Interpolatory Subdivision Curves with Local Shape Control

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ABSTRACT

In this paper we present a novel subdivision scheme that can produce a nice-looking interpolation of the control points of the initial polyline, giving the possibility of adjusting the local shape of the limit curve by choosing a set of tension parameters associated with the polyline edges. If compared with the other existing methods, the proposed model is the only one that allows to exactly reproduce conic section arcs of arbitrary length, create a variety of shape effects like bumps and flat edges, and mix them in the same curve in an unrestricted way. While this is impossible using existing 4-point interpolatory schemes, it can be easily done here, since the proposed subdivision scheme is non-stationary and non-uniform at the same time.

Keywords
Non-uniform and Non-stationary Subdivision, Curves, Interpolation, Tension, Locality.

1 INTRODUCTION

Curve-subdivision is an iterative process that defines a smooth curve as the limit of a sequence of successive refinements applied to an initial polyline. If, at each refinement level, new points are added into the existing polyline and the original points remain as members of all subsequent sequences, becoming points of the limit curve itself, the scheme is called interpolatory. In 1987 Dyn et al. [Dyn87a] introduced the first interpolatory subdivision scheme for curves, known in the literature as the "classical" 4-point scheme. This name derives from the fact that, starting from a coarse control polygon, the new points recursively introduced between each pair of old points are computed by a linear combination of the immediate four neighboring points. The proposed scheme can generate smooth interpolatory subdivision curves characterized by a tension parameter (denoted by ω) that can only be used to control the shape of the whole limit curve. Another disadvantage of this method is that the global parameter ω acts as a tension parameter only in a very restricted range. More precisely, if $0 \leq \omega_1 < \omega_2 \leq \frac{1}{16}$, the limit curve corresponding to ω_2 will be looser than the one corresponding to ω_1. But outside of the range $[0, \frac{1}{16}]$ nothing can be predicted. Recent works in interpolatory subdivision theory [Mar05a, Bec05a, Oht03a] improved the performance of the "classical" 4-point scheme, but, although providing users many choices to fit the different requirements in applications, they still suffer from some limitations to be very useful in future modelling systems. In fact, even though some of them provide degrees of freedom for controlling the shape of the limit curve, no interpolatory schemes have been designed yet to make user manipulations as intuitive as possible and allow the generation of a curve whose behavior can be locally modified in any desirable way. These considerations motivated the research reported in this paper, where we propose a novel scheme that contains [Dub86a, Dyn87a] as special cases and generalizes the non-stationary and uniform interpolatory 4-point scheme in [Bec05a] to possess local shape design parameters which have a visual interpretation on the screen. Due to its versatility, the scheme we are going to present can be used conveniently both for interactive design and for automatic curve fitting. Therefore it is very well-suited for design applications in various fields including Image Processing, CAGD and Computer Graphics.

The paper is organized as follows. In Section 2 we construct and analyze the novel scheme. In Section 3 we investigate its properties and we show its performance. Next in Section 4 we conclude the paper

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with some examples and propose some applications for which the novel scheme is very well-suited.

2 DEFINITION OF THE NOVEL INTERPOLATORY 4-POINT SCHEME

In this section we are going to present a novel interpolatory 4-point scheme that allows to adjust the shape of the limit curve by choosing a set of local tension parameters that, when increased within their span of definition, allow to appreciate considerable variations of tension in the corresponding regions of the limit curve. Such tension parameters are associated with the edges of the coarsest polyline and are opportunely updated at each refinement level. In this way, the mask which provides a rule to pass successively from one set of control points to the following, is simultaneously non-stationary (since it is different for each iteration of the subdivision process) and non-uniform (since it also changes along the curve). However, the scheme is very easy to implement because the same rule for generating the new tension parameters is used at each subdivision step.

