A Variational Representation for Efficient Noisy Segmentation

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ABSTRACT

This paper focuses on a novel technique *VRENS*, (Variational Representation for Efficient Noisy Segmentation) to segment universal images (i.e. containg also textures) under noise. It is based on a hierarchical representation obtained by a combination of the classical *weak membrane* and a simpler *region competition* method. *VRENS* seems to be an interesting algorithm because of its robustness to additive gaussian noise along with a low computational cost.

Keywords: Image Segmentation, Variational Models, Textures

1 INTRODUCTION

Image segmentation is fundamental in various fields and a large number of techniques has been proposed during the past years. It consists in splitting an image in uniform regions in agreement with the human perception [Berge93, Lovel92]. In the middle of eighty, an interesting approach to image segmentation based on a variational formulation, has been proposed [Blake87]. In fact, it has shown some interesting characteristics, such as multi scale detection and, mainly, selective smoothing, i.e. the elimination of noise with a preservation of the discontinuities representing information. The promising results obtained by such an approach on edge detection, led the scientific community to extend it to textured images. For instance, in [Zhu96] an elegant formulation has been proposed, where a good segmentation can be obtained minimizing a generalized Bayes/Minimum Description Length criterion using the variational principle. This framework, called *Region Competition*, allows to obtain a minimum, using the competition among different regions looking at their pixels' statistics. In [Koepf94], Koepfler, Lopez and Morel propose a very fast algorithm with a pyramidal architecture, able to compute a hierarchy of segmentations. Lee, Mumford and Yuille in [Lee92] present a combination of the classical variational approach with a Gabor-wavelet based representation, that gives good quality results but resulting very time expensive.

In a previous paper [Roman01], an alternative approach to extend the variational formulation to segment images under noise containing textures has been proposed. Strictly speaking, the hypothesis (see [Jain86] p. 268 and [Tebou98]), used also in this paper, that $f = gu+\eta$ has been made; f is the noisy image, u the noiseless one and g and η are two kinds of distortion. We assume that $g = Id$, where Id is the identity operator so that we have only additive noise. We assume that the only gray level mean is not able to segment a very general image, where there may be textures too. In other words, there are many pratical situations where a given pictorial scene is composed by some textured parts. In this sense, *EVRIST* (Efficient Variational Representation Based Image Segmentation Technique) is able to achieve good results [Roman01]. Nonetheless this model, that will be explained more in detail later, presents some drawbacks: it is not able to segment images where the components have a generic shape since Split and Merge that uses a quadtree partitioning,

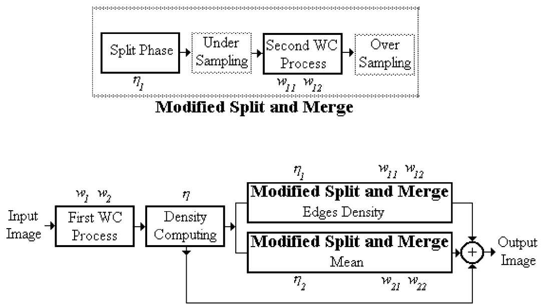

Figure 1: *EVRIST*'s block scheme with its thresholds.

deals with small area regions where the probability of a good detection is low under noise. This led us to propose *VRENS* (Variational Representation for Efficient Noisy Segmentation), where the model is based on detecting the edges as "contact point" between two test regions, like in [Zhu96]. Our model doesn't use active contours, is very fast and eliminates the "seeds'" problem typical of the Zhu-Yuille model. The obtained results seem to be interesting since comparable to other existing techniques, on some smooth test images and better for textures under noise with a low computational effort.

2 FROM *EVRIST* TO *VRENS*

EVRIST is based on the following colored characteristic function $\tilde{\chi} : \Omega \to R$:

$$\tilde{\chi}(x,y) = \begin{cases} u(x,y) & \text{if } (x,y) \text{ is an edge point} \\ 0 & \text{otherwise.} \end{cases} \tag{1}$$

where both the function u and the edges points are computed by means of a classical weak membrane or weak continuity (WC) process[Mumfo89, Blake87]. $\tilde{\chi}(\Omega)$ is a hierarchical representation (still an image) of the input image. Starting from it, a Modified Split and Merge (MSM) is used to find the input image components, considering the local mean and the edges density (i.e. the textures coarseness, similarly to [Rosen75]) as similarity criteria. In particular, while a classical Split phase is performed on the image obtained by $\tilde{\chi}$, the Merge phase is substituted by a second WC process to obtain the final result. The block scheme in Fig. 1 shows the phases of the technique.

