

Computer Graphics Of Kinematic Surfaces

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ABSTRACT

This work deals with some problems of constructing and graphical displaying for a class of kinematic surfaces generated by mutual moving ruled surfaces touching one another along their common ruling in each position [1,2]. Exactly, we are dealing with the surfaces generated by movement of a cone fixed ruling during combined movement of the cone with respect to a torse surface [2]. The movement is defined as a cone rotating with simultaneous movement of the cone vortex along the torse edge-of-regression so that both surfaces, the torse and the cone, touch one another along the common ruling [3-5]. The methods and program modules developed allow using a wide range of curves as a torse edge-of-regression and a cone guiding line. Since the program modules are implemented using some numerical methods the application of a new-type curve is reduced to define the curve within the program module. This computer implementation includes cylindrical circular and elliptical helixes, and lines based on circular and elliptical helixes as the edge-of-regression, and circle and ellipse curves as the cone guiding line. Surfaces obtained are displayed on a screen in a 'frame' (Fig. 1) or 'solid' (Fig. 2) style in any user-selected projection.

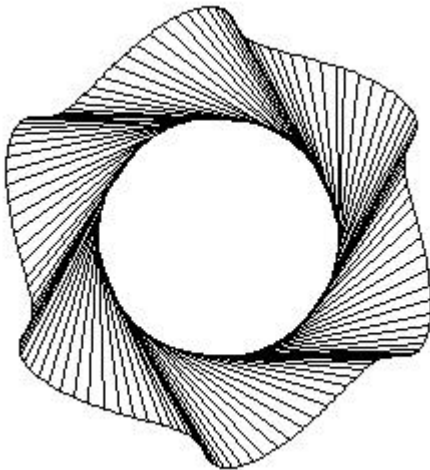


Figure 1. Frame style (basic model)



Figure 2. Solid style (basic model)

Keywords

Computer graphics, mathematical model, kinematic surface, ruled surface, cone, torse, Visual Basic.

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1. MATHEMATICAL MODEL AND METHODS OF SOLUTION

A torse edge-of-regression curve, s , is defined in the main fixed coordinates by the following parametric equations:

$$x = x_s(t), \quad y = y_s(t), \quad z = z_s(t).$$

A cone is defined by the parametric equations of a flat closed curve (the cone guiding line):

$$x = x_k(\mathbf{j}), \quad y = y_k(\mathbf{j}), \quad z = h \text{ and its vortex in the coordinate origin.}$$

It should be noted that the cone might be either the right one or the oblique one.

To define the movement process it is necessary to define a range, $[t_1, t_2]$, for the torse edge-of-regression parameter, t , and an initial value \mathbf{j}_0 and a value \mathbf{j}_C which defines position of a fixed cone ruling, defining the kinematic surface of interest, for the cone parameter, \mathbf{j} .

Based on the definition of movement, its main criterion is keeping equal values of total angles on the torse surface and on the cone involute between the corresponding common rulings for a current edge-of-regression point. The angle on torse surface is calculated by integration of a curvature function of torse edge-of-regression.

The problem for a cone is solved by incorporation of a spherical guiding line, generally, a spatial curve generated by intersection of the cone and the sphere having the center in the cone vortex and the radius, l .

The equation of the spherical guiding line:

$$x = x_{kl}(\mathbf{j}) = \frac{lx_k(\mathbf{j})}{\sqrt{x_k^2(\mathbf{j}) + y_k^2(\mathbf{j}) + h^2}}$$

$$y = y_{kl}(\mathbf{j}) = \frac{ly_k(\mathbf{j})}{\sqrt{x_k^2(\mathbf{j}) + y_k^2(\mathbf{j}) + h^2}}$$

$$z = z_{kl}(\mathbf{j}) = \frac{lh}{\sqrt{x_k^2(\mathbf{j}) + y_k^2(\mathbf{j}) + h^2}}$$

Therefore, we have a circular sector with radius l on the cone involute and we need only to calculate the arc length of the relevant section of the spherical guiding line. Now, the main criterion of movement is defined by the following (1):

$$\int_{t_1}^{t_2} k(t) \sqrt{(x_s')^2 + (y_s')^2 + (z_s')^2} dt = \frac{1}{l} \int_{\mathbf{j}_1}^{\mathbf{j}_2} \sqrt{(x_{kl}')^2 + (y_{kl}')^2 + (z_{kl}')^2} d\mathbf{j}$$

where $k(t)$ is a function of curvature [6] of the torse edge-of-regression in a current point $M(x_M, y_M, z_M)$: $x_M = x_s(t)$; $y_M = y_s(t)$; $z_M = z_s(t)$.

