3D Visibility of 4D Convex Polyhedra

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ABSTRACT

In mathematics we can easily generalize Euclidean 3D space to n-dimensional one for arbitrary n>3. The task, how one can express n-dimensional objects in 3D or even in 2D, arises. In the paper the generalization of the back-volume culling algorithm is analyzed.

Keywords

visibility, higher-dimensional polyhedra

1. INTRODUCTION

The output information from the physically based models is very often in the form of spatial data set with an internal structure (e.g. vector field, tensor field). It means that the set of the values in defined point is transformed according to defined rules when the point is moved to the different position. In the paper we consider the simplest case of such structure – only n-dimensional Euclidean space with rotation.

In mathematics we can easily generalize Euclidean 3D space to n-dimensional one for arbitrary n>3. The task, how one can express n-dimensional objects in 3D or even in 2D, arises. In [Agu04] the unraveling approach is used, in [Hol91] Depth-Cueing of 4d bodies is applied.

In the paper the generalization of the visibility criterion for convex polyhedra is analyzed. Criterion is formulated for 4D case.

2. THE FIRST APPROACH TO THE VISIBILITY

Visualization of n-dimensional convex polyhedra we can realize it in two steps: **1.** projection of the body vertices to 3D, **2.** construction of the convex hull of 3D projections. As we consider convex polyhedra only, the scheme is correct. This schema is used e.g. in [Agu04], [Holl91]. However, this approach doesn't use any information about the structure of the body (faces) and its projections.

Our solution is based on the well-known criterion – back-volume culling algorithm (BVCA).

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3. ORIENTATION AND VISIBILITY

When we use BVCA, we must distinguish between external and internal side of the faces. So, the orientation of the space must be introduced. In 3D case the ordered triplet of base vectors $(\vec{e}_1, \vec{e}_2, \vec{e}_3)$ is right oriented, if

$$[\vec{e}_1, \vec{e}_2] = \vec{e}_3, \quad [\vec{e}_2, \vec{e}_3] = \vec{e}_1, \quad [\vec{e}_3, \vec{e}_1] = \vec{e}_2 \qquad (1)$$

In 4D case we can introduce the vector product in similar way as in 3D case:

$$[\vec{u}, \vec{v}, \vec{w}] = \begin{vmatrix} \vec{e}_1 & \vec{e}_2 & \vec{e}_3 & \vec{e}_4 \\ u_1 & u_2 & u_3 & u_4 \\ v_1 & v_2 & v_3 & v_4 \\ w_1 & w_2 & w_3 & w_4 \end{vmatrix}$$

Here the ordered quadruplet of base vectors $(\vec{e}_1, \vec{e}_2, \vec{e}_3, \vec{e}_4)$ is right-oriented if the relations below are fulfilled:

$$\begin{aligned} &[\vec{e}_1, \vec{e}_3, \vec{e}_2] = \vec{e}_4, \quad [\vec{e}_1, \vec{e}_2, \vec{e}_4] = \vec{e}_3, \\ &[\vec{e}_1, \vec{e}_4, \vec{e}_3] = \vec{e}_2, \quad [\vec{e}_2, \vec{e}_3, \vec{e}_4] = \vec{e}_1. \end{aligned} \qquad (2)$$

Here $\vec{e}_i = (e_{i1}, e_{i2}, \dots e_{in}), \; e_{ii} = 1, \; e_{ij} = 0, \; i \neq j, .$

Following example demonstrates the consistent orientation of volumes:

Let the 4D simplex is defined on the vertices 0=(0,0,0,0), 1=(1,0,0,0), 2=(0,1,0,0), 3=(0,0,1,0), 4=(0,0,0,1). Orientation of the triplets of the base vectors 1-0, 2-0, 3-0, 4-0 in (2) is illustrated as oriented triangles in Fig.1. For oriented tetrahedron sim(a,b,c,d) defined on triplet of base vectors b-a,c-a, d-a, the complementary base vector e-a defines 'the external normal vector' (arrows in Fig.1).

