# Geometric modeling and computer graphics of kinematic ruled surfaces on the base of *complex moving* one axoid along another
## (one-sheet hyperboloid of revolution as fixed and moving axoids)


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## ABSTRACT
In concordance with logical structure “Geometrical Model of Interaction of two contacted surfaces during the movement of one ruled surface along another – Corresponding Mathematical Transformations of Surfaces – New Kinematic Ruled Surfaces” [1-3] a new geometrical model of *complex moving* one axoid along another for the case of one-sheet hyperboloid of revolution as fixed and moving axoids has been proposed. The main condition of constructing kinematic ruled surfaces is that moving axoid contact with fixed axoid along one their common generating line in each of their positions during *complex moving* one axoid along another. A case when the axes of fixed and moving axoids are crossed (Fig. 1,2,3), has been considered in this research. Analytical development and computer graphics of the new kinematic surfaces are realized for three types of *complex moving*. (1)The outside surface of the fixed axoid is revolved *slipping-free* by the outside surface of the corresponding moving axoid (Fig. 1). (2)The interior surface of the fixed axoid is revolved *slipping-free* by the outside surface of the corresponding moving axoid (Fig. 2). (3)The outside surface of the fixed axoid is revolved *slipping-free* by the interior surface of the corresponding moving axoid (Fig. 3). Computer graphics of the constructed surfaces (Fig. 1*a*,2*a*,3*a*) have been performed by the previously developed software application [4].



Figure 1.


Figure 1*a*.


Figure 2.


Figure 2*a*.


Figure 3.

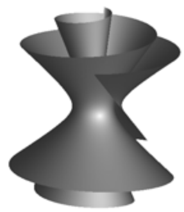
Figure 3*a*.


### Keywords
Geometric Modeling, Computer Graphics, Kinematic Surfaces.


## 1. INTRODUCTION
Kinematic ruled surfaces are constructed by the movement of a generating line of one (moving) axoid during its moving along another (fixed) axoid [1]. The main condition of constructing kinematic ruled surfaces is that moving axoid contact with fixed axoid along one their common generating line in each of their positions during movement of one axoid along another. In some cases such moving as rolling one axoid along another is sufficient to meet this main condition. The examples of these cases are well-known kinematic ruled surfaces constructed on the base of rolling one cylinder along another cylinder or one cone along another cone [1]. At the same time such moving as rolling one axoid along another in the case of one-sheet hyperboloid of revolution as fixed and moving axoids is insufficient to meet the main condition of constructing kinematic ruled surfaces. However, as follow from this research

in this case task of constructing kinematic ruled surfaces move to feasible solution on the base of *complex moving* one axoid along another. *Complex moving* can be represented as a combination of concerted movement such as rolling one axoid along another and translational movement of the moving axoid along the common generating line of both axoids The similar case of geometrical modeling of *complex moving* one axoid along another on the example of moving a cone along a torse was described earlier [2, 3]. In that case *complex moving* was represented as a superposition of three interrelated elementary movements: rotational movement of the cone around its axis; turn of the cone axis; translational movement of the cone vertex along the torse edge-of-regression. Similar task of constructing geometrical model of *complex moving* one-sheet hyperboloid of revolution along another one as the base for generating new kinematic ruled surfaces was solved in this research.

## 2. PRINCIPAL GEOMETRICAL MODEL OF *COMPLEX MOVING*

One-sheet hyperboloid surface of revolution is a ruled surface, which can be constructed by the straight line rotation about the axis, if the straight line is crossed with the axis of rotation. A case of two interacting axoids contacted along mutual ruling when the axoids' axis are crossed (Fig. 1, 3, 5), has been considered in this research. In this case, *complex moving* one axoid along another can be represented as a superposition of three interrelated elementary movements: rotational movement of the moving axoid around its axis *OZ* in the moving coordinate system *OXYZ* connected with the moving axoid, rotational movement of the moving axoid axis *OZ* around the fixed axoid axis *oz* in the fixed coordinate system *oxyz* connected with the fixed axoid, and translational movement of the moving axoid along the common generating line of both axoids. The origin of the fixed coordinate system *oxyz* is located in the center of the waist circle of the fixed axoid. The origin of the moving coordinate system *OXYZ* is located in the center of the waist circle of the moving axoid.

