Computer composition of transformed classical surfaces as the ways and means for constructing visual models of realistic objects (New software application "ArtMathGraph")

Galina S. Rachkovskaya
107 Pushkinskaya Str. # 25
344007, Rostov-on-Don, Russia
galina.rachkovskaya@ verizon.net

Yuriy N. Kharabayev
1Б Budennovskiy Ave. # 46
344008, Rostov-on-Don, Russia
kharabayev@aaanet.ru

Natalya S. Rachkovskaya
1006 Moorefield Hill Pl
22180, Vienna, VA, USA
natashar@verizon.net

ABSTRACT

The new software application "ArtMathGraph" (AMG*)* is developed for constructing a visual representation of predetermined complicated geometrical forms as visual models of realistic objects, including objects of Nature. This application is based on the methods of Analytical Geometry and Computer Graphics only.

The AMG application includes two basic methodical parts: 1 - the analytical modeling and computer visualization of multiple different sets of new mathematical transformed surfaces as components for a consequential computer superposition of these surfaces; 2 - interactive visual modeling of predetermined geometrical forms by using the computer superposition of constructed mathematical transformed surfaces. By experience, consecutive use of all means of the AMG application allows obtaining various compositions of visual images of realistic objects (Figures 1, 2).

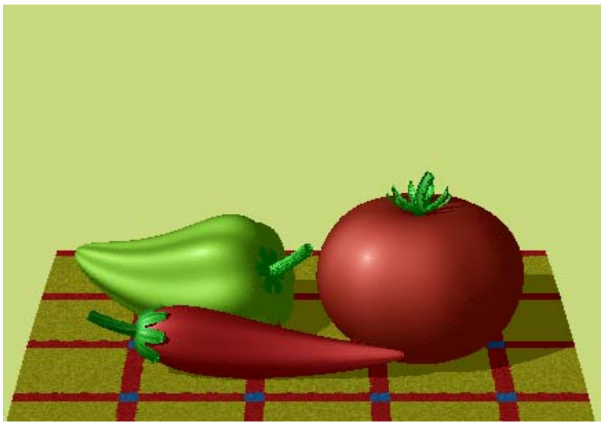

Figure 1

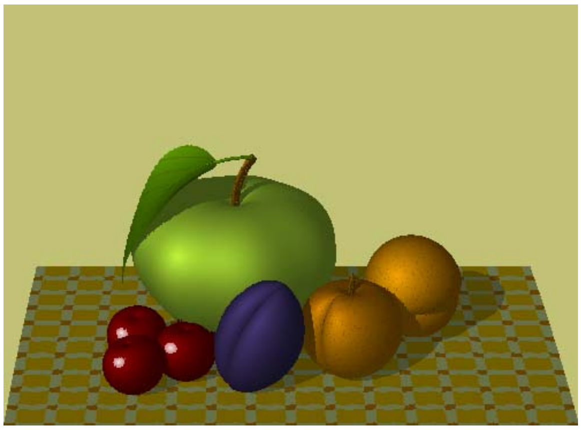

Figure 2

Keywords

Computer graphics, analytical modeling, visualization.

1. INTRODUCTION

The problems in constructing visual models of realistic objects, including objects of Nature, can be solved by the means of computer compositions of mathematically transformed classical surfaces. This solution is implemented in the new software application "ArtMathGraph" (AMG). The AMG application is developed as a computer mean for constructing visual images of complicated geometrical forms as models of realistic objects.

The AMG application includes:

1) basic set of initial well-known classical curves and surfaces;
2) some groups of mathematical transformations;
3) means for computer construction and visualization of the mathematical transformed surfaces;
4) means for interactive visual mathematical modeling of predetermined complicated geometrical forms as visual models of realistic objects;
5) means for obtaining various compositions of visual images of realistic objects.

2. ANALYTICAL MODELING AND COMPUTER VISUALIZATION OF MATHEMATICAL TRANSFORMED CLASSICAL SURFACES

The mathematical transformed surfaces are a result of applying the mathematical transformations to the analytical representations of initial well-known classical surfaces such as plane, cone, cylinder, sphere, ellipsoid, etc.

It is obvious that it is possible to construct multiple different sets of new mathematical transformed surfaces by combining the available values of variable parameters in the equations of Initial Curves (or Initial Surfaces) and/or of mathematical transformations. The analytical statement and software application for constructing and computer visualizing a large number of new mathematical transformed surfaces was developed previously based on conical transformations of torses as examples of Initial Surfaces [1-4]. It should be noted that the surfaces obtained using this software application are displayed on a screen in a 'frame' or 'solid' style [3]. Surfaces in the "solid" style are displayed in a user-selected color using variable color brightness to create a 3D effect. As example, the "frame" and the "solid" styles in the AMG application are shown in the Figures 3, 4.

