$\sqrt{3}$ Subdivision and 3 connected meshes with creases, boundaries and holes.

Olivier Guillot   **Jean-Paul Gourret**

Laboratoire Informatique, Image, Interaction (L3i) Université de La Rochelle

IUT department d'informatique
15 rue François de Vaux de Foletier

17026 La Rochelle Cedex 1

olivier.guillot@univ-lr.fr   jean-paul.gourret@univ-lr.fr

ABSTRACT

A project of surface meshing for still and animated images and associated software has been developed at L3i for several years. The software ([KG04]) implements a meshing technique of deformable objects based on triangular meshes or on polygons with only 3 connected vertices. In this paper, we present a technique of subdivision of triangular meshes and its adaptation to 3 connected meshes. The technique insures Cn continuity (n = 1 or 2), takes into account discontinuities such as creases, darts and boundaries, and allows approximation or interpolation of surfaces.

Keywords
Mesh – subdivision - surface.

1. INTRODUCTION

The subdivision is based on the insertion of a new vertex in each triangular face. For several years we have been interested by deformable meshes based on physical laws. Then we have introduced 3 connected meshes, because deformation calculations are faster than with other kind of meshes. The rapidity is due to the minimal number of vertices and connections, and because the mechanical structure is less rigid. A 3 connected mesh allows the fast meshing of a cloud of points with interactive guidance of calculations. We are concerned by the compression of shape information in the context of object deformation and not in terms of file size.

In this paper we start from a control mesh at subdivision level j=0. The control mesh does not have to be 3 connected. It can represent a closed surface of genus 0 or greater than 0, it can present discontinuities (creases, darts, boundaries) and holes.

Internally our software handles only triangular meshes, but we have developed an algorithm that can convert a subdivided triangular mesh into a 3 connected mesh, with fewer vertices and faces.

2. MESH SUBDIVISION BY INSERTION OF A VERTEX ON A FACE

The $\sqrt{3}$ subdivision pertains to this subdivision family. A vertex is inserted in each face. Then new faces are created joining the new vertices to the initial vertices and to the new vertices in immediate neighborhood.

Doing this, a vertex shares 6 faces. From an even level j, we obtain an odd level j+1. It is an "odd subdivision". From the odd level j+1, we obtain an even level j+2. It is an even subdivision. These two successive applications of the algorithm on a triangle, gives 9 triangles. Two applications of the butterfly method, by insertion of points on edge would create 16 triangles [DLG90]. For our purpose the $\sqrt{3}$ subdivision method is better than the butterfly method, because it creates less faces between two even steps. We have more control on the final number of faces. Without creases, darts or boundaries, we use two $\sqrt{3}$ subdivision methods: Approximation of Kobbelt [K00] or Interpolation of Labsik-Greiner [LG00]. We developed new

algorithms in order to compute the vertices near discontinuities.

First of all, we have to choose a mask for the calculus. It defines which vertices of the mesh are going to be used to compute the position of the inserted vertex. Of course, the bigger the mask is, the smoother the limit surface will be, i.e. the limit surface will be Cn, with a higher n. It is confirmed by the calculation of the coefficients for different masks.

We use the method that J.P.Gourret used in [G87] to compute the potential and the electric field near surfaces of reversed biased PN junctions.

3. TRANSFORMATION INTO 3-CONNECTED MESH

To transform a mesh into a 3 connected mesh, we keep only the vertices that have been generated at the last subdivision, and we create a face for each (removed) vertex. The face is made up of all vertices connected to the removed vertex.

4. CREASES

Subdivision software would be useless without creases handling. A crease is a serie of vertices that model a natural discontinuity of the surface. So we cannot take a mask crossing a crease. The article [HDD94] references 3 kinds of natural discontinuities. Crease : it is a deformable crease (it can be smoothed). Corner : it is a non deformable crease. Dart : it is the extremity of a crease or corner. Our software will handle open creases (with 2 darts) or closed creases (cyclical). For example in the article we find the case shown on Fig.1a. The resulting mesh after our 4 subdivisions is shown on figure 1 b&c.
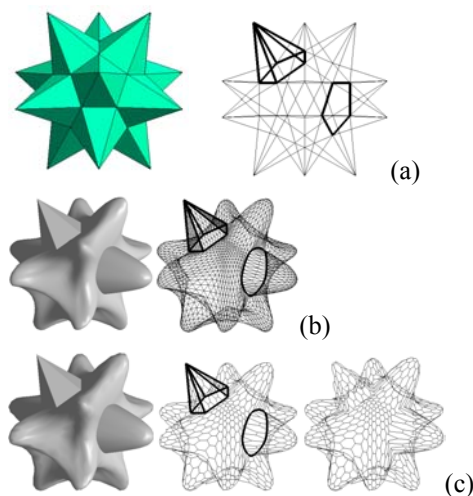

(a)

(b)

(c)

Figure 1: kepler (a) control mesh, subdivision with creases (b) triangular, (c) 3-connected

5. BOUNDARIES

Boundaries allow us to handle open meshes as the one shown in figure 2.
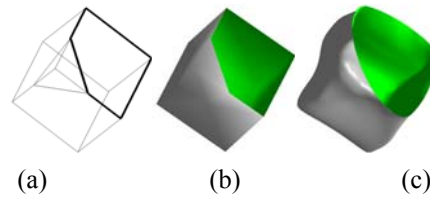

(a)   (b)   (c)

Figure 2: a&b) open mesh (c) after 4 subdivisions

A boundary is a limit, always closed. We can handle as many boundaries as we want, but they should not cross themselves. The idea we use to handle boundaries is to consider it as a crease and create a temporary vertex for each boundary in order to "close" the mesh, so we can reuse the same algorithm than without boundaries.

6. CONCLUSION AND WORK IN PROGRESS

We create a subdivisor that handles creases corners, darts and boundaries. The successive subdivisions of the impulse response of a n connected vertex are shown on Fig. 3. It can be considered as scale functions to establish wavelet functions in a multiresolution analysis. So we are establishing a compression method based on this multiresolution representation by wavelets taking into account creases, corners, darts and boundaries.
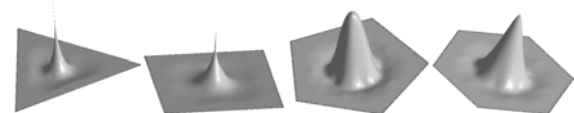

Fig.3: impulse response for a vertex n-connected

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