Consider an oriented polyline $P^0 = \{p_i^0\}_{i\in\mathbb{Z}}$ (where the upper index 0 indicates that no subdivision steps have been applied yet) and assign a tension value $v_{i,0} \in]-1,+\infty[$ to each edge $\overline{p_i^0 p_{i+1}^0}$. The subindex $(i,0)$ underlines that the tension value $v_{i,0}$ is associated with the i-th edge of the polyline defined at the $0-th$ subdivision level. Next, for any $k \geq 1$, update the tension parameters $v_{i,k-1}$ into $v_{i,k}$ through the relation

$$v_{i,k} = \sqrt{\frac{1+v_{i,k-1}}{2}} \tag{1}$$

and successively compute the coefficients

$$w_{i,k} = \frac{1}{8v_{i,k}(1+v_{i,k})} \tag{2}$$

that will be used to map the polygon $P^{k-1} = \{p_i^{k-1}\}_{i\in\mathbb{Z}}$ to the refined polygon $P^k = \{p_i^k\}_{i\in\mathbb{Z}}$ by applying the following subdivision rules:

$$\begin{aligned} p_{2i}^k &= p_i^{k-1} \\ p_{2i+1}^k &= -w_{i,k}\,(p_{i-1}^{k-1}+p_{i+2}^{k-1}) \\ &+ \left(\frac{1}{2}+w_{i,k}\right)(p_i^{k-1}+p_{i+1}^{k-1}). \end{aligned} \tag{3}$$

Remark 1. Since after each round of subdivision one new point is inserted between two old ones, every edge of the old control polygon P^{k-1} is split into two new edges in the refined one. Thus, denoted by $v_{i,k-1}$ the tension parameter associated with the edge $\overline{p_i^{k-1}p_{i+1}^{k-1}}$, since such an edge is split into the two new edges $\overline{p_{2i}^k p_{2i+1}^k}$, $\overline{p_{2i+1}^k p_{2i+2}^k}$, according to (1) we will make them inherit respectively the tension values $v_{i,k}$ that is $v_{2i,k} = v_{2i+1,k} = v_{i,k}$.

Remark 2. Note that the initial tension values $v_{i,0}$ are updated before computing the coefficients $w_{i,1}$ used in the first subdivision step. Hence, for any choice of $v_{i,0} \in]-1,+\infty[$, (1) implies that $v_{i,k} \in]0,+\infty[\ \forall k \geq 1$ and (2) leads to $w_{i,k} > 0\ \forall k \geq 1$.

2.1 Convergence analysis

In this paragraph we are going to show that our novel interpolatory subdivision scheme always generates a C^0-continuous limit curve. More precisely, we will prove that, given an initial polyline P^0, the subdivision rules in (3) define an increasingly dense collection of polylines P^k that converge to a continuous curve. To show this, we will exploit two important properties of our scheme described in the following lemmas.

Lemma 1. *Given the initial parameter $v_{i,0} \in]-1,+\infty[$, the recurrence relation in (1) satisfies the property:*

$$\lim_{k\to+\infty} v_{i,k} = 1. \tag{4}$$

Proof 1. To prove this we recall that a monotonic and bounded sequence is always convergent and in particular: if it is non decreasing and upper bounded, then it converges to the upper bound of the values it assumes, whereas if it is non increasing and lower bounded, then it converges to the lower bound of the values it assumes. For the sequence defined by

$$\begin{cases} v_{i,0} \in]-1,+\infty[\\ v_{i,k} = \sqrt{\frac{1+v_{i,k-1}}{2}} \quad \forall k \geqslant 1 \end{cases} \tag{5}$$

it holds:

- if $v_{i,0} = 1$, then the sequence $\{v_{i,k}\}_{k\geq 1}$ is stationary;
- if $v_{i,0} \in]-1,1[$, then the sequence $\{v_{i,k}\}_{k\geq 1}$ is non decreasing;
- if $v_{i,0} \in]1,+\infty[$, then the sequence $\{v_{i,k}\}_{k\geq 1}$ is non increasing.

Therefore, in all these cases $v_{i,k}$ is convergent and converges to 1. In fact, called ℓ its limit, we have

$$\ell = \lim_{k\to+\infty} v_{i,k} = \lim_{k\to+\infty}\left(\sqrt{\frac{1+v_{i,k-1}}{2}}\right) = \sqrt{\frac{1+\ell}{2}}.$$

Thus, solving the last equation with respect to ℓ we get $\ell = 1$. □

Lemma 2. *Due to the recurrence (1), the parameters $v_{i,k-1}$ and $v_{i,k}$ satisfy the relation*

$$\frac{1-v_{i,k}}{1-v_{i,k-1}} < \frac{1}{2} \tag{6}$$

for any $v_{i,k} \in]0,+\infty[$, $k \geqslant 1$.