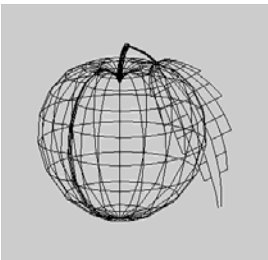

Figure 3

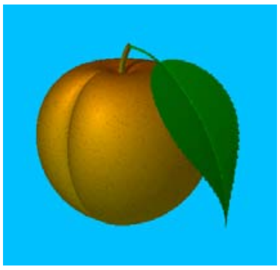

Figure 4

2.1. *Analytical Representation of Initial Surfaces*

Analytically the Initial Surfaces are represented by parametric equations:

$$x = x(u,v); \quad y = y(u,v); \quad z = z(u,v).$$

The Initial Surface equations are described in common form with use of their typical parameters. The selection of values for these parameters is available to the user and allows performing some deformations of the surfaces. On the whole, it is extension and compression.

The more significant surface modifications are achieved with the use of special transformations. There are two groups of mathematical transformations (the group "*Before*" and the group "*After*") differed in their impact on the analytical representation of surfaces.

2.2. *Mathematical Transformation Groups "Before" and "After"*

The mathematical transformation group "*Before*" is used for equations of surfaces on the level of arguments-parameters *u* and *v*; in essence, they change the analytical representation of surfaces.

With the help of these mathematical transformations the smooth surfaces can be performed cut, wavy, or lobulated. Actually, transformations of the group "*Before*" extend a basis set of Initial Surfaces. It is possible, for example, to derive a pyramid or prism by using cutting for a cone or cylinder accordingly.

Mathematical transformations of the group "*After*" are performed for the values *x, y, z* calculated by proceeding from parametrical equations subject to transformations of the group "*Before*". The group "*After*" contains mathematical transformations of the surfaces twisting about axis *oz* (which is usually the axis of rotation for surfaces of rotation) and bending along the arc of a circle in different directions.

2.3. *Mathematical Transformed Cone and Cylinder Surfaces (Analytical Representation and Computer Visualization)*

For methodical example, the result from some mathematical transformations of Cone or Cylinder Surfaces as initial surfaces is shown in the Figures 5-16.

Fig. 5 Fig. 6 Fig. 7 Fig. 8 Fig. 9 Fig. 10

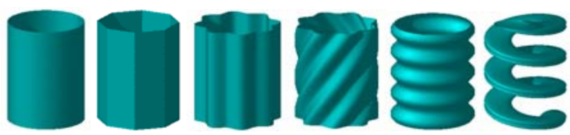

Fig. 11 Fig. 12 Fig. 13 Fig. 14 Fig. 15 Fig. 16

There are parametric equations of the Initial Cone and Cylinder Surfaces (Figures 5 and 11) and the Transformed Cone Surfaces (Figures 6-10) as results

 ISBN 978-80-86943-99-2

of applying the mathematical transformations of groups "*Before*" and "*After*" to the Initial Cone Surface cited below. (The parametric equations of Transformed Cylinder Surfaces and Transformed Cone Surfaces are analogous.)

<u>*Initial Cone Surface*</u> (Fig. 7):

$$x = av\cos u;$$
$$y = av\sin u;$$
$$z = v\,; \qquad (1)$$
$$0 < u \le 2\pi,\ \ 0 \le v \le h.$$

<u>*Initial Cylinder Surface*</u> (Fig. 13):

$$x = a\cos u;$$
$$y = a\sin u;$$
$$z = v\ \ ; \qquad (2)$$
$$0 \le u \le 2\pi,\ 0 \le v \le h.$$

<u>*Transformed Cone Surfaces*</u> (Fig. 8-12):

1. Cutting (Fig. 8) – group "*Before*"

$$x = a\frac{\cos\frac{\pi}{k}\cdot\cos u}{\cos(\frac{\pi}{k}-\varphi)}v;$$
$$y = a\frac{\cos\frac{\pi}{k}\cdot\sin u}{\cos(\frac{\pi}{k}-\varphi)}v;$$
$$z = v\,; \qquad (3)$$
$$\varphi = u - [\frac{k}{2\pi}\cdot u]\frac{2\pi}{k},$$
$$0 \le u \le 2\pi,\qquad 0 \le v \le h\ .$$

k –quantity of sides.

2. Sinus-waviness (Fig. 9) – group "*Before*"

$$x = av(1 + d\sin ku)\cos u;$$
$$y = av(1 + d\sin ku)\sin u;$$
$$z = v\,; \qquad (4)$$
$$0 < u \le 2\pi,\ \ 0 \le v \le h,$$

k –quantity of waves, d –height of a wave.

3. Quasi-epicycloidal lobularity with twisting along axis *oz* (Fig. 10) – group ("*Before*" + "*After*")

$$X\ = x\cos\omega z + y\sin\omega z;$$
$$Y = -x\sin\omega z + y\cos\omega z;$$
$$Z = z\,, \qquad (5)$$

ω – determines direction and degree of twisting, *x, y, z* are calculated according to the equations of quasi-epicycloidal lobularity – group "*Before*"

$$x = av((1+k)\cos u - \cos((1+k)u));$$
$$y = av((1+k)\sin u - \sin((1+k)u));$$
$$z = v\,; \qquad (6)$$
$$0 < u \le 2\pi,\ \ 0 \le v \le h,$$

k –quantity of lobes.