## 3. GEOMETRICAL MODEL №1 (ONE AXOID LOCATE ON THE OUTSIDE OF ANOTHER AXOID)

Geometrical model №1 is related to the case if the outside surface of the fixed axoid is revolved *slipping-free* by the outside surface of the corresponding moving axoid (Fig. 1). Model №1A is related to the case if moving axoid is the same as fixed axoid. Model №1B is related to the case of two contacted different axoids.

### 3.1 Geometrical Model №1A

Geometrical model №1A is related to the case if moving axoid (2) is the same as fixed axoid (1), i.e.
$a_1 = a_2 = a$ , $c_1 = c_2 = c$ , where
$a_1$, $c_1$ – parameters of the fixed axoid,
$a_2$, $c_2$ – parameters of the moving axoid.

Parameters $a$, $c$ – parameters of the canonical equation of one-sheet hyperboloid surface of revolution in the coordinate system *oxyz*:

$\frac{x^2}{a^2} + \frac{y^2}{a^2} - \frac{z^2}{c^2} = 1$ , where $a$ – radius of waist circle.

Parametric equations of surface generated by one of the ruling of the moving axoid in the system *OXYZ*:

$$X = R(v\cos(\varphi + \alpha) - (v-1)\cos(\varphi - \alpha));$$
$$Y = R(v\sin(\varphi + \alpha) - (v-1)\sin(\varphi - \alpha));$$
$$Z = -h(2v-1),$$

where $R$ – radius of circular section of one-sheet hyperboloid of revolution at $h$ distance from the waist section: $R = a\sqrt{1+(h/c)^2}$ ; $\alpha = arctg(h/c)$ ;
$\varphi$ – current value of the angle of rotation of the moving axoid around its axis, $v$ – input parameter.
The equations of transition from the coordinate system *OXYZ* to the coordinate system *oxyz*:

$$x = X\cos\varphi - (Y\cos\theta - Z\sin\theta)\sin\varphi + 2a\cos\varphi$$
$$y = X\sin\varphi + (Y\cos\theta - Z\sin\theta)\cos\varphi + 2a\sin\varphi$$
$$z = Y\sin\theta + Z\cos\theta,$$

where $\theta = 2arctg(a/c)$ .
The resulting parametric equations of the kinematic ruled surface in the fixed coordinate system *oxyz*:

$$x = (A+2a)\cos\varphi - (B\cos\theta + C\sin\theta)\sin\varphi;$$
$$y = (A+2a)\sin\varphi + (B\cos\theta + C\sin\theta)\cos\varphi;$$
$$z = B\sin\theta - C\cos\theta,$$

where

$$A = R(v\cos(\varphi + \alpha) - (v-1)\cos(\varphi - \alpha));$$
$$B = R(v\sin(\varphi + \alpha) - (v-1)\sin(\varphi - \alpha));$$
$$C = h(2v-1).$$

Pair of contacted axoids (Fig. 4)) and corresponding kinematic ruled surface (Fig. 4*a*) are shown below:

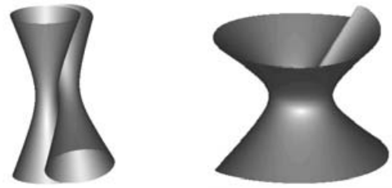

Figure 4. Figure 4*a*.

Figures of pairs of contacted axoids and kinematic ruled surfaces have been constructed with the help of the previously developed AMG ("ArtMathGraph") software application [3, 4].

### 3.2 Geometrical Model №1B

Geometrical model №1B is related to the case of two different axoids ($a_1 \neq a_2$, $c_1 \neq c_2$). As it will be shown below parametric condition for two contacted different axoids is $a_1^2 + c_1^2 = a_2^2 + c_2^2$.
This formula follows from model №1B (Fig. 5).

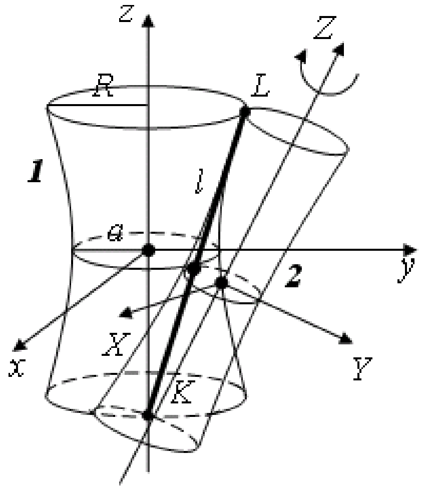


Figure 5.