Proof 2. Exploiting recurrence (1) we can write the ratio $\frac{1-v_{i,k}}{1-v_{i,k-1}}$ in the following way

$$\frac{1-v_{i,k}}{1-v_{i,k-1}} = \frac{1-\sqrt{\frac{1+v_{i,k-1}}{2}}}{1-v_{i,k-1}} = \frac{1}{2+\sqrt{2}\sqrt{1+v_{i,k-1}}}$$

and observe that since $\sqrt{2}\sqrt{1+v_{i,k-1}} > 0 \; \forall k \geqslant 1$, thus

$$\frac{1-v_{i,k}}{1-v_{i,k-1}} < \frac{1}{2}. \qquad \square$$

Now, exploiting the well-known theorem in [Dyn95a], that relates the convergence of a non-stationary scheme to its asymptotically equivalent stationary scheme, we are going to show the following result.

Proposition 3. *The locally controlled scheme in (3) converges and generates C^0-continuous limit curves.*

Proof 3. To prove our thesis we need to show that, given an initial polyline, the novel locally controlled scheme defined by the following mask

$$m^{k-1} = \Bigg[-\frac{1}{8v_{i-1,k}(1+v_{i-1,k})},\ 0,\ \frac{(2v_{i,k}+1)^2}{8v_{i,k}(1+v_{i,k})},\ 1,\ \frac{(2v_{i+1,k}+1)^2}{8v_{i+1,k}(1+v_{i+1,k})},\ 0,\ -\frac{1}{8v_{i+2,k}(1+v_{i+2,k})} \Bigg] \quad (7)$$

converges to a C^0-continuous limit curve.
Since from (4) it follows that

$$m^{\infty} \equiv \lim_{k\to+\infty} m^{k-1} = \left[-\frac{1}{16},\ 0,\ \frac{9}{16},\ 1,\ \frac{9}{16},\ 0,\ -\frac{1}{16}\right],$$

then the non-stationary subdivision scheme associated with (7) converges to the stationary scheme in [Dub86a], that is exactly the "classical" 4-point scheme in [Dyn87a] obtained with $\omega = \frac{1}{16}$.
Next, by computing $m^{k-1} - m^{\infty}$ we have that

$$\begin{aligned} \| m^{k-1} - m^{\infty} \|_{\infty} &= \frac{|1-v_{i-1,k}|(2+v_{i-1,k})}{16v_{i-1,k}(1+v_{i-1,k})} \\ &+ \frac{|1-v_{i,k}|(2+v_{i,k})}{16v_{i,k}(1+v_{i,k})} \\ &+ \frac{|1-v_{i+1,k}|(2+v_{i+1,k})}{16v_{i+1,k}(1+v_{i+1,k})} \\ &+ \frac{|1-v_{i+2,k}|(2+v_{i+2,k})}{16v_{i+2,k}(1+v_{i+2,k})} \end{aligned} \quad (8)$$

where $v_{i-1,k}, v_{i,k}, v_{i+1,k}, v_{i+2,k} \in]0,+\infty[\ , \forall k \geqslant 1$.
Now, by the well-known theorem in [Dyn95a], if

$$\sum_{k=1}^{+\infty} \| m^{k-1} - m^{\infty} \|_{\infty} < \infty, \quad (9)$$

then the two schemes are asymptotically equivalent, and since m^{∞} is C^0, we can conclude that the scheme associated with m^{k-1} is C^0 too. Thus, to show our thesis we only have to prove that relation (9) holds as the parameters $v_{i-1,k}$, $v_{i,k}$, $v_{i+1,k}$, $v_{i+2,k}$ vary in the interval $]0,+\infty[$. For space limitations, we just give a brief sketch of the proof here. Observe that, from the analytic properties of the series, once we have proved the convergence of each term in (8), relation (9) will immediately follow. To this aim, thanks to the results in lemma 1 and 2, we can repeat, for each term of the sum in (8), the calculations presented in [Bec05a] and claim that the schemes m^{k-1} and m^{∞} are asymptotically equivalent, which proves our thesis. $\square$

Remark 3. Note that the proposed scheme is asymptotically equivalent to the "classical" interpolatory 4-point scheme in [Dyn87a] with parameter $\omega = \frac{1}{16}$, that is C^1. Hence, since according to [Dyn95a], the continuity level of a non-stationary scheme is, in general, the same as that of the stationary scheme to which this scheme converges, we expect the limit curve generated by the scheme in (3) to be C^1.

