Image Deformation using Radial Basis Function Interpolation

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ABSTRACT

Image deformation technique is widely used in the field of computer animation, image editing, medical imaging, and other applications in 2D and 3D computer graphics. All the algorithms aim to provide simple user interface, most of which need the user to drag the control points, lines or polygon. The deformation process and the final position of the controlling points should be smooth and precise respectively, and it should also run in real-time.

This paper provides a simple image deformation method using the radial basis function interpolation in approximation theory. Radial basis function is a very popular and convenient tool for data representation problems. The proposed method using radial basis function is fast and easy to use more than previous deformation methods. Experiments indicate that the algorithm is stable and well performed.

Keywords

Image deformation, Radial basis functions, Interpolation /approximation, Image editing, Computer animation

1. INTRODUCTION

Image deformation has a number of uses from computer animation to morphing. It gives the user the ability to manipulate the object as if the user was handling it in real life. The image is given a set of handles in which the user can intuitively control its position and orientation.

Although there are many ways to deform image to make the desired shape, to perform these deformations the user selects a set of handles to control the deformation. These handles may take the form of points, lines, or even polygon grids. As the user modifies the position and orientation of these handles, the image should deform in an intuitive fashion.

The deformation techniques divide into two parts. One is to deform the shape using mesh based method, the other is to deform the shape using approximation method.

Igarashi et al. [Iga05a] and Weng et.al. [Wen06a] belongs to the first parts. Igarshi et al. presented an interactive system that allows the user to deform a 2D shape by manipulating a few points. The shape is represented by a triangle mesh and the user moves several vertices of the mesh as constrained handles. The system then computes the positions of the remaining free vertices by minimizing the distortion of each triangle. To make the problem linear, they present a two-step closed-form algorithm. The first step is to compute the rotation and the second step to compute the scale. This divides the problem into two least-squares minimization problems that can be solved sequentially. Weng et.al. presents a novel 2D shape deformation algorithm based on nonlinear least squares optimization. The algorithm have to preserve two local shape properties: Laplacian coordinates of the boundary curve and local area of the shape interior, which are together represented in a non-quadratic energy function. The result is an interactive shape deformation system that can achieve physically plausible results.

In approximation method, Arad et al. [Ara94a] has introduced radial basis function for image warping and Schaefer et al. [Sch06a] provided an image deformation based on moving least squares using various classes of linear functions including affine, similarity and rigid transformations. These deformations are realistic and give the user the impression of manipulation real-world objects.

Our paper builds on a recent paper by Arad et al. Therefore our paper provides a simple image deformation technology using the radial basis function interpolation in approximation theory. The radial basis function method is easier use, quicker to calculate, than the previous deformation method. Experiments indicate that the algorithm is stable and well performed.

The paper is organized as follows: chapter 2 is devoted to introduce the general theories on the radial basis function interpolation. In chapter 3, we explain image deformation using RBF. In chapter 4, we illustrate some resulting image deformation experiments, and conclude.

2. Radial Basis Function

Radial basis function interpolation [Lee06a, For99a, Toi08a] is a very popular and convenient tool for scattered data approximation problems. This chapter introduces the general setting and basic theory of radial basis function interpolation [Lee06a]. Suppose that a continuous function $f:\mathbb{R}^{d_1}\to\mathbb{R}^{d_2}$ is known only at a set of discrete points $U := \{u_1,u_2,\cdots,u_n\}$ and desired function values $V := \{v_1,v_2,\cdots,v_n\}$. The radial basis function interpolation to f on U starts with choosing a basis function ϕ, and then it defines an interpolant by

$$S_{f,U}(u)=\sum_{i=1}^{n}\alpha_i\phi(\|u-u_i\|)+\sum_{j=1}^{m}\beta_j p_j(u), \quad (1)$$

where $p_1,p_2,\cdots,p_m$ is a basis for $\Pi_r^{d_1}$ and the coefficients α_i are chosen so that

$$\sum_{i=1}^{n}\alpha_i p_j(u_i)=0, \qquad j=1,2,\cdots,m. \quad (2)$$

Here, Π_r^d is the space of polynomial of total degree r in d spatial dimensions, $m=\dim(\Pi_r^d)=\dfrac{(d+r)!}{d!r!}$.