Solving the Eq. (1) for \mathbf{j} for a current point M on the torse edge-of-regression s (Fig. 3) allows to determine a point $P(x_P, y_P, z_P)$ on the cone spherical guiding line (Fig. 4) which defines a position of a cone ruling touching with the torse surface. Then, a point $C(x_C, y_C, z_C)$, the curvature center of the spherical guiding line in a point P , and the angle \mathbf{b} between the curvature axis, OC , and the cone guiding line, OP (Fig. 4), are calculated. This is valid for the spherical guiding line because the curvature axis for the point P actually passes through the cone vortex and coincides with the cone instantaneous rotation axis for its ruling OP .

It is convenient to take the point $N(x_N, y_N, z_N)$:

$$x_N = x_{kl}(\mathbf{j}_0), \quad y_N = y_{kl}(\mathbf{j}_0), \quad z_N = z_{kl}(\mathbf{j}_0)$$

on the spherical guiding line:

$$x_N = x_{kl}(\mathbf{j}_0), \quad y_N = y_{kl}(\mathbf{j}_0), \quad z_N = z_{kl}(\mathbf{j}_0)$$

as a fixed point on the cone (for $\varphi = \varphi_0$).

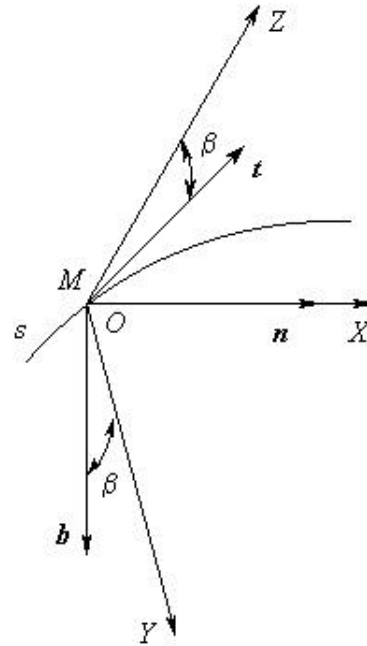


Figure 3. Movable coordinates (OXYZ)

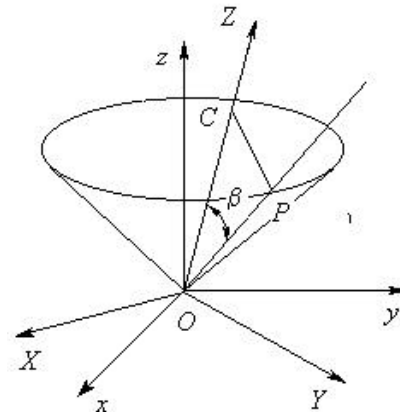


Figure 4. Directions of the movable coordinates

It is clear that using the line segment ON for displaying of the kinematic surface automatically provides for displaying in the form of the spatial band with fixed width l .

It remains just to add a movable coordinates related to the current point of the torse edge-of-regression, to place the cone within it so that the current cone position corresponds its movement conditions and to transform into the coordinates of the point N to the base coordinates $oxyz$ (in which the equation of the curve of torse edge-of-regression is defined).

The movable coordinates (OXYZ) is defined on the base of directing vectors of edges of the accompanying trihedron rotated around the main normal for an angle \mathbf{b} (Fig. 3).

The direction cosines for the coordinates $OXYZ$ are following:

$$\cos(X, x) = n_x;$$

$$\cos(X, y) = n_y;$$

$$\cos(X, z) = n_z;$$

$$\cos(Y, x) = b_x \cos \mathbf{b} - t_x \sin \mathbf{b};$$

$$\cos(Y, y) = b_y \cos \mathbf{b} - t_y \sin \mathbf{b};$$

$$\cos(Y, z) = b_z \cos \mathbf{b} - t_z \sin \mathbf{b};$$

$$\cos(Z, x) = t_x \cos \mathbf{b} + b_x \sin \mathbf{b};$$

$$\cos(Z, y) = t_y \cos \mathbf{b} + b_y \sin \mathbf{b};$$

$$\cos(Z, z) = t_z \cos \mathbf{b} + b_z \sin \mathbf{b},$$

where $t_x, t_y, t_z, n_x, n_y, n_z, b_x, b_y, b_z$ are direction cosines for the tangent (t), the main normal (n) and the binormal (b), respectively.

To connect the movable coordinates $OXYZ$ with the base coordinates of the cone they are directed as follows:

the OZ axis along the curvature axis vector OC ;

the OY axis along the curvature radius vector CP (see Fig. 4).

Therefore, the direction cosines of the $OXYZ$ axes in the coordinates of the cone are the following:

$$\cos(Z, x) = \frac{x_C}{l \cos \mathbf{b}}$$

$$\cos(Z, y) = \frac{y_C}{l \cos \mathbf{b}}$$

$$\cos(Z, z) = \frac{z_C}{l \cos \mathbf{b}}$$

$$\cos(Y, x) = \frac{x_C - x_P}{l \sin \mathbf{b}}$$

$$\cos(Y, y) = \frac{y_C - y_P}{l \sin \mathbf{b}}$$

$$\cos(Y, z) = \frac{z_C - z_P}{l \sin \mathbf{b}}$$

$\cos(X, x), \cos(X, y), \cos(X, z)$ – they are defined under requirements of the $OXYZ$ is a right orthogonal coordinates.