3 THE LOCAL TENSION PARAMETERS

The novel scheme described by (3) has a very powerful and intuitive geometric interpretation. If we write the second line of (3) in the form

$$\begin{aligned} p^k_{2i+1} &= \frac{p^{k-1}_i + p^{k-1}_{i+1}}{2} \\ &+ 2w_{i,k}\left(\frac{p^{k-1}_i + p^{k-1}_{i+1}}{2} - \frac{p^{k-1}_{i-1} + p^{k-1}_{i+2}}{2}\right) \end{aligned} \quad (10)$$

it is clear that the new point p^k_{2i+1} turns out to be the mid-point of the edge $\overline{p^{k-1}_i p^{k-1}_{i+1}}$, corrected by a vector $2w_{i,k}e$, where e denotes the vector connecting the mid-points of the edges $\overline{p^{k-1}_i p^{k-1}_{i+1}}$ and $\overline{p^{k-1}_{i-1} p^{k-1}_{i+2}}$ (see Figure 1).

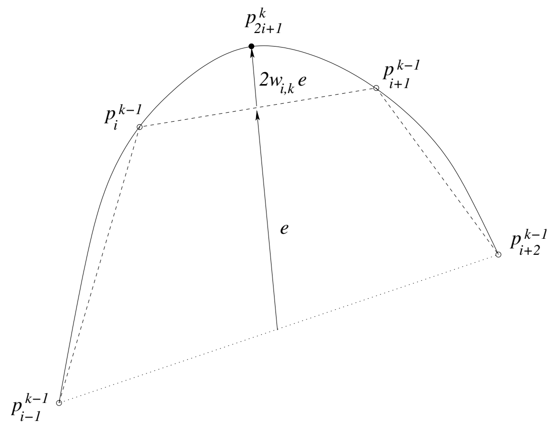

Figure 1: Geometric interpretation of the local parameter $w_{i,k}$.

The value of $w_{i,k}$ in (10) clearly depends on the choice of the initial tension parameter $v_{i,0}$. For different values of $v_{i,0}$ ranging from -1 to $+\infty$, we will have different values of $w_{i,1}$ in $]0,+\infty[$. The important cases to be mentioned are $v_{i,0}=1$ ($w_{i,1}=\frac{1}{16}$) and $v_{i,0} \rightarrow +\infty$ ($w_{i,1} \rightarrow 0$). In the first case, in fact, the subdivision scheme becomes stationary and the limit curve we obtain coincides with the one generated by [Dub86a] and by [Dyn87a] choosing the parameter ω equal to $\frac{1}{16}$. In the second case, instead, the portion of curve confined to the endpoints of the i-th edge is pulled towards the linear interpolant of those two points, since when $w_{i,1}$ is exactly zero, the scheme exactly generates the linear interpolant to the initial control points.
In particular, progressively increasing the value of $v_{i,0}$, the portion of curve confined to the endpoints of the i-th edge will become progressively tighter and tighter.
It is interesting to note also that, by setting all initial tension parameters equal to the same value v_0, we get the uniform tension-controlled interpolatory scheme defined in [Bec05a].

3.1 Local tension parameters provided by the user

The local tension parameters $v_{i,0}$ that define the interpolatory scheme presented in Section 2, can be either arbitrarily provided by the user to intuitively model the limit shape, or automatically set to let the limit curve fit special requirements in applications.
In Figure 2 we show an example of interactive design where the local tension parameters play an important role in the overall display of the final shape. The top left curve has been obtained by setting all the parameters $\{v_{i,0}\}_{i=0,\dots,6}$ to $\gamma=\cos(\frac{2\pi}{7})$. As explained in [Bec05a], such a configuration of tension values, applied to the regular heptagon denoted by the dashed line, generates the exact circle interpolating its vertices $\left(\cos(\frac{2j\pi}{7}),\sin(\frac{2j\pi}{7})\right)$, $j=0,\dots,6$. The consecutive five interpolants demonstrate the role of the local tension parameters specified in the array v, by showing their influence on the resulting shapes. Comparing the six figures, we can see that, by opportunely modifying the tension values γ, we can obtain different ways of achieving local shape control on the limit curve.
These results and many others, obtained using different initial polylines and various local tension parameters, confirm our conjecture about the C^1-smoothness of the curve generated by the refinement scheme in (3).
Additionally, they point out that the model we have proposed exhibits many properties that a subdivision scheme should include to become useful for geometric modelling applications. Indeed, it allows:

- **Intuitive shape deformations**
  Among all the shape parameters that we can find in existing subdivision models, the ones that provide a tension effect indeed appear totally intuitive and possess a direct visual interpretation on the screen (see Figure 2).

