

# A Local Error Bound Approach to Simplifying Complex Geometric Models

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## ABSTRACT

This paper presents a new error bound simplification algorithm for complex geometric models. A lower polygon count approximation of the input model is generated by performing edge collapse operations. The collapse vertex is constrained to lie within a localised tolerance volume built around the edge collapse neighbourhood. This constraint ensures that all points on the simplified surface are within a user specified distance from the surface after the previous edge collapse.

## Keywords

Simplification, level of detail, tolerance volume.

## 1. INTRODUCTION

This paper presents a new simplification algorithm for geometric models. The algorithm develops upon the simplification envelope approach proposed in [Coh95] and [Bre00], by creating an arbitrarily tight, localised tolerance volume built around the edge collapse neighbourhood. Constraining the collapse vertex to lie within this tolerance volume guarantees that all points on the simplified surface lie within a user specified distance from the surface after the previous edge collapse operation.

## 2. THE TOLERANCE VOLUME

The tolerance volume is built around the triangles making up the collapse neighbourhood (e.g. those triangles sharing either vertex of the collapsing edge) and represents a subset of collapse vertex positions that preserve a user specified bound on simplification error. Firstly, the convex kernel is created from the boundary edges (edges not sharing a vertex with the collapsing edge) within the collapse neighbourhood by constructing boundary planes lying orthogonal to its corresponding boundary triangle and passing through its boundary edge. The convex kernel of the collapse neighbourhood is then intersected with a set

of prisms created for each triangle within the collapse neighbourhood. The prisms are formed by offsetting each triangle along its positive and negative surface normal by a distance equal to the error bound. The open sides of the prisms are capped with quadrilaterals. A typical prism is shown in Figure 1.

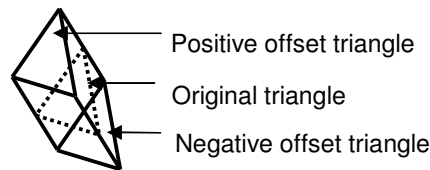


Figure 1. Prism used in the construction of the tolerance volume.

The set of prisms defined above may contain illegal vertex positions. A position is illegal if it results in any part of the simplified surface breaching the error bound. This problem is overcome by partitioning the prisms into legal and illegal sub-regions with partition sets.

### 2.1 Partition Sets

Partition sets are created for the offset triangles within each prism. There are three basic varieties of partition: those for boundary prisms (created for triangles sharing only one vertex with the collapsing edge), those for internal prisms (created for triangles sharing both vertices of the collapsing edge) and those where the boundary vertex used to construct the partition is external to the prism used in the construction of the partition.

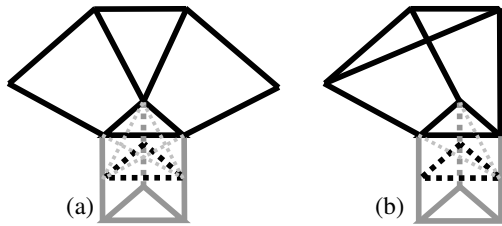
#### 2.1.1 Boundary Prisms

Partitions are created for the offset triangles within each boundary prism. The vertices belonging to the internal edges (e.g. those sharing one of the offset collapsing vertices) on each offset triangle are displaced along the vector formed between the opposite boundary vertex (on the surface triangle

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used to construct the prism) and the vertex to be displaced. The vertices need to be displaced by a large enough distance to ensure that the partition is capable of intersecting all prisms. The vertices of the internal edge together with the displaced vertices define a quadrilateral partition. Since there are two internal edges on an offset triangle within a boundary prism, two quadrilateral partitions are formed. The gap between the two partitions is capped to form a triangle. The offset triangle itself is added to the set making a total of four partitioning polygons. The complete partitioning set for a boundary prism is illustrated in Figure 2(a).



**Figure 2. Partition set for a boundary (a) and internal (b) prism**

### 2.1.2 Internal Prisms

The partition sets for internal prisms are formed in an analogous fashion to those of boundary prisms. The difference lies in the fact that the offset triangles within internal prisms have three internal edges. A partitioning set for an internal prism is illustrated in Figure 2 (b).

### 2.1.3 Exterior Partitions

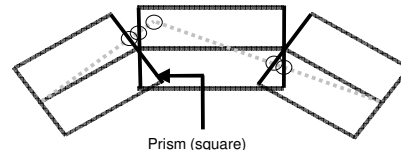
Exterior partitions are created for both boundary and internal prisms in a similar fashion to those of internal prisms except that the boundary vertex lies within another prism. Partition sets are created for all boundary vertices lying on the inside (with respect to the prism) of the half-space formed from any internal edge on an offset triangle and the corresponding edge on the surface triangle used to construct the prism.

Each partition set represents a convex region defined by four half-spaces. The partition set divides space into a legal and an illegal region, with the prism used to construct the partition defined as being in the legal region. Each partition set is used to partition the prisms and the illegal regions are discarded. The remaining regions represent a sub-set of all legal collapse vertex positions.

## 2.2 Measuring Simplification Error

Surface approximation error is measured by projecting the simplified surface onto the original surface and calculating the maximum distance

between corresponding points on the two surfaces. The simplified surface within each prism is projected by parallel orthographic projection onto the surface triangle used to construct the prism. The distance between the simplified surface and the original surface changes linearly within each prism. Hence, the maximum error will occur at a vertex on the simplified surface within a prism or at an intersection point between the simplified surface and the sides of a prism. An illustrative two-dimensional example is given in Figure 3 which shows 2D quadrilateral analogues to prisms. The target vertices and intersection points are highlighted with circles in the diagram. To calculate the maximum error, it is only necessary to project these target points onto the original surface within the appropriate prism (quadrilateral) and search for the maximum distance between corresponding points.



**Figure 3. Illustrative example of a 2D edge collapse showing quadrilateral analogues of prisms and the simplified surface (dotted).**

## 3 RESULTS

The local error bound algorithm has been implemented to perform all legal edge collapses within a user specified global error bound. The global error bound is set manually to some fraction of the diagonal of the bounding box of the model. The error bound used in the construction of tolerance volumes is set to a small fraction of the global error bound and all legal collapses are performed. The error bound is gradually increased until the desired triangle count is reached, or the error bound exceeds the user specified global error bound. The results have shown that the algorithm is capable of large scale polygon reduction while preserving important surface features.

## 4 REFERENCES

- [Bre00] Bremer, P., Kreylos, O., Hamann, B., Wolter, F., Simplification of closed triangulated surfaces. M.L. Haungs, ed., Proc. of the 2000 UC Davis Student Workshop on Computing, TR CSE-2000-9, California, 2000.
- [Coh95] Cohen, J., Varshney, V., Manocha, D., Turk, G., Weber, H., Agarwal, P., Brooks, F., Wright, W., Simplification envelopes. Proc. SIGGRAPH, pp. 119-128, 1995.