In the developed geometrical model one axoid is revolved *slipping-free* by the another axoid. In this case any circle (waist circle and all others) of the fixed axoid (1) is revolved *slipping-free* by the corresponding circle of the moving axoid (2). Hence it follows that a relation length of each pairs of contacted circles of contacted axoids must be equal. The formula of dependence $R(l)$ was obtained on the base of model №1B (Fig. 5), where $l$ – length of the segment of the common generating line of both axoids from the waist circle to the circle of radius $R$:

$$R = a\sqrt{1 + \frac{l^2}{a^2 + c^2}} \ .$$

As stated above the relation

$$\frac{R_2(l)}{R_1(l)} = \frac{a_2\sqrt{1 + \frac{l^2}{a_2^2 + c_2^2}}}{a_1\sqrt{1 + \frac{l^2}{a_1^2 + c_1^2}}}$$ must be $l$ -independent.

It is possible if $a_1^2 + c_1^2 = a_2^2 + c_2^2$.

The result parametric equations of the kinematic ruled surface have been derived in the system oxyz:

$$x = (A + a_1 + a_2)\cos(\varphi / n) -$$
$$-(B\cos\theta + C\sin\theta)\sin(\varphi / n);$$
$$y = (A + a_1 + a_2)\sin(\varphi / n) +$$
$$+(B\cos\theta + C\sin\theta)\cos(\varphi / n);$$
$$z = B\sin\theta - C\cos\theta,$$ where

$$A = R(v\cos(\varphi + \alpha) - (v-1)\cos(\varphi - \alpha));$$
$$B = R(v\sin(\varphi + \alpha) - (v-1)\sin(\varphi - \alpha));$$
$$C = h(2v - 1);$$
$$n = a_2 / a_1 \ ; \ \theta = arctg(a_1 / c_1) + arctg(a_2 / c_2);$$
$$R = a_2\sqrt{1 + (h / c_2)^2} \ ; \ \alpha = arctg(h / c_2).$$

Some matched pairs of contacted different axoids (Fig. 1, 6, 7) and corresponding generated kinematic ruled surfaces (Fig. 1*a*, 6*a*, 7*a*) are shown. (Figure 6: $(a_1 / a_2) = 2/1$; Figure 7: $(a_1 / a_2) = 1/2$).

Figure 6.

Figure 6*a*.

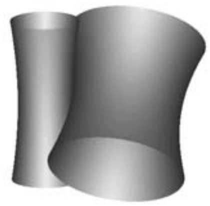

Figure 7.

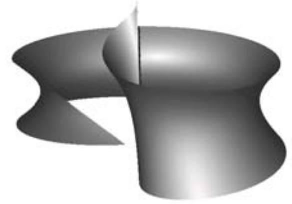

Figure 7*a*.

## 4. GEOMETRICAL MODEL №2 (ONE AXOID LOCATE IN THE INTERIOR OF ANOTHER AXOID)

Geometrical model №2 of *complex moving* one axoid along another is related to the two cases:

- the interior surface of the fixed axoid is revolved *slipping-free* by the outside surface of the corresponding moving axoid (model №2A),
- the outside surface of the fixed axoid is revolved *slipping-free* by the interior surface of the corresponding moving axoid (model №2B).