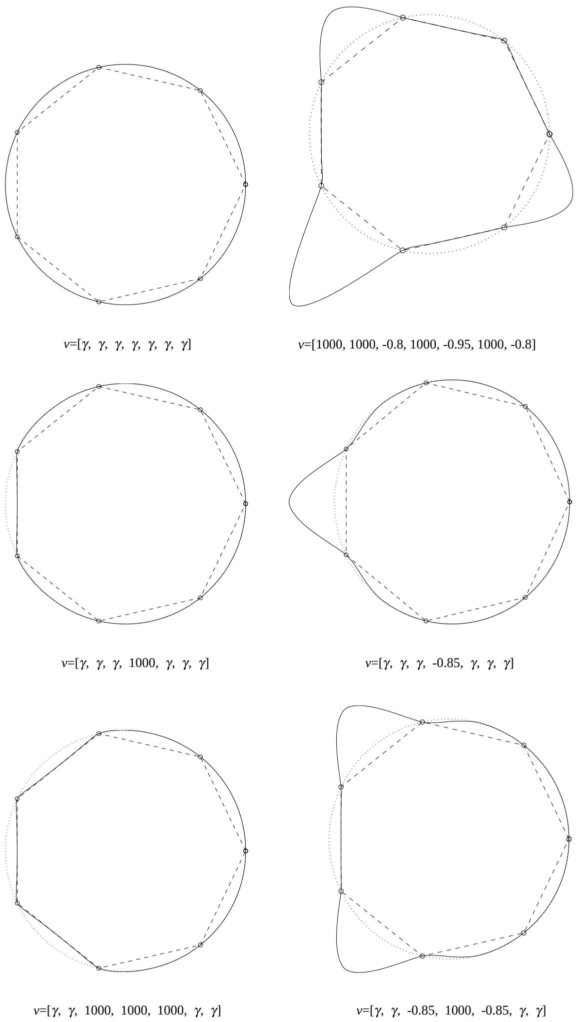

Figure 2: Closed subdivision curves produced by interpolation of the vertices of a regular heptagon using the tension parameters in the indicated array v where $\gamma=\cos(\frac{2\pi}{7})$.

- **Local control**
  Each tension parameter, associated with a specified edge of the initial polyline, influences the shape of the limit curve only in a restricted zone, corresponding to the related edge and the neighboring parts of its two adjacent edges. This allows a very powerful local shape control (see Figure 2).

- **Warping, flattening and mixed effects**
  Warping is a tool which can be used to deform a local segment of a curve pulling out a bump; flattening is a tool which allows to easily introduce straight line segments into curves. The local tension parameters can reproduce both warping and

flattening behaviors as well as provide a variety of interesting shape effects. The subdivision curves that we can generate, in fact, are able to incorporate both round shapes and flat shapes, like bumps and localized flat edges. Additionally such effects can be mixed in the same curve in an unrestricted way, always allowing soft transitions between them (see Figure 2).

3.2 Local tension parameters automatically set

In this subsection we will show the capability of the proposed subdivision scheme to fit certain classes of curves widely used in CAGD applications, like polynomials, conic sections and piecewise cubic Bézier curves.

Since an interpolatory subdivision process defines a smooth curve as the limit of a sequence of successive refinements applied to an initial polyline, the approximating algorithm we are going to describe requires to select an adequate set of points on the reference curve C to start the locally controlled refinement procedure described in Section 2. Note that, the greater the number of selected points is, the more the approximating shape tends towards the original curve C, but the more the shape descriptor size and the number of computations increase. To balance these two extreme scenarios, the strategy adopted for determining the initial polyline to which apply the refinement process should select the minimum number of points needed to get a visually precise reconstruction. By the term *visually precise* we mean that, if you compare our interpolatory subdivision curve with the original one, you can hardly spot the differences between them (see Figures 4, 5, 6, 7).