The $OXYZ$ origin coincides with the origin of the coordinates of the cone.

So, to determine the coordinates of the point N in the base coordinates it is necessary to transform them from the coordinates of the cone into the movable coordinates $OXYZ$ and then to transform the results into the base coordinates $oxyz$. The line segment MN depicts a ruled kinematic surface during movement.

In addition to discussed classical version, on the base of the above method some kinematic surfaces were generated by movement of a “crest”-curve situated in the plane containing the fixed cone ruling, with this

plane being perpendicular to the cone surface. Sinusoid, circle, lemniscate of Bernoulli as well as curves generated from them by compression or stretching were used as “crest”-curves.

2. COMPUTER APPLICATION

Using the Visual Basic 6.0 an application was developed for construction and graphical representation of kinematic surfaces of interest.

The application interface include more than 20 controls for selection and adjusting different settings of calculations, styles and color palette of displaying. Among the interface features are the following:

- Selection of a torse edge-of-regression from the available list as well as its parameters, for example, sizes of ellipse semi-axes and a lead of helix for the elliptical helix;

- Adjustment of cone parameters (ellipse semi-axes and a cone height for the elliptical cone);

- Selection of a form and parameters of the “crest”;

- Selection a range of the parameter t for functions of the torse edge-of-regression as well as initial and fixed point positions on the cone surface;

- Selection of a frequency of frame guiding lines for graphical representation of a kinematic surface;

- Adjustment of a length of the cone fixed ruling, i. e. a width of graphic display band;

- Selection of a position and scale of screen display;

- Selection of foreground and background colors.

The most part of the interface features is implemented using scroll-bar control, which allows a user to change settings in a continuous way, and a new image appears either automatically or by user’s command depending on the corresponding flag setting.

As mentioned above, two styles, “frame” and “solid” are available for displaying kinematic surfaces.

In the “frame” style, a surface is displayed as a series of rulings.

Taking into account a fan-like arrangement of frame rulings for surfaces of interest a step of their output is defined by a fixed angle increment. This allows obtaining uniform covering of the frame by rulings including the case of edge-of-regression curves or cone guiding lines with variable curvature.

Graphics in the “solid” style is constructed point by point. To obtain proper displaying overlapped surface parts the graphics output to the screen is performed by layers using a matrix which size is determined in dynamical way after preliminary constructing of the surface in the “frame” style.

It is warranted by the following facts: firstly, time to construct graphics in the “frame” style is much less than in the “solid” style; secondly, preliminary calculations in the “frame” style provides not only required sizes of a matrixes, but index boundaries,

which as can be readily appreciated corresponds to coordinates of displayed surface points.

Correctness and quality of graphical display depend on increment values of function variables used in calculations of integral sums and determining a frequency of output of rulings of a cinematic surface to the screen, a step of graphical representation of the ruling itself which may be a curve (in the case of a “crest”), and certain parameters of a screen display.

In addition, application provides for constructing and graphical displaying of kinematic surfaces for a case cone rotating in reverse direction by comparison with basic model (Fig. 2).

Surfaces in the “solid” style are displayed in a user-selected color using variable color brightness to create a 3D effect.

An image can be done more attractive by using angular or radial alternation of colors (Fig. 5, 6); a user with the interface selects a color bandwidth and colors themselves.

Obviously, any surface representation on the screen is a representation of a surface point projection onto a certain plane.

The application includes a dedicated control “trio” to allow a user to select any projection plane.

This control has a window form (see Fig. 7) and provides the following possibilities:

- Adjustment of coordinate axes by the “drag-and-drop” method and a surface image can be redrawn during mouse movement (for the “frame” style);
- Rotation of the coordinates around any axis for a user selected angle.

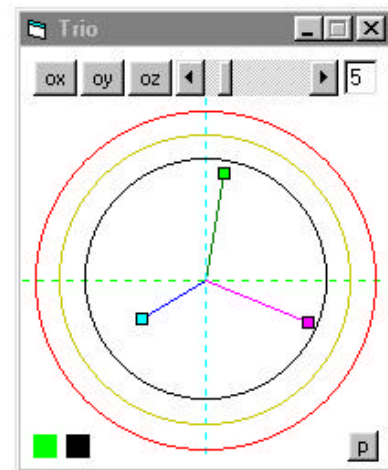


Figure 7. “Trio” control



Figure 5. Angular alternation of colors

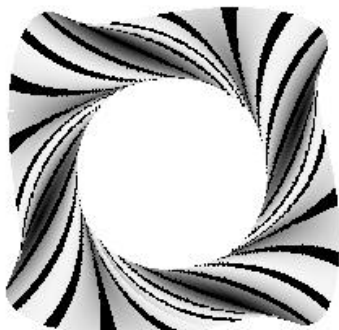


Figure 6. Radial alternation of colors

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