For a wide choice of function ϕ and polynomials in Π_r^d, the coefficients of $S_{f,U}(u)$ that are required to satisfy the $(n+m)\times(n+m)$ system of linear equations, which can be written in a matrix form as

$$\begin{pmatrix} A & P \\ P^T & 0\end{pmatrix}\begin{pmatrix}\alpha \\ \beta\end{pmatrix}=\begin{pmatrix}\mathrm{v} \\ 0\end{pmatrix}, \quad (3)$$

where A and P are the $n\times n$ and $n\times m$ matrices that have the elements $A_{ij}=\phi(\|u_i-u_j\|)$ and $P_{ij}=p_j(u_i)$ respectively.

This can be written as

$$A=\begin{pmatrix}\phi(\|u_1-u_1\|) & \phi(\|u_1-u_2\|) & \dots & \phi(\|u_1-u_n\|)\\ \phi(\|u_2-u_1\|) & \phi(\|u_2-u_2\|) & \dots & \phi(\|u_2-u_n\|)\\ \vdots & \vdots & \ddots & \vdots \\ \phi(\|u_n-u_1\|) & \phi(\|u_n-u_2\|) & \dots & \phi(\|u_n-u_n\|)\end{pmatrix}$$

and $P=\begin{pmatrix}p_1(u_1) & p_2(u_1) & \dots & p_m(u_1)\\ p_1(u_2) & p_2(u_2) & \dots & p_m(u_2)\\ \vdots & \vdots & \ddots & \vdots\\ p_m(u_n) & p_m(u_n) & \dots & p_m(u_n)\end{pmatrix}$.

Further $\alpha\in\mathbb{R}^{n\times d_1}$ and $\beta\in\mathbb{R}^{m\times d_2}$ are the vectors of coefficients of $S_{f,U}(u)$ and components of v are the data v_j with $j=1,2,\cdots,n$.

In a fundamental paper by Micchelli [Mic86a], the existence and uniqueness of the solution of the linear system is ensured when the basis function ϕ is a conditionally positive definite function.

3. Image Deformation using RBF

We construct a deformation function $S_{f,U}(u)$ relates the u_i points in the set of control points to their v_i-u_i to the counter parts in the deformed position where v_i-u_i is difference vector (Figure 1).

If we have given two sets data $U:=\{u_1,u_2,\cdots,u_n\}$ and $V:=\{v_1-u_1,v_2-u_2,\cdots,v_n-u_n\}$.

Given a point u in the image, we solve for the RBF interpolation $S_{f,U}(u)$, satisfying

$$S_{f,U}(u_i)=v_i-u_i, i=1,2,\cdots n. \quad (4)$$

Finally, we obtain a deformed position v of u

$$v=u+S_{f,U}(u), \quad (5)$$

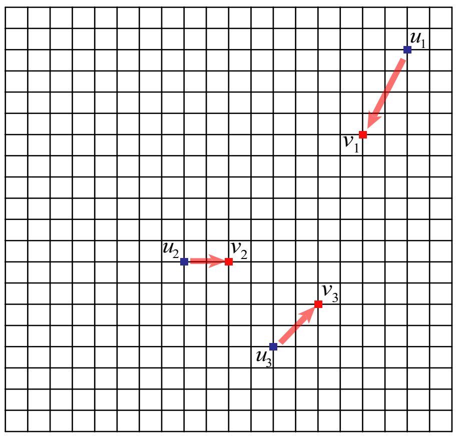

Figure 1. An Image Deformation Data Set.

Figure 2. shows the deformation result according to polynomial degree. (a) shows a change by using RBF without polynomial terms when one control-points were moved. In case of constant term($m=1$), if we click on the shape to place one control-point and then drag the control-point to its new position, only translation occurs from old position to new position as shown in (b). In case of a linear term($m=3$), deformation occurs when over 3 points are selected. If they are 3, the result with both rotation and shearing is obtained in (c) .In case of a quadratic terms($m=6$), the use of over 6 points makes deformation, resulting in curved surface as shown in (d) because it has the characteristics of quadratic function.

4. Experimental Result

We have tested some of the images using Gaussian and Wendland's [Wen95a] functions as basis functions, respectively, to evaluate the proposed algorithm. Both characteristic feature of the function monotonically decreases with distance from the centre.