Let $P^0 = \{p_i^0\}_{i\in\mathbb{Z}}$ be the initial sequence of points opportunely sampled on C. Due to (10), the points p_{2i+1}^1 computed at the first subdivision step, will be placed on the line r passing through the mid-points of the edges $\overline{p_i^0 p_{i+1}^0}$ and $\overline{p_{i-1}^0 p_{i+2}^0}$, at a distance from $\frac{p_i^0+p_{i+1}^0}{2}$ inversely proportional to the tension value $v_{i,0}$. Therefore, in order to achieve a visually precise fitting, each point p_{2i+1}^1, defined in correspondence of the i-th edge $\overline{p_i^0 p_{i+1}^0}$ of the coarsest polygon P^0, should be placed exactly on the curve C. According to this request, the initial tension value $v_{i,0}$ will be determined in such a way the point p_{2i+1}^1 turns out to be the intersection point between the line r and the curve C. Repeating this procedure for each curve segment, we will be able to automatically determine a tension value $v_{i,0}$ that, used in the refinement process (3), generates the interpolatory subdivision curve that approximates the given curve C. Although we have no guarantee that in the following refinement levels the subdivision process will insert all the new points on the curve C, whenever we start from a sequence of points $P^0 = \{p_i^0\}_{i\in\mathbb{Z}}$ suitably sampled on C, all our experiments showed visually precise reconstructions.

The following algorithm implements an automatical procedure to get an interpolatory subdivision curve that turns out to be visually precise in respect to the original curve C.

Algorithm 1

1. *Determine the initial polyline $P^0 = \{p_i^0\}_{i\in\mathbb{Z}}$;*
2. *compute the tension parameters $v_{i,0}$ associated with the edges $\overline{p_i^0 p_{i+1}^0}$, in such a way the points p_{2i+1}^1, inserted through (3), lie on C (for each edge of the initial polyline, this requires to solve the system of equations deriving by forcing the point p_{2i+1}^1 to be placed on C);*
3. *for $k = 1$ to the desired refinement level*
   - *3.1 compute the tension values $v_{i,k}$ through (1);*
   - *3.2 compute the coefficients $w_{i,k}$ through (2);*
   - *3.3 apply the refinement rules (3);*
4. *stop.*

Remark 4. Note that, due to the rule in the second line of equation (3), to insert a new point p_{2i+1}^k in the refined polyline P^k, it is necessary to possess a well defined two-neighborhood (given by two points on its left and right side in the coarsest polyline P^{k-1}). Hence, if C is an open curve, the initial polyline P^0 should be extended to contain two more sample points both at the beginning and at the end of the initial sequence $P^0 = \{p_i^0\}_{i\in\mathbb{Z}}$.

In the following we will show how the proposed algorithm leads to some useful applications of our locally controlled interpolatory 4-point scheme.

The first one is related to two desirable properties of subdivision schemes, namely polynomial precision and conics reproduction. These two terms usually refer to the capability of exactly reproducing a specified curve by applying the subdivision scheme to a set of control points uniformly sampled on it. As far as we know, existing interpolatory subdivision schemes are able to reproduce neither polynomials nor conic sections starting from unevenly sampled points.

Exploiting Algorithm 1 we can show that our interpolatory scheme turns out to be exact for linear polynomials even if the initial data are unevenly sampled on them. Additionally, a visually precise fit to points unevenly sampled on any quadratic or cubic polynomial and any arbitrary conic section, can be achieved too (see Figure 3).