4. Thickening (Fig. 11) – group "*After*"

$$X\ = \frac{x}{1 + p(v - v_c)^2};$$
$$Y = \frac{y}{1 + p(v - v_c)^2};$$
$$Z = z\,, \qquad (7)$$

x, y, z are calculated according to the equations (1),
p – determines degree of thickening,
v_c – determines position of thickening along axis *oz*.

5. Bending along axis *oy* (Fig. 12) – group "*After*"

$$X\ = q - (q - x)\cos\frac{z}{q};$$
$$Y = y;$$
$$Z = (q - x)\sin\frac{z}{q}, \qquad (8)$$

x, y, z are calculated according to the equations (1),
q - determines direction and degree of bending.

3. MEANS OF INTERACTIVE VISUAL MATHEMATICAL MODELING OF PREDETERMINED COMPLICATED GEOMETRICAL FORMS

The AMG application includes the means of interactive visual mathematical modeling of predetermined complicated geometrical forms. It allows the user, guided by a screen view, creating visual images, adjusting parameters, as well as deriving realistic three-dimensional images. During the process of interactive visual mathematical modeling the user can change object's location and color, background color, and direction of light.

Consecutive mathematical transformations of the Initial Surfaces and computer superposition of the constructed transformed surfaces allows obtaining visual images of Complicated Geometrical Forms as models of various realistic objects (Fig. 1, 2, 17-19).

Figure 17

Figure 18

Figure 19

4. MEANS OF OBTAINING ARTISTIC COMPOSITIONS OF VISUAL IMAGES

The following operations are implemented in the AMG application for obtaining artistic compositions:

- constructing visual images of individual geometrical forms as models of realistic objects;
- combining individual images into a composition of derivable visual images;
- arranging each individual image and a composition of individual images in the space with its 3D-representation on the screen;
- choosing color for individual images;
- choosing direction of light and constructing corresponding distribution of light and shade;
- considering perspective as a system of representation of visual images on the screen.

Several "Mathematical Pictures" are shown in the Figures 1, 2, 17-22 as some illustrations of the AMG application possibilities.

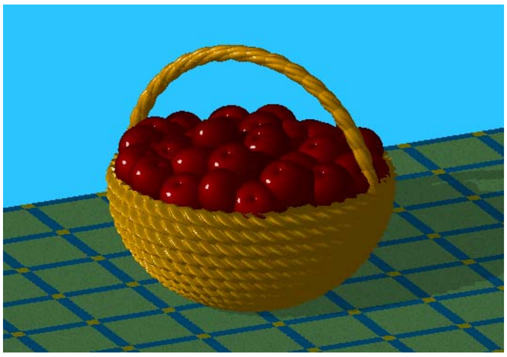

Figure 20

Figure 21

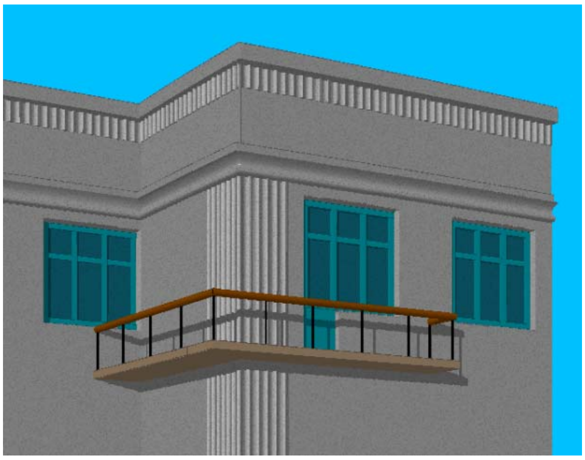

Figure 22

By experience, consecutive use of all means of the AMG application allows obtaining various artistic compositions of visual images of realistic objects by the methods of computer superposition of the mathematical transformed classical surfaces.

REFERENCES

[Con02a] Rachkovskaya, G.S. and Kharabayev, Yu.N. Mathematical modelling of kinematics of ruled surfaces based on conical transformations of torses. Proceedings of the 10th International Conference on Geometry and Graphics. Kyiv, Ukraine, Vol.1, pp. 283-286, 2002.

[Con02b] Rachkovskaya, G.S. and Kharabayev, Yu.N. The computer graphics of modelling of kinematic linear surfaces based on rolling a cone along a torse. Proceedings of the 12th International Conference GraphiCon' 2002, N.Novgorod, Russia, pp. 153-156, 2002.

[Con04a] Rachkovskaya, G.S., Kharabayev, Yu.N., and Rachkovskaya, N.S. Computer graphics of kinematic surfaces. Proceedings of the 12-th International Conference in Central Europe on Computer Graphics, Visualization and Computer Vision 2004, Plzen, Czech Republic, p.p. 141-144, 2004.

[Con04a] Rachkovskaya, G.S., Kharabayev, Yu.N., and Rachkovskaya, N.S. The computer modelling of kinematic linear surfaces (based on the complex moving a cone along a torse). Proceedings of the International Conference on Computing, Communications and Control Technologies, Austin (Texas), USA, Vol.1, p.p. 107-111, 2004.