Using the geometric model №2 the result parametric equations of the kinematic ruled surface have been derived in the coordinate system *oxyz*:

$$x = (A + |a_1 - a_2|)\cos(\varphi / n) +$$
$$+(B\cos\theta + C\sin\theta)\sin(\varphi / n);$$
$$y = -(A + |a_1 - a_2|)\sin(\varphi / n) +$$
$$+(B\cos\theta + C\sin\theta)\cos(\varphi / n);$$
$$z = B\sin\theta - C\cos\theta,$$ where

$$A = R(v\cos(\varphi + \alpha) - (v-1)\cos(\varphi - \alpha));$$
$$B = R(v\sin(\varphi + \alpha) - (v-1)\sin(\varphi - \alpha));$$
$$C = h(2v - 1);$$
$$n = a_2 / a_1; \ \theta = |arctg(a_1 / c_1) - arctg(a_2 / c_2)|;$$
$$R = a_2\sqrt{1 + (h / c_2)^2} \ ; \ \alpha = arctg(h / c_2).$$

 ISBN 978-80-86943-95-4

## 4.1 Geometrical Model №2A

For example, the kinematic ruled surfaces (Fig. 2*a*, 8*a*) for matched pairs of contacted axoids ($(a_1/a_2) = 6/1$) are constructed. The axis of the fixed axoid (1) is located vertically on the Fig. 2, 8.

Figure 8.

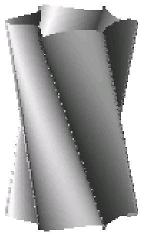

Figure 8*a*.

## 4.2 Geometrical Model №2B

The types of kinematic ruled surfaces constructed on the base of the model №2A or model №2B are different (Fig. 8*a* or 9*a*). In this case the type of surfaces is defined by the proportion $(a_1/a_2)$. The surfaces shown in Fig. 2*a*, 8*a* are corresponded to the proportion $(a_1/a_2) = 6/1$, and the surfaces shown in Fig. 3*a*, 9*a* are corresponded to the proportion $(a_1/a_2) = 1/6$. The axis of the fixed axoid (1) is located vertically in Fig. 3, 9.

Figure 9.

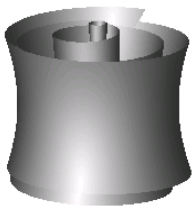

Figure 9*a*.

It is necessary to note that all described above matched pairs of contacted axoids fulfill geometric condition of overlap-free *complex moving* one axoid along another. However, fulfillment of this condition is optional for constructing kinematic ruled surfaces.

## 5. MODEL OF *COMPLEX MOVING* WITH OVERLAPPING TWO CONTACTED AXOIDS

The example of matched pair of axoids (Fig. 10) and corresponding kinematic ruled surface (Fig. 10*a*) as the case of *complex moving* one axoid along another with overlapping two contacted axoids is shown.

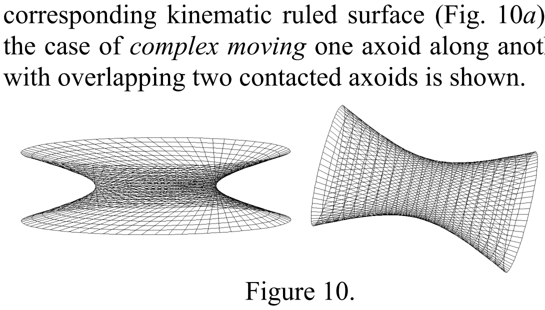

Figure 10.

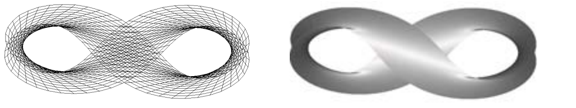

Figure 10*a*.

At the first sight the shown above pair of axoids (Fig. 10) looks like an "inappropriate pair" of contacted axoids. Nevertheless, this axoids pair was selected in correspondence with condition $a_1^2 + c_1^2 = a_2^2 + c_2^2$ as well. The axial angle of the contacted axoids is $\theta$ :

$$\theta = arctg(a_1/c_1) + arctg(a_2/c_2) = 102^\circ .$$

In correspondence with value $\theta = 102^\circ$ fixed and moving axoids are located in Fig. 10.

It should be emphasized, that the mathematical transformations as well as the resulting parametric equations of the kinematic ruled surfaces are independent of the type of *complex moving* - with or without overlapping two contacted axoids.

## 6. CONCLUSIONS

Thus, the proposed principal geometrical model of *complex moving* one axoid along another for the case of one-sheet hyperboloid of revolution as fixed and moving axoids has been assumed as a basis for analytical development of the new logical mean for constructing kinematic ruled surfaces. The main result of this analytical treatment in the combination with graphic ability of the previously developed software application gives a good opportunity for computer search of desirable kinematic surfaces.

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