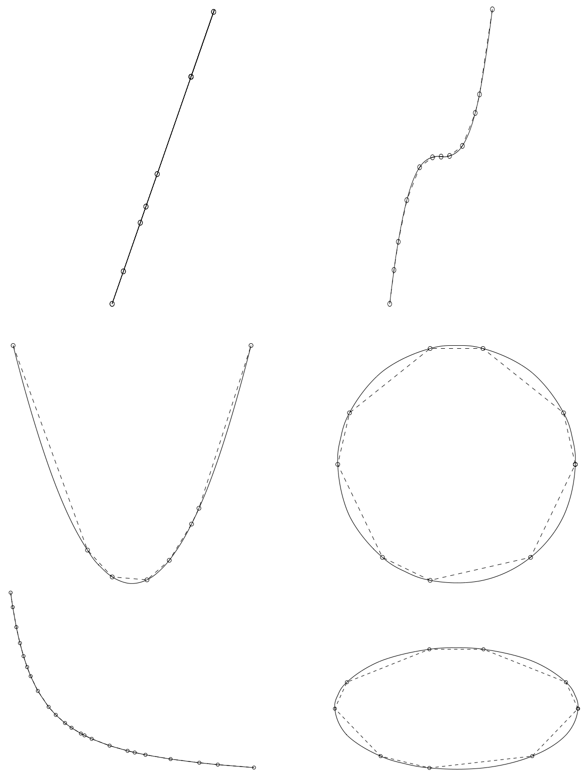

Figure 3: Representation of polynomials and conic sections via the interpolatory subdivision curve with local shape control (dashed lines: polylines; solid lines: subdivision curves).

On the other hand, it can be easily verified that, when we consider sample points uniformly spaced, the proposed scheme is exact for cubic polynomials. In fact, performing the computations described in Algorithm 1, we get that the same tension parameter $v_{i,0} = 1$ should be applied on the whole curve. This value coincides with the global tension parameter introduced in [Bec05a] to exactly reproduce cubic polynomials. Analogous results can be found repeating the same computations when the initial data are evenly sampled on a conic section [Bec05a]. Therefore, when starting from uniformly distributed points, we can conclude that by applying Algorithm 1 we can work out the initial tension parameters that allow the subdivision scheme in (3) to be exact for cubic polynomials and to exactly reproduce all conic sections.

Since in CAGD applications curves are frequently represented in the piecewise cubic Bézier form, we want to show now an additional application of our algorithm which allows to obtain a very good representation of a curve C in the piecewise cubic Bézier form. Denoted by Q^0 the sequence of control points defining the piecewise cubic Bézier curve C, we select from Q^0 the subsequence consisting exactly of all the junction points of the single Bézier segments. Next, to define the initial polyline P^0 for generating the interpolatory subdivision curve, we add supplementary points wherever two consecutive selected points (so far computed) turn out to be too distant. According to our experiments this strategy always ensures a visually precise reconstruction.

4 APPLICATIONS

The proposed subdivision scheme was designed to make our model a candidate of choice for many applications. An example consists in proposing this novel class of interpolatory subdivision curves as an efficient and economical representation of the outline curves used to describe fonts as well as output images of tracing algorithms [Sel03a]. In general, a vector outline describes a digitalized image via a family of piecewise cubic Bézier curves that may join either C^0 or C^1 continuously. Our subdivision scheme provides a mathematical description of vector outlines that turns out to be simpler and more efficient than the one currently used in Postscript files and tracing algorithms. In fact, although in vectorial outlines one switches frequently between round shapes and flat shapes, by using only one subdivision curve with local shape control, we will be able to make a good job (see Figure 4).

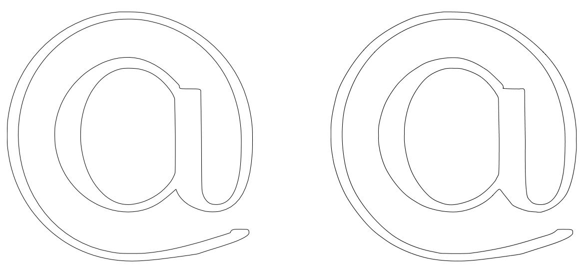

Figure 4: Two piecewise cubic Bézier curves describing the outline of the character "@" - 73 cubic segments (left); two single-piece interpolatory subdivision curves - 75 points, 73 tension parameters (right).

Remark 5. Note that, due to the smoothness properties of the scheme, to make the job perfectly, and then allowing the possibility of reproducing sharp corners between flat regions and round ones, we need to generate distinct subdivision curves and stitch them together in the corners (see Figure 5).

Now we point to a number of advantages that could make this novel class of interpolatory subdivision curves the standard representation of vectorial outlines.