It is worth spending some words about some aspects of *EVRIST*. The choice of considering two only features for the segmentation has been made taking into account both computational time and robustness to the noise. More complicate features may fail in these conditions and do not constitute the object of this paper (for a good and recent review see [Rande99]). Moreover, *edge density* shows a high robustness to the noise as explained in Appendix A, yelding a right segmentation up to 90% of gaussian zero-mean noise on many images.

Though the good achieved results, there are some drawbacks using MSM. The reason stems in the fact that it (as matter of fact also classical Split and Merge has the same behavior) shows some limits for non regular shapes under noise, i.e. shapes that are not squared regions or compositions of them: this limit is true only under noise. Moreover, small windows lead to wrong results under high amplitude noise, since the local information in the image is missed. The limit of detection (which is inversely tied to localization) takes into account the *uncertainty principle* [Wilso84]. The problem is not new at all and has been dealt with, with some differences, in [Zhu96] along with another problem. We know that for high amplitude noise we have, as much as possible, to avoid little regions, but what must be the shape of two competing regions? Zhu and Yuille propose "elliptical windows with their major axes parallel to the boundary" ([Zhu96] pag. 893). This choice is due to some requirements for the functional they use. Here, we use the following way. We know that a given region has to be wide enough. Thus, we have to maximize its area. On the other hand, we have to avoid interferences due to other regions, i.e. we want that each test region is completely included inside one and only one homogeneous component of the image to be segmented. Starting from the hypotheses that i) we don't know *a priori* the shape of each component of the image, ii) we must minimize the contact region between the two test regions for getting accurate edges, the simplest shape for the test regions is a circle. For computational reasons we will adopt the diamond (see Fig. 2). In other words, the point that represents the common vertex of the two competing regions will be an edge point if the homogeneity criteria are not satisfied for the two regions themselves. This choice leads us to avoid the problem of *seeds* as in the region competition model where we must know the number of the uniform regions (and their disposition) inside the input image. Summing up, in order to design *VRENS* we preserve the hierarchical representation in 4) which has revealed very efficacious substituting MSM by the technique previously mentioned. In the next section

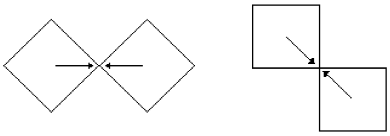

Figure 2: Competing regions have an only common point as candidate edge.

Image	*% noise*	w_1	w_2	η_1	η_2	ρ
Fig. 5	10%	2	.05	1	6.5	9
Fig. 5	30%	.01	.01	1	7	13
Fig. 6	10%	2	1	.3	20	31
Fig. 6	90%	.5	.5	.8	9.3	31

Table 1: Thresholds' values for some simulations. w_1 and w_2 are relative to the first *weak membrane* (scale level and noise sensitivity), η_1 and η_2 are relative to the similarity for respectively edges density and mean, while ρ is the diagonal value of the diamonds.

we will show some results and will outline some aspects of this combination.

3 EXPERIMENTAL RESULTS

VRENS has been tested on many images. Nonetheless the results we show in this section are oriented to outline both potentialities and limits of the proposed model. The first example we use is the test image proposed in [Liu00]. As shown in Fig. 3, *VRENS* achieves results comparable to other existing techniques as, for instance Perona-Malik, and works a bit worse than Liu-Wang-Ramirez algorithm (see [Liu00] for a visual comparison with other techniques).

As regards the textured images the situation is strongly different. Liu-Wang-Ramirez algorithm has been built in a way that it works very well for smooth regions but not for textured regions. In fact, on the same composition they present (shown in Fig. 4), whereas their algorithm fails to achieve right segmentation on the noiseless composition. *VRENS* is able to obtain good results up to 90% of noise.