According to (2) we obtain right-oriented 3D sub-simplexes sim(0,1,3,2), sim(0,1,2,4), sim(0,1,3,4), sim(0,2,3,4) with external normal vectors 4-0, 3-0, 2-0, 1-0.

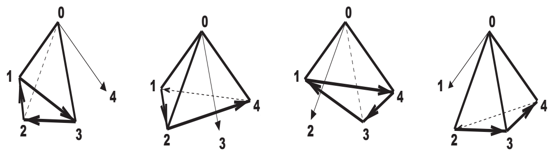

Figure 1. Right-oriented sub-simplexes.

4. VISIBILITY OF 4D BODIES

Let's consider 4D cube. Its 3D projections

$$C_{i,0} = \left\{ (x_1, x_2, x_3, x_4) \in \langle 0,1 \rangle^4 : x_i = 0 \right\},$$
$$C_{i,1} = \left\{ (x_1, x_2, x_3, x_4) \in \langle 0,1 \rangle^4 : x_i = 1 \right\}.$$

are in the Fig.2. Orientation of these 3D projections is the same as in the Fig.1. E.g. we can see that $C_{4,0}$ is in the subspace $\vec{e}_1, \vec{e}_2, \vec{e}_3$ so the sim(0,1,3,2) defines it's orientation.

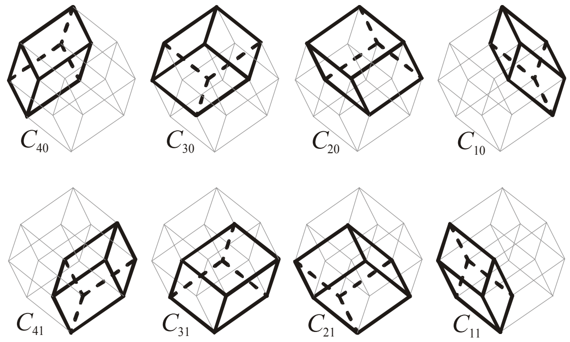

Figure2. Oriented 3D projections of 4D cube.

Let us choose the view vector so that $C_{4,0}$, $C_{2,0}$ are visible and $C_{3,0}$, $C_{1,0}$ are invisible. It can be proved that the orientation of $C_{i,0}$,$C_{i,1}$ are mutually opposite.

BVCA criterion connects the visibility of the body with the visibility of its *(n*-1*)*D projections. But we cannot see the whole 3D visible volumes − we can see only some of their faces. Moreover, there exists a 3D-visible volume of 4D-cube, which doesn't take place in resultant 2D-visible projection. So, we cannot use directly BVCA criterion when 4D-body is visualized.

We shall use next important construction – contour.

2D contour of 3D body consists of edges, which are the intersections of visible and invisible faces.

In similar way we can introduce the 3D-contour of 4D-body as a set of faces, which are the intersections of 3D-visible and 3D-invisible projections. The resultant criterion can be formulated:

face is visible <=> face is intersection of visible and invisible 3D-projections of body.

In the most general case the 3D-contour of the 4D-cube obtains 12 contour faces ($C_{i,0} \cap C_{i,1} = \emptyset$). So, the most general resultant projection of the 4D-cube is dodecahedron – see Fig. 3.

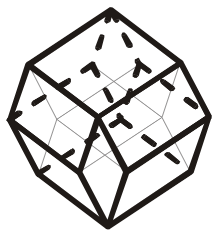

Figure 3. The most general resultant 3D representation of 4D cube.

It similar way we can formulate the visibility criterion in *n*-dimensional case:

2D-face is visible <=> face is intersection of visible and invisible (n-1)-D projections of body.

5. CONCLUSIONS

Representation of 4D cube in "usual habit" reduces original body very significantly, in similar way as the substitution of the 3D cube with its hexagonal contour.

Visibility criterion, which connects visibility of boundary faces of the body with the visibility of its n-1 dimensional projection, is introduced.

6. ACKNOWLEDGMENTS

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7. REFERENCES

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