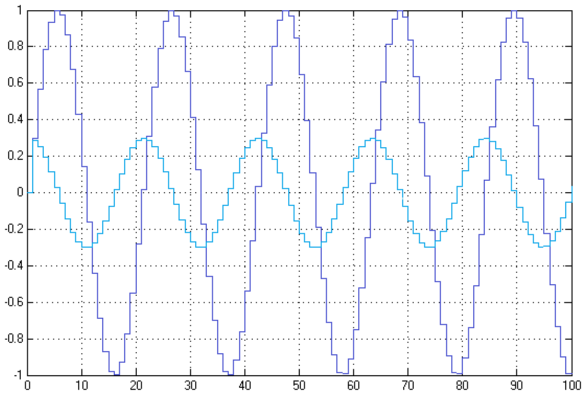

Fig. 5: Raw generated and oversampled data

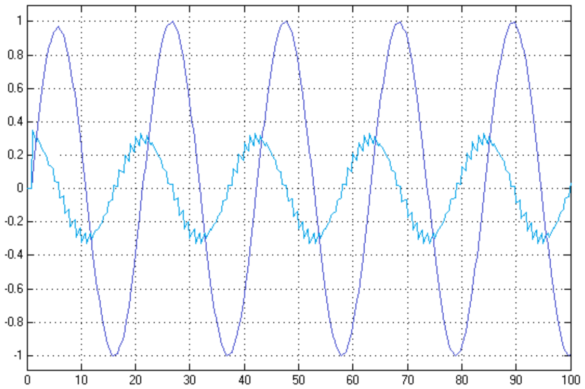

Fig. 6: Generated data after eKF

In Fig. 5 and Fig. 6, the dark line represents the position. The light line represents the velocity, which is the position time derivative. It is possible to see a significant improvement of the position data quality and serrate deformation of velocity data in Fig. 6. This serrate deformation is caused by reconstruction of the oversampled data. This result fulfills the condition of the differentiation and integration, which is the important quality for the further processing.
The enhanced Kalman Filter has been also tested on samples of the measured data.

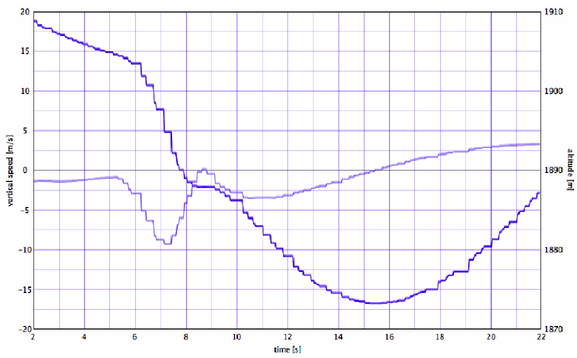

Fig. 7: Raw measured data.

In Fig. 7 and Fig. 8, the bold line represents the height above the mean sea level (altitude). The thin line represents the vertical speed, which is an altitude (position) time derivative. It is possible to see a significant improvement of the position data smoothness and very small deformation of velocity data in Fig. 8.

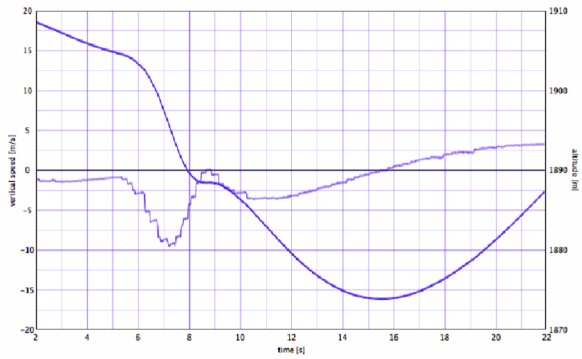

Fig. 8: Measured data after eKF.

To decrease deformation, it is possible to use a dynamic change of the covariance matrix $\mathbf{Q}$ in order to reflect the frequency of updating of the measured samples from the GPS. The GPS has much lower frequency of updating with comparison to frequency of output data.

CONCLUSION

In this paper, the enhancement of the Extended Kalman Filter is proposed to derive the velocity data from the filtered position data. The major advantage of the presented solution is the consistency between the position data and the velocity data so that data fulfill the condition of the differentiation and integration.
Proposed enhanced Kalman Filter could be used for the enhancement of the consistency and the precision of the GPS data sampled on the higher frequencies. The presented approach could be used as an out-of-the-box solution. The filtered GPS data can then be used as an input for visualization in MEDS (Multifunction Electronic Display Subsystem) with higher frequency data for the smoother animation; or can decrease the lags of FMS (Flight Management System).
For the purposes of the performance evaluation, it is necessary to test the algorithm in real–time applications. For the future research, it is envisioned to measure the performance and precision changes from a dynamic extension of the Extended Kalman Filter. This dynamic extension could decrease the deformation of the velocity data. The deformation of velocity intervention is the biggest disadvantage of presented algorithm.

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