The primary advantage of interpolatory subdivision curves with local shape control over piecewise cubic Bézier curves has to do with the physical storage

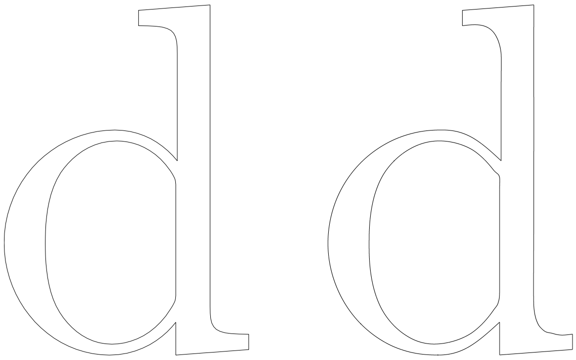

Figure 5: Two piecewise cubic Bézier curves describing the outline of the character "d" - 24 cubic segments (left); the corresponding piecewise interpolatory subdivision curves - 9 single-piece subdivision curves, 33 points, 24 tension parameters (right).

of vector outlines in files. In fact, as the set of local tension parameters act as handles to model the contour that best fits the original image, exploiting a small number of points per contour, we can obtain a very compact numerical format for representing outline curves. Since the necessary information to represent the outline curve are 1 endpoint and 1 tension parameter per segment, we manage to remarkably compress the data to be stored in the file and optimize them for speed and for handling contour images with large character sets.
Another consistent advantage is that our technique turns out to be also very convenient for computing the points required for the rasterization of the final curve, which consists in converting the outline to a pattern of dots (whether it's screen pixels or the dots of a laser, inkjet or wire-pin printer) on the grid of the output device.
All these observations let us notice that indeed there are several key improvements which may swing the future in curve-subdivision's favor. While subdivision surfaces have not been widely adopted by the CAGD community [Gon01], maybe some day subdivision curves could intelligently substitute the use of piecewise cubic Bézier curves in describing Postscript fonts as well as the output images of tracing algorithms.
Exploiting the procedure described in Algorithm 1, very good results have been obtained over a range of outline images. We show here the reconstructions we got by testing the algorithm on the data obtained from the outline of a Type1, cmr10 Postscript font (Figures 4, 5), an hand-drawn sketch (Figure 6) and a detail of a university logo (Figure 7). As it appears, if you compare Figures 4, 5, 6, 7 (right) with the corresponding outlines of piecewise cubic Bézier curves in Figures 4, 5, 6, 7 (left), you can hardly spot the differences between them.

5 CONCLUSIONS AND FUTURE WORK

Stimulated by the observation that a "good" local interpolatory scheme does not seem to be presently available, we have proposed a novel subdivision scheme for interpolatory curves which, in contrast to existing models, gives the possibility of generating a big variety of good quality curves modifying the tension parameters associated with the edges of the initial polyline. To our knowledge, this is the first interpolatory subdivision scheme, easy to implement and computationally economical, that includes many classical properties as well as several original features considered vital or simply desirable in applications. For example, the proposed scheme can generate a limit curve that can be locally modified with the help of shape control parameters. Namely, it is able to produce a locally controlled interpolatory curve which can incorporate a variety of shape effects like bumps and flat edges, and can mix them in an unrestricted way, always allowing a soft transition between them. Additionally, instead of being altered by the user for interactive design purposes, the tension parameters can be automatically set in such a way the limit curve generated can elegantly fit curves extremely used in geometric modelling systems, such as conic section arcs and piecewise cubic Bézier curves. As an application, we have proposed our interpolatory subdivision curves with local shape control as an efficient and economical representation of the vector outline of a scanned image or a letter-form shape. This makes it possible to introduce subdivision techniques in document description languages as Postscript and encourages the CAGD community to swing in curve-subdivision's favor.

The authors are looking, as a future work, to extend the curve subdivision scheme to surfaces. This could be quite useful for various applications in different fields of studies.

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Figure 6: Outline of Loxie and Zoot cartoon: 236 piecewise cubic Bézier curves - 3767 cubic segments (left); reconstruction via interpolatory subdivision curves with local control - 236 single-piece subdivision curves, 4003 points, 3767 tension parameters (right).

Figure 7: Outline of a detail of the University of Bologna logo: 561 piecewise cubic Bézier curves - 6679 cubic segments (left); reconstruction via interpolatory subdivision curves with local control - 561 single-piece subdivision curves, 7240 points, 6679 tension parameters (right).

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