Wendland's functions $\phi_{3,1}(r)=(1-\frac{r}{c})_+^4+(4\frac{r}{c}+1)$ are known as basis functions with compact-support basis function. To help illustrate the behavior of a number of the basis functions consider the cat image shown in Figure 3. The same locality parameter was used for each of the localized basis functions. Figure 3. (b) and (c) are images using Gaussian function and Wendland's basis function, respectively. To expend only the cat's ears, that is, to deform image locally, the Gaussian function have to use 5 control-points while Wendland's one does only 2-control points. That's due to compactly supported property.

Figure 4 shows the comparison results of our algorithm (b) and Schaefer et al of algorithm (c). (c) have to set many control-points on the image in order to avoid image translation. As a result, deformation brings the lack of smoothness while our algorithm can get smoother result than Figure 4. (c) only with fewer control points.

The deformation of fish image is shown in Figure 5 and its continuous frames made by moving a tail can take effect as if the fish swim.

5. Conclusion

In this paper, we implemented the deformation based on RBF. As you saw in experimental results, the various results of deformation were obtained by a term of controlling polynomial of degree. The proposed method is faster by simple calculation and its result is better than the previous methods. The term of controlling polynomial degree has many possibilities for various application fields. Especially, in field of animation, we can select it according to global or partial changes. In this paper, we used two basis functions. Further research will be extended to uses of other basis functions and line or curve segments instead of points as control attribute.

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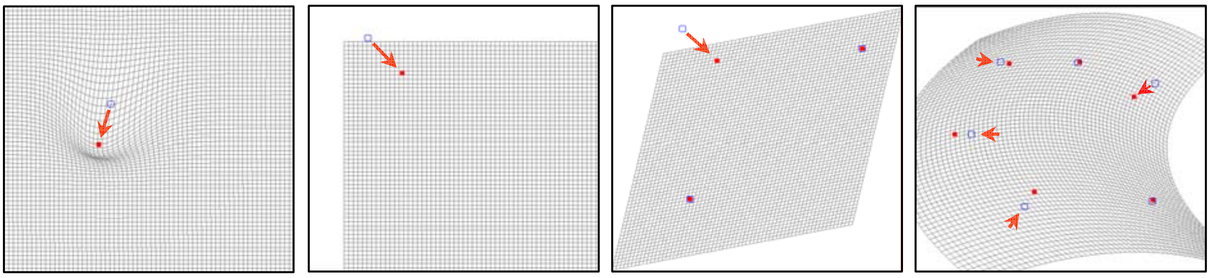

(a) without polynomial (b) with constant (c) with linear (d) with quadratic

Figure 2. Comparison result between different polynomial degree about moving the control-points.

(a) Original image (b) Gaussian basis (c) Wendland's basis

Figure 3. The effects of different radial basis functions.

(a) Original image (b) Our algorithm (c) Schaefer et al [Sch08a].

Figure 4. Comparison between our algorithm and [Schaefer et al].

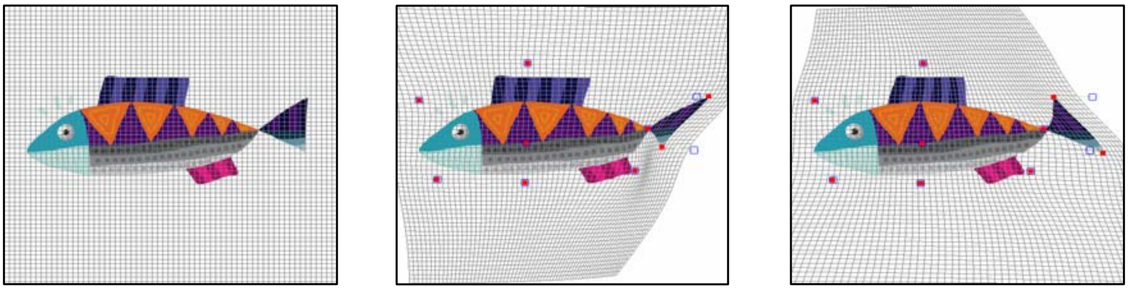

Figure 5. Deformation of a fish image. Left : original image ; Middle, Right : deformation results