More in general, there are more than one reason why *VRENS* works better than Liu-Wang-Ramirez's algorithm. Firstly, it doesn't take into account of the textel size, i.e. with a fixed size of the smoothing windows (even if the algorithm may be easily modified), so that we expect that for more complicate textures, (like that contained

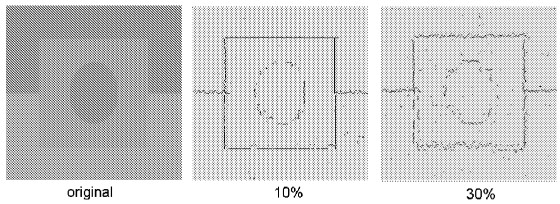

Figure 3: 256 × 256 Liu-Wang-Ramirez's test image. *VRENS*'s segmentation at different percentages of additive zero-mean gaussian noise.

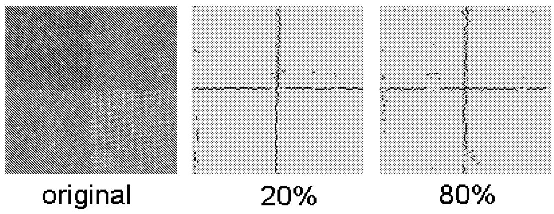

Figure 4: 128 × 128 Liu-Wang-Ramirez's textured test image. *VRENS*'s segmentation at different percentages of additive zero-mean gaussian noise.

in Fig. 3 in [Roman01]) this algorithm is not able to achieve good results. Second, after any smoothing algorithm, the only available feature is the average gray level: but this is generally not enough for image segmentation involving textures. Finally, the computational time is very high so that a parallel implementation is recommended by authors. Few seconds are generally required on a risk workstation to obtain a segmentation using *VRENS*. In particular, for the image shown in Fig. 4, the computational time in C Language on a workstation Octane/SI R10000 175 MHz/1Mb cache is 3.23 secs at 30% of noise. Summing up, for smooth regions *VRENS* achieves results comparable to other existing techniques, while works better for textured regions resulting very fast.

4 CONCLUSIONS

In this paper we have presented *VRENS*, a novel technique for segmenting very general images, containing smooth and textured regions, under noise. It looks to be interesting since, besides its robustness to the noise, it is very fast employing a low computational time. Future research will consist in using more complicated features along with an adaptative size of the area of the competing regions.

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APPENDIX A: ABOUT TEXTURES

Let's study the *edges density* behavior under noise, considering N_i discontinuities in the noiseless image $f(x, y,) : \Omega \to R$, relative to the texture under study for a given threshold and a fixed scale level. Adding noise we have: $\tilde{f}(x, y) = f(x, y) + \eta(x, y)$ where $\eta(n)$ is the added noise with distribution $N(0, \sigma)$. The situation can be roughly split in two extreme case: i) low percentage and ii) high percentage, with respect to the image information. In the first case the discontinuities have trivially a great probability of being discontinuities too. In the second case let's suppose that we have two noiseless textures t_1, t_2 with densities ρ_1, ρ_2 and $|\rho_1 - \rho_2| = \bar{\rho}$. Examining only t_1, we have that $\rho_1 = \frac{od_1}{|W_1|}$ where $od_1 \equiv$ are the original discontinuities, i.e., without noise. When noise is added, keeping on considering only t_1 we have : $\tilde{\rho_1} = \frac{\tilde{od_1} + sp_1}{|W_1|}$ where $\tilde{od_1}$ are od_1 under noise, i.e. some discontinuities may disappear, so that $od_1 \leq \tilde{od_1}$, while sp_1 are the discontinuities due to the noise, i.e. localized where od_1 is not and W_1 is the area under study. If W_1 and W_2 are wide enough, the two effects: "missing good egdes" and "new spurious edges" are opposite, and are generally similar on both the regions. So, from the considerations above, *VRENS* works till $\tilde{\rho_1} \simeq \tilde{\rho_2}$ and obviously, the higher is $\bar{\rho}$, the higher is *EVRIST*'s robustness to the noise.